ODTK
A Technical Summary*

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Contents

1 Introduction **3**
1.1 Optional States . . . 3
1.2 Optional Force Models . . . 4
1.3 Optional Measurement Models . . . 5
1.4 Measurement Model Options . . . 5

I Orbit Determination Science **6**

2 Methods of Orbit Determination **6**
2.1 Initial Orbit Determination . . . 6
2.2 Least Squares Differential Corrections . . . 6
2.3 Sequential Processing . . . 6
2.4 ODTK . . . 7

3 Optimal Orbit Determination **7**
3.1 Mathematical Operators for SP Methods . . . 7
3.1.1 Subscript Notation . . . 7
3.1.2 Nonlinear Operators . . . 8
3.1.3 Linear Operators . . . 8
3.2 Definitions . . . 8
3.2.1 State Estimate Reference for Linearization of SP Methods . . . 8
3.2.2 Local Linearization . . . 9
3.2.3 Global Linearization . . . 9
3.2.4 Observability . . . 9
3.2.5 Completeness . . . 9
3.2.6 Optimal Orbit Determination . . . 9
3.3 Discussion . . . 10
3.3.1 Sherman's Theorem . . . 10
3.3.2 Complete State Estimate . . . 11
3.3.3 Local Linearization . . . 11

4 Measurements **11**
4.1 Ground Station . . . 11
4.2 Space Based . . . 11
4.3 TDRSS . . . 12
4.4 GPS . . . 12

5 Optimal Sequential Filter 12
5.1 State Estimate Error Model 12
5.2 Integral Equation 12
5.3 State Error Covariance for Filter Time Update 12
5.4 Solutions for the Optimal State Estimate Error Model 13
5.4.1 Gravity Solution 13
5.4.2 Air-Drag Solution 13
5.4.3 Solar Pressure Solution 14
5.5 Filter Time Update Algorithm 14
5.6 Filter Measurement Update Algorithm 14
5.7 Filter Measurement Update and Filter Time Update 15
5.8 Measurement Editing 15
5.8.1 Kalman Measurement Editor Limitation 15
5.8.2 Filter Initialization Problem 15
5.8.3 The Dynamic Editor 15
5.9 Filter Initialization 15
5.10 Filter Divergence 16
5.10.1 Kalman Measurement Editor 16
5.10.2 Dynamic Editor 16
5.10.3 Unmodeled Thrust Maneuvers 16

6 Fixed Interval Sequential Smoother 16
6.1 Smoother Initialization 16
6.2 Notation for Smoother Nonlinear State Transition 16
6.3 Smoother Sequential Equations 17
6.3.1 Transition Smoothed State Estimate Backwards 17
6.3.2 Incorporate Filter Estimate and Covariance at Time t_k 17
6.3.3 Prepare to Calculate Smoother Covariance 17
6.3.4 Smoother Covariance 17
6.4 Filter-Smoother Consistency Test 17
6.4.1 Test 17

7 Least Squares 18

II Computer Science 18

8 Software Architecture 18

9 AGI Technology Platform (ATP) 19

10 ODTK Engine 20

11 ODTK GUI 20

12 Summary 20

1 Introduction

ODTK is an optimal orbit determination capability, where optimality is defined in the technical discussion in Section 3 below. ODTK is an integrated set of software modules, using the same astrodynamics software library and integrators as STK, using modern C++ programming techniques, designed to support real-time satellite operations, non-real-time post-mission analysis, and design and trade studies. The software architecture is described in Part II, below.

The main modules in ODTK are a Filter, a Smoother, and a Simulator. The Filter is an optimal sequential filter, as described below. The Filter is a particular nonlinear extension to the linear Kalman Filter[4], and is significantly different in the way that process noise is set by the user and implemented in the Time Update. The ODTK approach improves stability, improves covariance realism, and provides the user with physically intuitive process noise controls. The ODTK Smoother is a nonlinear extension to the Meditch[5] linear fixed-interval smoother.

The Simulator can deviate initial orbit conditions, force model coefficients, facility and transponder biases, clock behavior, and tracking facility locations, all under analyst controls.

1.1 Optional States

The following list of estimable states reflects those quantities that ODTK can deviate during simulation and estimate during filter and smoother operations. Any number of states from any number of satellites and any number of trackers can be simulated and/or estimated simultaneously with ODTK.

- Satellite
  - Position and velocity
  - Drag coefficient
  - Local atmospheric density
  - Solar radiation pressure coefficient(s)
    - One coefficient for most satellites
    - Two coefficients for GPS Satellites
- Measurement biases
  - Time-varying time-correlated bias on each measurement type
  - Time-varying time-correlated bias for each transponder including
    - Relay satellite transponder
    - SGLS-type transponder
    - TDRS-type and BRTS-type transponder
    - TDOA / FDOA-type transponder
    - GEO transponder for GPS signal
  - Facility location
  - Troposphere scale correction
- Space based GPS receiver
  - Clock phase, frequency, and (optional) aging
  - Antenna location in body frame
  - Center-of-mass location in body frame
- GPS Satellite transmitter
  - Clock phase, frequency and (optional) aging
- Finite maneuver magnitude and/or direction

1.2 Optional Force Models

The following force models are provided, each implementation conforms to international standards, as defined in IERS Conventions and other comparable sources.

- Gravitational perturbations
  - Several Earth models, which the user can supplement, including:
    - EGM96
    - GEM-series models
    - GGM01C
    - GGM02C
  - Moon, Sun, and all planets (using the barycenter of respective planet – moon systems)
  - Solid earth tide perturbations
  - Ocean tide perturbations
  - Relativistic accelerations
- Atmospheric drag perturbations
  - Spherical body model
  - Plug-in point for user-provided complex force model
  - Several atmospheric density models including
    - CIRA 1972
    - MSIS 1990
    - NRLMSIS 2000
    - Jacchia 1971
    - Jacchia-Roberts
- Photon pressure perturbations
  - Spherical body model for solar photon pressure
  - Spherical body model for earth albedo pressure
  - GPS-specific block-specific models for GPS satellites
  - Plug-in point for user-provided complex force model
  - Eclipse model for earth shadow and lunar shadow
  - User selectable file for Earth albedo model coefficients
- Thrust perturbations
  - Impulsive delta-V
  - Finite maneuvers (constant thrust or complex acceleration profiles)
  - Grouped thrust events to reflect repeated use of the same thrusters

1.3 Optional Measurement Models

The following measurement types represent capabilities of ODTK. Format is not an issue, since a measurement provider plug-in point is provided which will accommodate any format. All measurement types that can be filtered can also be simulated.

- Simple ground based tracking
  - Range, azimuth, elevation, Doppler
  - Right ascension and declination
- Multiply relayed ground tracking
  - Bistatic ranging
  - TDOA & FDOA, singly differenced TDOA & FDOA, TDOA dot
  - TDRS 4-legged round-trip range, 5-legged Doppler, Return-link Doppler
  - GPS 2-legged pseudo-range (GPS to GEO to ground)
- GPS receiver tracking of GPS signal
  - C/A pseudo-range & L/A phase (also called ADR)
  - L1 & L1 Phase
  - P1 & P2 pseudo-range
  - Dual frequency corrected pseudo-range and/or phase
  - Single-differenced pseudo-range and phase
  - Double-differenced pseudo-range and phase
- Space based tracking
  - Range, azimuth, and elevation
  - Right ascension & declination
  - Space based TDOA & FDOA

1.4 Measurement Model Options

Various high fidelity models are employed to support high accuracy programs.

- Ground-based station motion models
  - Solid earth tides
  - Polar tide
  - Ocean loading
  - Tectonic plate drift
- Ground based physical models
  - Tropospheric refraction models
    - SCF model
    - Marini-Murray (for laser ranging)
    - Saastamoinen with Niell hydrostatic mapping function
- Satellite body motion models

  - Antenna location defined in body coordinates
  - Body orientation defined by
    * Attitude rules
    * Attitude profile (quaternions)
    * Attitude model (GPS satellites)

Part I
Orbit Determination Science

2 Methods of Orbit Determination

Orbit determination methods are sharply partitioned by three classes: Initial Orbit Determination (IOD), Least Squares (LS), and Sequential Processing (SP). Operationally, the order in which these methods are used defines a dependency tree: IOD output is LS input, and LS output is SP input:

$$\text{IOD} \Longrightarrow \text{LS} \Longrightarrow \text{SP}$$

2.1 Initial Orbit Determination

IOD methods input tracking measurements with tracking platform locations, and output spacecraft position and velocity estimates. *No* a priori orbit estimate is required. Associated output orbit estimation error magnitudes are large. IOD methods are nonlinear methods, and are relatively trivial to implement. Measurement editing is typically not performed during IOD calculations. Operationally, the orbit determination process is frequently begun, or restarted, with IOD. IOD methods were derived by various authors: LaPlace, Poincaré, Gauss, Lagrange, Lambert, Gibbs, Herrick, Williams, Stumpp, Lancaster, Blanchard, Gooding, and Smith. ODTK provides two methods to solve initial orbit determination problems: The Herrick-Gibbs method for range and angles measurements, and the Gooding method for angles-only.

2.2 Least Squares Differential Corrections

LS methods input tracking measurements with tracking platform locations and an a priori orbit estimate, and output a refined orbit estimate. An a priori orbit estimate *is* required. Associated output error magnitudes are small when compared to IOD outputs. LS methods consist of a sequence of linear LS corrections where sequence convergence is defined as a function of tracking measurement residual RMS (Root Mean Square). Each linear LS correction is characterized by a minimization of the sum of squares of tracking measurement residuals. LS methods produce refined orbit estimates in a batch mode, together with error covariance matrices that are optimistic; i.e., orbit element error variances are typically too small by at least an order of magnitude. Operationally, LS may be the only method used, or it may be used to initialize SP. LS methods frequently require inspection and manual measurement editing by human intervention. LS algorithms therefore require elaborate software mechanisms for measurement editing. The LS method was derived first by Gauss[1] in 1795, and then independently by Legendre. ODTK provides the accurate QR factorization and triangularization method with orthogonal Householder transformations to solve the LS equation.

2.3 Sequential Processing

SP methods input tracking measurements with tracking platform locations, input an a priori state estimate (inclusive of orbit estimate), and input an a priori state error covariance matrix. An a priori state estimate *is* required, and an a priori state error covariance matrix *is* required. SP methods output refined state estimates in a sequential mode. SP filter methods are forward-time recursive sequential machines consisting of a repeating pattern of *filter time update* of the state estimate and *filter measurement update* of the state estimate. The *filter time update* propagates the state estimate forward, and the *filter measurement update* incorporates the next measurement. The recursive pattern includes an important interval of filter initialization. SP smoother methods are backward-time recursive sequential machines consisting of a repeating pattern of state estimate refinement using filter outputs and backwards transition. Time transitions for both filter and smoother are dominated most significantly by numerical orbit propagators. The search for sequential processing was begun by Wiener, Kalman[4], Bucy[7], and others. ODTK provides a unique sequential filter-smoother to solve the optimal orbit determination problem.

2.4 ODTK

ODTK provides methods for IOD, LS, and SP. In particular, IOD and LS are used to initialize SP.

3 Optimal Orbit Determination

Orbit determination refers to the estimation of orbits of spacecraft (or natural satellites or binary stars) relative to primary celestial bodies, given applicable measurements. All useful orbit determination methods produce orbit estimates, and all orbit estimates have errors. But what is *optimal* orbit determination?

By itself, the adjective *optimal* refers [44] to most desirable, most favorable, or most satisfactory. But most satisfactory to whom? There are choices to make from available orbit determination methods. The fastest methods are the least accurate. Should we prefer sequential methods to batch methods? Should we minimize the size of measurement residuals or the size of orbit errors? How should we model measurement residuals and orbit errors?

All orbit determination problems are multidimensional and nonlinear. Should we attempt a multidimensional nonlinear solution directly? Or should we use a linearization method? If so, is there a preferred method for linearization?

The purpose of this section is to answer the question: What is *optimal* orbit determination? A definition requires the use of mathematical operators for sequential processing (SP) methods.

3.1 Mathematical Operators for SP Methods

3.1.1 Subscript Notation

State Matrices The state estimate $\hat{X}$ is referred to two times with the notation[5]:

$$\hat{X}_{j|i} \equiv \hat{X}\left(t_j|t_i\right) \tag{1}$$

where i, $j \in \{0, 1, 2, \ldots\}$. The time t_j to the left of the vertical bar denotes the epoch for $\hat{X}$, and is driven by the filter time update function. The time t_i to the right of the bar denotes the time-tag of the last measurement processed to form $\hat{X}$, and is driven by the filter measurement update function. Examples: $\hat{X}_{7|6}$ refers to the state estimate at time t_7, given the last measurement processed at time t_6, whereas $\hat{X}_{7|7}$ refers to the state estimate at time t_7, given the last measurement processed at time t_7. Evidently, $\hat{X}_{7|6}$ was obtained by filter time update of $\hat{X}_{6|6}$ from t_6 to t_7.

Similar notation is used for the state estimate correction:

$$\Delta\hat{X}_{j|i} \equiv \Delta\hat{X}\left(t_j|t_i\right) \tag{2}$$

and the state estimate error covariance matrix:

$$P_{j|i} \equiv P\left(t_j|t_i\right) \tag{3}$$

Measurement Matrices Denote a measurement at time t_j with y_j, and denote a measurement estimate (representation) at time t_j with $\hat{y}_{j|h}$. The subscript h denotes the time t_h of last measurement incorporated by the filter.

3.1.2 Nonlinear Operators

Nonlinear operators are required in the state estimate time update and the state estimate measurement update for SP methods of orbit determination.

State Propagation Let φ denote a nonlinear operator that propagates the state estimate $\hat{X}_{i|h}$ from time t_i to time t_j :

$$\hat{X}_{j|h} = \varphi\left\{t_j; \hat{X}_{i|h}, t_i\right\} \tag{4}$$

Measurement Representation Let $y\left(\cdot\right)$ denote a nonlinear operator that calculates the measurement representation $\hat{y}_{j|h}$, given the state estimate $\hat{X}_{j|h}$:

$$\hat{y}_{j|h} = y\left(\hat{X}_{j|h}, t_j\right) \tag{5}$$

3.1.3 Linear Operators

State Estimate Error Propagation Let $\Phi_{j,i} \equiv \Phi\left(t_j, t_i\right)$ denote the linear operator that propagates the state error estimate $\Delta\hat{X}_{i|h}$ from time t_i to time t_j

$$\Delta\hat{X}_{j|h} = \Phi_{j,i}\Delta\hat{X}_{i|h} \tag{6}$$

where

$$\Phi_{j,i} = \left[\frac{\partial X_j}{\partial X_i}\right]_{\hat{X}_{j|h}} \tag{7}$$

and where evaluation derives from $\hat{X}_{j|h}$.

Measurement Residual Let Δy_j denote the linear operator that defines the measurement residual at time t_j:

$$\Delta y_j = y_j - \hat{y}_{j|h} \tag{8}$$

Measurement/State Partials Jacobian Let H_j denote the jacobian of measurement to state partial derivatives at time t_j:

$$H_j = \left[\frac{\partial y_j}{\partial X_j}\right]_{\hat{X}_{j|h}} \tag{9}$$

where evaluation derives from $\hat{X}_{j|h}$.

3.2 Definitions

3.2.1 State Estimate Reference for Linearization of SP Methods

Evaluation of the measurement representation $\hat{y}_j$ defined by Eq. 5 requires the use of some a priori state estimate reference $\hat{X}_{j|h}$, where $t_j \geq t_h$. A similar requirement is associated with Eqs. 7 and 9. $\hat{X}_{j|h}$ is called the state estimate *reference* for linearization.

3.2.2 Local Linearization

Let t_k and $t_{k+1} \geq t_k$ be the time tags of adjacent ordered measurements y_k and y_{k+1}, for $k \in \{0, 1, 2, \ldots\}$. That is, there exist no measurements between y_k and y_{k+1}. Then the use of $\hat{X}_{k+1|k}$ as the state estimate *reference* for all linearizations at time t_{k+1} defines *local* linearization at time t_{k+1}.

The use of any state estimate reference other than $\hat{X}_{k+1|k}$ at time t_{k+1} for linearization is a non-local linearization at time t_{k+1}.

3.2.3 Global Linearization

Given the integer variable $k \in \{0, 1, 2, \ldots\}$ and any fixed non-negative integer j, then the use of $\hat{X}_{k|j}$ as the state estimate *reference* for linearization at time t_k for each k is *global* linearization.

3.2.4 Observability

A particular parameter is *observable* to a particular measurement if and only if the sequential processing of that measurement reduces the estimate error variance on that parameter.

3.2.5 Completeness

The state estimate structure is *complete* if and only if all parameters, that are both unknown and observable, are contained in the state estimate structure.

3.2.6 Optimal Orbit Determination

By optimal orbit determination [24], we mean that the method used to calculate the state estimate (containing the orbit estimate) satisfies the following eight conditions:

1. Sequential processing (SP) is used to account for force modeling errors and measurement information in the time order in which they are realized.

2. Sherman's Theorem (summary) [2],[3],[5],[4]: The optimal state estimate correction matrix $\Delta\hat{X}$ is the expectation of the state error matrix ΔX given the measurement residual matrix Δy. That is: $\Delta\hat{X} = E\left\{\Delta X|\Delta y\right\}$.

3. Linearizations of state estimate time transition and state to measurement representations are *local* in time, *not global*.

4. The state estimate structure is *complete*

5. All state estimate models and state estimate error model approximations are derived from appropriate force modeling *physics,* and measurement sensor performance

6. All measurement models and measurement error model approximations are derived from appropriate sensor hardware definition and associated physics, and measurement sensor performance

7. Necessary conditions for real data:

   - Measurement residuals approximate Gaussian white noise [5],[8]
   - McReynold's [29],[24] filter-smoother consistency test is satisfied with probability 0.99

8. Sufficient conditions for simulated data: The state estimate errors agree with the state estimate error covariance function.

The first six conditions define standards for optimal algorithm design, and the creation of a *realistic* state estimate error covariance function. The last two conditions enable validation: They define realizable test criteria for optimality.

3.3 Discussion

3.3.1 Sherman's Theorem

Sherman's Theorem [5] is applicable to a *linear* state estimate; i.e., a condition where the state estimate is a linear combination of available measurements. But in all orbit determination problems, the orbit (substate) estimate is a *nonlinear* function of available measurements. We must linearize in order to use Sherman's Theorem.

Optimal orbit determination requires measurement linearization about the local state estimate $\hat{X}_{k+1|k}$ to produce a local measurement *residual*:

$$\Delta y_{k+1} = y_{k+1} - y\left(\hat{X}_{k+1|k}\right)$$

and linearization about the same local state estimate $\hat{X}_{k+1|k}$ to produce a local state *error* estimate $\Delta\hat{X}_{k+1|k+1}$, given Δy_{k+1}. Local linearization enables a linear relation between each state error estimate $\Delta\hat{X}_{k+1|k+1}$ and each measurement residual Δy_{k+1}. With *local* linearization, one applies Sherman's Theorem anew to each scalar measurement residual, never simultaneously to a batch of measurement residuals.

Let $\Delta X_{k+1|k} = X_{k+1|k} - \hat{X}_{k+1|k}$ define the error in state estimate $\hat{X}_{k+1|k}$, and let $\delta X_{k+1|k} = \Delta X_{k+1|k} - \Delta\hat{X}_{k+1|k}$ define the error in $\Delta\hat{X}_{k+1|k}$. Ideally $\delta X_{k+1|k} = 0$, and the state error estimate $\Delta\hat{X}_{k+1|k}$ is perfect. When $\delta X_{k+1|k} \neq 0$, assign a penalty (or loss) function $L = L\left(\delta X_{k+1|k}\right)$ with four admissibility requirements:

- L is a scalar-valued function of the N state estimate variables.
- $L(0) = 0$, where the first 0 is an $N \times 1$ matrix of zeros. No loss is assigned when the state error estimate is perfect.
- $L\left(\delta X^a_{k+1|k}\right) \geq L\left(\delta X^b_{k+1|k}\right)$ whenever $\rho\left(\delta X^a_{k+1|k}\right) \geq \rho\left(\delta X^b_{k+1|k}\right)$, where ρ is a scalar-valued, non-negative, convex function of N variables. Thus L is defined to be a non-decreasing function of distance ρ from the origin. The closer $\delta X_{k+1|k}$ is to zero, the smaller the loss.
- $L\left(\delta X_{k+1|k}\right) = L\left(-\delta X_{k+1|k}\right)$. That is, $L(\cdot)$ is symmetric about the origin.

Performance $J\left[\delta X_{k+1|k}\right]$ is defined as the expectation of the loss; i.e., as the mean value of loss. Our goal is to minimize $J\left[\delta X_{k+1|k}\right]$:

$$J\left[\delta X_{k+1|k}\right] = E\left\{L\left(\delta X_{k+1|k}\right)\right\} \tag{10}$$

Denote the conditional probability distribution function of $\Delta X_{k+1|k}$ given Δy_{k+1} by:

$$P\left\{\Delta X_{k+1|k} \leq \xi | \Delta y_{k+1}\right\} = F\left\{\xi | \Delta y_{k+1}\right\} \tag{11}$$

The reader is referred to Chapter 5.0, Section 5.2, of Meditch [5] for the following theorems.

Most General Form Given any admissible loss function $L\left(\delta X_{k+1|k}\right)$, and any conditional probability distribution function $F\left\{\xi | \Delta y_{k+1}\right\}$ such that $F\left\{\xi | \Delta y_{k+1}\right\}$ is:

- Symmetric about its mean $\bar{\xi}$
- Convex for all $\xi \leq \bar{\xi}$

then:

$$\Delta\hat{X}_{k+1|k+1} = E\left\{\Delta X_{k+1|k} | \Delta y_{k+1}\right\} \tag{12}$$

Application of the conditional mean $E\left\{\Delta X_{k+1|k} | \Delta y_{k+1}\right\}$ generates a global minimum to the performance function $J\left[\delta X_{k+1|k}\right]$. This is true for all combinations of admissible loss functions and symmetric and convex conditional probability distribution functions. Proof is due to Sherman[2][3].

Gaussian Distribution Given any admissible loss function $L\left(\delta X_{k+1|k}\right)$, and Gaussian random variables $\Delta X_{k+1|k}$ and Δy_{k+1}, then:

$$\Delta \hat{X}_{k+1|k+1} = E\left\{\Delta X_{k+1|k} | \Delta y_{k+1}\right\} \tag{13}$$

Application of the conditional mean $E\left\{\Delta X_{k+1|k} | \Delta y_{k+1}\right\}$ generates a global minimum to the performance function $J\left[\delta X_{k+1|k}\right]$, even for asymmetric loss functions. Proof is due to Doob[14].

Mean Square Error If $L\left(\delta X_{k+1|k}\right) = \left(\delta X_{k+1|k}\right)^T \left(\delta X_{k+1|k}\right)$, then:

$$\Delta \hat{X}_{k+1|k+1} = E\left\{\Delta X_{k+1|k} | \Delta y_{k+1}\right\} \tag{14}$$

The loss function $\left(\delta X_{k+1|k}\right)^T \left(\delta X_{k+1|k}\right)$ is referred to as the mean square state error. Minimization of the performance function $E\left\{\left(\delta X_{k+1|k}\right)^T \left(\delta X_{k+1|k}\right)\right\}$ results, in part, in the minimization of mean square orbit error. Application of the conditional mean $E\left\{\Delta X_{k+1|k} | \Delta y_{k+1}\right\}$ generates a global minimum to the performance function. In this case the conditional probability distribution function need not be either symmetric or convex. Proof is due to Doob[14].

3.3.2 Complete State Estimate

Consider any case where the state estimate structure is incomplete. The observable parameter neglected in the state estimate structure will *alias* into the estimated orbit elements, significantly degrading them. Thus one needs an appropriate place in the state estimate structure to put every observable effect.

3.3.3 Local Linearization

Local linearization has obvious benefits relating to the use of small local a priori estimation error magnitudes. Another benefit becomes obvious when one considers gravity resonance effects. Local linearization immediately transfers locally acquired orbit measurement information to the local state estimate for numerical integration by a rigorous nonlinear complete formulation of the equations of motion across all time intervals. Thus the new information relating to all significant resonance effects is rigorously accounted for by numerical integration of the local orbit estimate.

Global linearization puts the burden of modeling resonance effects, in part, into the stochastic process for the gravity error model. The construction of a global stochastic model for gravity errors referred to resonance is daunting – no one has even attempted it.

4 Measurements

4.1 Ground Station

- Two-way range
- Two-way Doppler (carrier phase count)
- Azimuth and elevation angles
- Right ascension and declination angles

4.2 Space Based

- Right ascension and declination angles
- Two-way range

4.3 TDRSS

- Two-way range with four legs
- Two-way Doppler with five legs
- One-way Doppler return link with three legs
- Two-way BRTS with four legs

4.4 GPS

- Single frequency C/A code pseudo-range
- Two frequency P-code pseudo-range
- Two frequency ionosphere removal
- USER clock effects removal by first differences
- Two frequency Doppler carrier phase count measurements

5 Optimal Sequential Filter

5.1 State Estimate Error Model

The optimal model equation for the state error is defined by the linear stochastic differential equation:

$$\frac{d}{dt}\delta X\left(t\right) = A\left(t\right)\delta X\left(t\right) + B\left(t\right)\delta u\left(t\right) \tag{15}$$

where $\delta X\left(t\right)$ is an n x 1 matrix, where $\delta u\left(t\right)$ is a 3 x 1 matrix-valued correlated Gaussian random error, where $A\left(t\right)$ is an n x n time dependent matrix, and where $B\left(t\right)$ is an n x 3 time dependent matrix. For sequential orbit determination, $\delta u\left(t\right)$ is always serially correlated (non-white) due to correlation in modeling errors for gravity[22] [23], air-drag, and solar pressure.

5.2 Integral Equation

Eq. 15 has an integral:

$$\delta X\left(t_{k+1}\right) = \Phi\left(t_{k+1}, t_k\right)\delta X\left(t_k\right) + \int_{t_k}^{t_{k+1}} \Phi\left(t_{k+1}, \tau\right) B\left(\tau\right)\delta u\left(\tau\right) d\tau \tag{16}$$

The right-hand side of this equation presents the sum of two nx1 matrix terms. The first term propagates initial condition state estimate errors $\delta X\left(t_k\right)$, and the second term accumulates and propagates acceleration modeling errors $\delta u\left(t\right)$ (state error process noise).

5.3 State Error Covariance for Filter Time Update

Form the outer product on $\delta X\left(t_{k+1|k}\right)$ using Eq. 16, and take its expectation to define:

$$P_{k+1|k} = E\left\{\delta X\left(t_{k+1|k}\right)\delta X\left(t_{k+1|k}\right)^T\right\} \tag{17}$$

where:

$$E\left\{\delta X\left(t_{k+1|k}\right)\delta X\left(t_{k+1|k}\right)^T\right\} = \Phi\left(t_{k+1}, t_k\right) E\left\{\delta X\left(t_{k|k}\right)\delta X\left(t_{k|k}\right)^T\right\}\Phi\left(t_{k+1}, t_k\right)^T +$$

$$I^C_{k+1,k} + I^L_{k+1,k} + I^R_{k+1,k} \tag{18}$$

and where:

$$I_{k+1,k}^{C} = \int \int_{t_k}^{t_{k+1}} H\left(t_{k+1}, \tau\right) E\left\{\delta u\left(\tau\right) \delta u^T\left(t\right) H^T\left(t_{k+1}, t\right)\right\} d\tau dt \tag{19}$$

$$I_{k+1,k}^{L} = \Phi\left(t_{k+1}, t_k\right) \int_{t_k}^{t_{k+1}} E\left\{\delta X\left(t_{k+1}|t\right) \delta u^T\left(t\right)\right\} H^T\left(t_{k+1}, t\right) dt \tag{20}$$

$$I_{k+1,k}^{R} = \int_{t_k}^{t_{k+1}} H\left(t_{k+1}, \tau\right) E\left\{\delta u\left(\tau\right) \delta X^T\left(t_{k+1}|\tau\right)\right\} d\tau \ \Phi^T\left(t_{k+1}, t_k\right) \tag{21}$$

$$H\left(t, \tau\right) = \Phi\left(t, \tau\right) B\left(\tau\right) \tag{22}$$

Then:

$$P_{k+1|k} = \Phi\left(t_{k+1}, t_k\right) P_{k|k} \Phi\left(t_{k+1}, t_k\right)^T + P_{k+1,k}^{\int\int} \tag{23}$$

where:

$$P_{k|k} = E\left\{\delta X\left(t_{k|k}\right) \delta X\left(t_{k|k}\right)^T\right\} \tag{24}$$

and:

$$P_{k+1,k}^{\int\int} = I_{k+1,k}^{C} + I_{k+1,k}^{L} + I_{k+1,k}^{R} \tag{25}$$

Eq. 23 formally specifies the method for moving the optimal state error covariance $P_{k|k}$ from time t_k to time t_{k+1}, and for the accumulation of acceleration modeling errors, to get $P_{k+1|k}$.

Examine Eq. 23 to see that optimal state error covariance propagation provides the structure $P_{k+1,k}^{\int\int}$ to accomodate random force modeling errors, whereas the least squares model has no such structure. This term is referred to as process noise covariance. Notice from Eq. 23 that the optimal filter covariance time update is time *sequential*. When $P_{k+1,k}^{\int\int}$ is significant and measurements are sparce with time, optimal estimation requires a time *sequential* update so as to incorporate $P_{k+1,k}^{\int\int}$ between measurement times t_k and t_{k+1}. Simultaneous batch processing of measurements eliminates the *sequential* update and thereby destroys optimality. If δu were white noise, then the process noise covariance would reduce to Kalman's $Q_{k+1,k}$ matrix.

From the definition of optimality: All state estimate modeling and modeling errors are derived from appropriate force modeling physics and sensor performance. That is, the contents of $P_{k+1,k}^{\int\int}$ are not arbitrary. We must seek appropriate results from force modeling physics.

5.4 Solutions for the Optimal State Estimate Error Model

Force modeling errors $\delta u\left(t\right)$ are captured via the state estimate error covariance function. Most significantly they derive from errors on the gravity model, the air-drag model, the solar pressure model, and the spacecraft rocket engine thrust model.

5.4.1 Gravity Solution

A solution to stochastic gravity modeling errors is given by Wright [22] [23]. This solution is derived by an extension to Kaula's gravity error covariance model [21].

5.4.2 Air-Drag Solution

A new solution to stochastic air-drag modeling errors is presented in the AGI math specification [25] for ODTK. In this innovation we sequentially estimate corrections to atmospheric density, with an autonomous filter gain that is defined (in part) by historical F_{10} and K_P data collected across two solar cycles, and by near real-time values of F_{10} and K_P.

5.4.3 Solar Pressure Solution

Stochastic solar pressure error modeling is presented in the AGI math specification [25] for ODTK. A geometric two-cone model is used for umbra-penumbra definition [19], with Earth disc diameter modification to compensate for atmospheric refraction effects [38]. In the absence of spacecraft specific surface and attitude modeling, the coefficient of diffuse reflection for a spherical surface is modeled [34].

5.5 Filter Time Update Algorithm

Let t_k be the time of last measurement. Given the state estimate $\hat{X}_{k|k}$, state estimate error covariance matrix $P_{k|k}$, and a new scalar measurement y_{k+1} at time $t_{k+1} \geq t_k$, calculate:

$$\hat{X}_{k+1|k} = \varphi \left\{ t_{k+1}; \hat{X}_{k|k}, t_k, u\left(\hat{X}\left(\tau|t_k\right), \tau \right), t_{k+1} \leq \tau \leq t_k \right\} \tag{26}$$

$$P_{k+1|k} = \Phi_{k+1,k} P_{k|k} \Phi^T_{k+1,k} + P^{\int\int}_{k+1,k} \tag{27}$$

where $P^{\int\int}_{k+1,k}$ is a sum inclusive of gravity acceleration error covariance, air-drag acceleration error covariace, solar pressure acceleration error covariance, and thrust acceleration error covariance.

5.6 Filter Measurement Update Algorithm

The Kalman filter [4] measurement update theorem, and associated algorithm, was a first order breakthrough for orbit determination. Although its implementation is associated with minor numerical difficulties (see Bucy and Joseph [7], page 141 and Chapter 16), its content has enabled, in part, the realization of optimality.

Let t_k be the time of last measurement y_k. Given a new scalar measurement y_{k+1} at time $t_{k+1} \geq t_k$, its scalar non-zero measurement error variance R_{k+1}, the state estimate $\hat{X}_{k+1|k}$, and the state estimate error covariance matrix $P_{k+1|k}$, calculate:

$$\Delta y_{k+1} = y_{k+1} - y\left(\hat{X}_{k+1|k}\right) \tag{28}$$

$$H_{k+1} = \left[\frac{\partial y\left(X\right)}{\partial X} \right]_{\hat{X}_{k+1|k}} \tag{29}$$

$$\tilde{R}_{k+1} = H_{k+1} P_{k+1|k} H^T_{k+1} + R_{k+1} \tag{30}$$

$$\text{If } \left|\Delta y_{k+1}\right| < 3\sqrt{\tilde{R}_{k+1}}, \text{ Continue} \tag{31}$$

$$K_{k+1} = P_{k+1|k} H^T_{k+1} / \tilde{R}_{k+1} \tag{32}$$

$$\hat{X}_{k+1|k+1} = \hat{X}_{k+1|k} + K_{k+1} \Delta y_{k+1} \tag{33}$$

$$P_{k+1|k+1} = \left(I - K_{k+1} H_{k+1}\right) P_{k+1|k} \tag{34}$$

$$\text{Else discard } y_{k+1}, \text{ acquire } y_{k+2},\ k+1 \rightarrow k+2 \tag{35}$$

Multiple measurements $y_{k+1} = y^j_{k+1}$, $j \epsilon \{1, 2, ...\}$, may be processed at time t_{k+1} by repetition of this sequence. Eqs. 31 and 35 define the Kalman measurement editor, naturally embedded in the Kalman measurement update algorithm.

Notice that $\tilde{R}_{k+1}$ is a scalar: H_{k+1} is $1xN$, $P_{k+1|k}$ is NxN, and H^T_{k+1} is $Nx1$. And note that $\tilde{R}_{k+1}$ is always non-zero because R_{k+1} is always non-zero. ODTK provides this scalar measurement update as the default, but also provides the capability to perform one simultaneous measurement update for cases when there are multiple measurements at one epoch.

5.7 Filter Measurement Update and Filter Time Update

We use the optimal measurement update algorithm due to Kalman and the optimal time update algorithm due to Wright. They are used recursively in a repeating sequential pattern without reprocessing any measurement.

The optimal measurement update algorithm always processes, or rejects completely, a new measurement; i.e., it is never iterated on the same measurement. Optimal iteration on the same measurement would require use of an auto-correlation model between the error on each measurement and the same error on the same measurement when reprocessed.

5.8 Measurement Editing

Given realistic covariance matrices and the successful completion of filter initalization, the Kalman measurement editor defined by Eqs. 31 and 35 provides a remarkable capability. But there is one situation where the Kalman editor can fail.

5.8.1 Kalman Measurement Editor Limitation

When the time update function propagates the state error covariance over an extended time interval the error variances can grow very significantly due to force modeling errors. Large state error variances are mapped into measurement error variances, and this opens wide the filter editor threshold. If the first measurement processed is a significant outlier and its residual falls within the editor threshold, then the outlier is accepted and incorporated by the measurement update, and state error variances are suddenly reduced. The state estimate is thereby significantly corrupted. If the the second measurement and measurements immediately following are good, it is likely that the measurement editor will reject these good measurements. Eventually the state error variances will grow to the point where the filter editor will accept one or more measurements. If these are good measurements, then the filter is autonomously reinitialized and it continues happily along.

5.8.2 Filter Initialization Problem

Typically the state error covariance is unrealistic during the filter initialization time interval because the error covariance matrix associated with the a priori orbit estimate is unknown. Thus the measurement editing thresholds are also unrealistic – usually too small. In this case the measurements are thrown out by the Kalman editor, and no measurements can be processed by the filter measurement update function.

5.8.3 The Dynamic Editor

ODTK provides a Dynamic Editor that removes, in part, the Kalman editor limitation. Let N_{RT} (default 10) denote an integer count reject threshold used to trigger the expansion of the measurement editor threshold, and let N_{AT} (default 3) denote an integer count accept threshold used to trigger the contraction of the measurement editor threshold. Integers N_{RT} and N_{AT} are defined by the user. Let $N_{\sigma LO}$ (default 3) denote the ordinary number of sigmas for the Kalman editor threshold, and let $N_{\sigma HI}$ (default 100) denote the extrordinary number of sigmas for the Dynamic Editor.

$N_{\sigma LO}$ is used for the nominal Kalman editor. During operation of the sequential filter if N_{RT} measurements are sequentially rejected, then $N_{\sigma HI}$ is used if the user has turned on the Dynamic Editor. If $N_{\sigma HI}$ is in use, and if N_{AT} measurements are sequentially accepted, then $N_{\sigma HI}$ is switched to $N_{\sigma LO}$.

5.9 Filter Initialization

The Dynamic Editor may be used for filter initialization, where N_{AT} is set to a large integer value. After filter initialization, the filter restart capability should be used to modify or eliminate the Dynamic Editor, and the filter should be restarted.

Filter divergence is not defined during filter initialization.

5.10 Filter Divergence

Filter divergence is intimately related to both the Kalman measurement editor and the complementary editor. Let N_D denote the number of complete measurement sets sequentially rejected by an editor to define filter divergence, where the N_D integer value is set by the user. A complete measurement set refers to all measurments defined at the same time tag.

5.10.1 Kalman Measurement Editor

Given completion of filter initialization, and given that the Dynamic Editor is not in use, then filter divergence is defined after N_D complete measurement sets have been sequentially rejected.

5.10.2 Dynamic Editor

Given completion of filter initialization, and given the use of the Dynamic Editor, filter divergence is defined after $(N_D + N_{RT})$ complete measurement sets have been sequentially rejected.

5.10.3 Unmodeled Thrust Maneuvers

Any significant unmodeled thrust maneuver will cause the filter to immediately diverge. When tracking hostile spacecraft this is a tremendous advantage, for it enables us to determine unknown thrust maneuver schedules for hostile spacecraft. When tracking friendly spacecraft, always tell the filter about upcoming thrust maneuvers. This will enable the filter to optimally ride across the maneuver without a filter restart.

6 Fixed Interval Sequential Smoother

Inputs to the fixed interval sequential smoother are outputs from the sequential filter. These filter outputs must therefore be stored while running the filter, for use in the smoother. The last filter output is the first smoother input and serves to initialize the smoother. The filter runs forward with time. The smoother runs backwards with time.

For this chapter only, we need notation to distinguish state estimates produced by the filter from state estimates produced by the smoother. Both are used in the same equations. Then let $\hat{X}$ and $\hat{P}$ denote state estimate and covariance output by the *filter*, and let $\tilde{X}$ and $\tilde{P}$ denote state estimate and covariance output by the *smoother*. The hats $\hat{X}$ and $\hat{P}$ denote filter, and the tildas $\tilde{X}$ and $\tilde{P}$ denote smoother. Let t_0 denote the first filter time in the fixed interval $\{t_0, t_L\}$, and let t_L, where $t_0 < t_L$, denote the last filter time in the fixed interval $\{t_0, t_L\}$.

6.1 Smoother Initialization

At the last time t_L in the fixed interval $\{t_0, t_L\}$, set:

$$\tilde{X}_{L|L} = \hat{X}_{L|L} \tag{36}$$

$$\tilde{P}_{L|L} = \hat{P}_{L|L} \tag{37}$$

6.2 Notation for Smoother Nonlinear State Transition

The left-hand side of Eq. 38 provides shorthand notation for propagation of the smoothed state estimate $\tilde{X}_{k+1|L}$ backwards in time from t_{k+1} to $t_k < t_{k+1}$ to get $\varphi_k\left(\tilde{X}_{k+1|L}\right)$:

$$\varphi_k\left(\tilde{X}_{k+1|L}\right) = \varphi\left(t_k; \tilde{X}_{k+1|L}, t_{k+1}, u\left(\tau\right); t_k \le \tau \le t_{k+1}\right) \tag{38}$$

where $k \in \{L-1, L-2, \ldots, 1, 0\}$. Note that:

$$\tilde{X}_{k|L} \ne \varphi_k\left(\tilde{X}_{k+1|L}\right) \tag{39}$$

In order to accomodate the smoother state estimate transition function for orbit substates, the VOP trajectory propagator will run backwards with time.

6.3 Smoother Sequential Equations

For $k \in \{L-1, L-2, \ldots, 1, 0\}$:

6.3.1 Transition Smoothed State Estimate Backwards

$$\varphi_k\left(\tilde{X}_{k+1|L}\right) = \varphi\left(t_k; \tilde{X}_{k+1|L}, t_{k+1}, u(\tau); t_k \leq \tau \leq t_{k+1}\right) \tag{40}$$

6.3.2 Incorporate Filter Estimate and Covariance at Time t_k

$$\tilde{X}_{k|L} = \hat{X}_{k|k} + \hat{P}_{k|k}\left[\hat{P}_{k|k} + \hat{\Phi}^{-1}_{k+1,k}\hat{P}^{\int\int}_{k+1,k}\left(\hat{\Phi}^{-1}_{k+1,k}\right)^T\right]^{-1}\left[\varphi_k\left(\tilde{X}_{k+1|L}\right) - \hat{X}_{k|k}\right] \tag{41}$$

6.3.3 Prepare to Calculate Smoother Covariance

$$A_{k,k+1} = \hat{P}_{k|k}\hat{\Phi}^T_{k+1,k}\hat{P}^{-1}_{k+1|k} \tag{42}$$

6.3.4 Smoother Covariance

$$\tilde{P}_{k|L} = \hat{P}_{k|k} + A_{k,k+1}\left[\tilde{P}_{k+1|L} - \hat{P}_{k+1|k}\right]A^T_{k,k+1} \tag{43}$$

The matrix subtraction in Eq. 43 is delicate; i.e., we must guarantee that $\tilde{P}_{k|L}$ has no negative eigenvalues due to numerical round-off.

6.4 Filter-Smoother Consistency Test

Calculate the $N \times N$ difference matrix $\bar{P}_{k|L}$ between the filtered covariance matrix $\hat{P}_{k|k}$ and the smoothed covariance matrix $\tilde{P}_{k|L}$ for time t_k:

$$\bar{P}_{k|L} = \hat{P}_{k|k} - \tilde{P}_{k|L} \tag{44}$$

for each $k \in \{0, 1, 2, \ldots, L\}$. The difference matrix $\bar{P}_{k|L}$ should have no negative eigenvalues. Denote the square root of the i^{th} main diagonal element of the $N \times N$ difference matrix $\bar{P}_{k|L}$ as $\sigma^i_{k|L}$. Also calculate the $N \times 1$ difference matrix $\bar{X}_{k|L}$ between filtered state estimate $\hat{X}_{k|k}$ and smoothed state estimate $\tilde{X}_{k|L}$ for time t_k:

$$\bar{X}_{k|L} = \hat{X}_{k|k} - \tilde{X}_{k|L} \tag{45}$$

Denote the i^{th} element of the $N \times 1$ difference matrix $\bar{X}_{k|L}$ as $\bar{X}^i_{k|L}$. Now calculate and graph the ratio :

$$R^i_{k|L} = \bar{X}^i_{k|L}/\sigma^i_{k|L} \tag{46}$$

for each $i \in \{1, 2, \ldots, N\}$ and for each $k \in \{0, 1, 2, \ldots, L\}$.

6.4.1 Test

If for each $i \in \{1, 2, \ldots, N\}$ and for each $k \in \{0, 1, 2, \ldots, L\}$ we have:

$$\left|R^i_{k|L}\right| \leq 3 \tag{47}$$

then McReynolds' filter-smoother test is satisfied globally. If for each $i \in \{1, 2, \ldots, N\}$ and for each $k \in \{0, 1, 2, \ldots, L\}$ we have:

$$\left|R^i_{k|L}\right| > 3 \tag{48}$$

then McReynolds' filter-smoother test is failed globally. For each i for which inequality 47 is satisfied McReynolds' filter-smoother test is passed for that state estimate element, and for each i for which inequality 48 is satisfied McReynolds' filter-smoother test is failed for that state estimate element.

7 Least Squares

Orbit determination using a batch of measurements from a fixed time interval could use the classical least squares normal equation:

$$A^T A \, \Delta X = A^T b \tag{49}$$

where A is the given triple product of an $m \times m$ root diagonal matrix $W^{1/2}$ of measurement weights, an $m \times n$ matrix of partial derivatives, and an $n \times n$ state error transition matrix. Matrix A has rank n with $m \geq n$. The given $m \times 1$ matrix b is the product of matrix $W^{1/2}$ with the $m \times 1$ matrix Δy of measurement residuals. ΔX is the unknown $n \times 1$ matrix to be calculated. Conventional solution:

$$\Delta X = \left(A^T A\right)^{-1} A^T b \tag{50}$$

But dense range measurement values may have variations only in the last few significant decimals. Given double precision calculation with 15^+ decimal mantissas, generally half of the 15^+ decimal significance is lost due to the squaring operation in the calculation of $A^T A$. This significance is regained by solving instead, the equation:

$$A \, \Delta X = b \tag{51}$$

for ΔX. This seems trivial when $m = n$:

$$\Delta X = A^{-1} b \tag{52}$$

For $m > n$, the solution of Eq. 51 with greatest numerical stability requires the use of orthogonal Householder transformations (Lawson & Hanson [40], page 121) for triangularization of matrix A. It is important to note that Eqs. 49 and 51 are theoretically equivalent in the sense that each implies the other. Our problem is to calculate an acceptable solution of Eq. 51 when $m > n$.

Triangularization TA of matrix A is obtained with the calculation of an $m \times m$ orthogonal matrix T such that the upper $n \times n$ matrix of the $m \times n$ matrix TA is upper triangular, and the lower $(m-n) \times n$ matrix of the $m \times n$ matrix TA is zero. Multiply Eq. 51 through by matrix T to get:

$$TA \, \Delta X = Tb \tag{53}$$

This is easily solved for ΔX with back substitutions because TA is upper triangular.

Part II
Computer Science

8 Software Architecture

The ODTK Application software design consists of a multi-tiered, layered architecture. Key design goals are to:

- Reuse and share astrodynamics functionality with the STK code libraries to achieve consistent mathematical modeling between both the STK and ODTK products;
- Maintain separation between the Graphical User Interface (GUI) and engine software to minimize complexity and improve portability;

- Provide enhanced user interface controls and a consistent look and feel with other AGI products.

The figure and sections below describe the ODTK architecture in more detail.

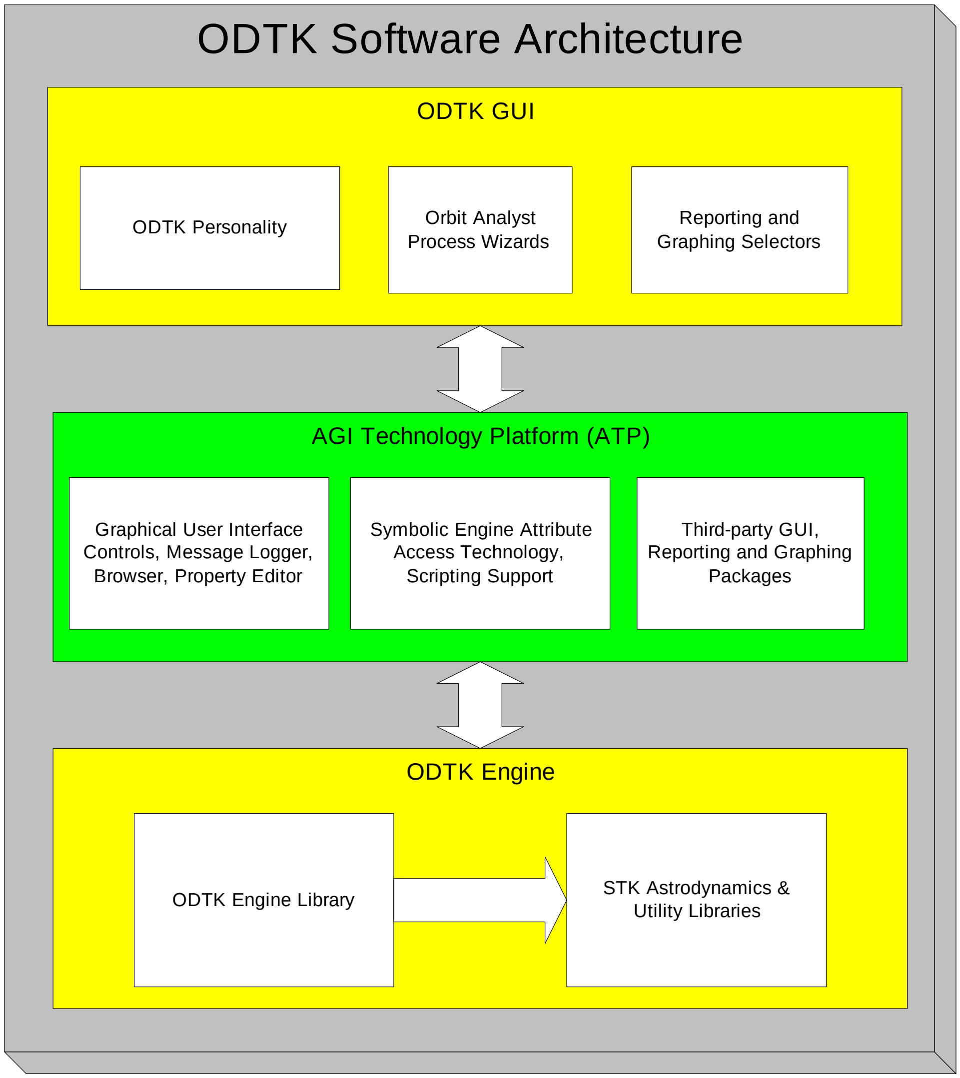

Figure 1: ODTK Software Architecture

9 AGI Technology Platform (ATP)

The ATP provides an expandable application and component framework for the ODTK software. ODTK makes use of the following ATP Technologies:

- Application "personality"
- Object browser - for viewing the OD configuration objects
- Property editor - for viewing and editing the OD object attributes
- Enhanced GUI controls,

- Attributes: an advanced symbolic access functionality that provides a consistent and logical object-based scheme for getting and setting internal data.
- Scripting automation support
- Reporting and graphing
- Object serialization (file save/load) in XML

Through the use of "personality" components the ATP can use a common application architecture to support different GUI requirements (and appearances) running on top of the same engine. The ODTK Personality loads and controls the ODTK GUI engine components.

The Object Browser provides the ability to graphically see the organization of an ODTK scenario. Selecting an object in the browser allows the user to edit its attributes in the Property Editor window.

Enhanced GUI controls support date/time editing and unit conversion.

The ATP Attributes technology exposes the engine data and control to the user interface and scripting. The ODTK engine exposes attributes for each object that the user may edit or view (in the case of read-only attributes). The user interface and scripts may also command objects to perform certain exposed functions. The exposed attributes can then be serialized on user command via file saving and loading.

Finally, AGI has incorporated into the framework best-of-breed third-party reporting and graphing packages. The reporting package provides What-You-See-Is-What-You-Get (WYSIWYG) visual editing of report styles. Additionally, the report designer may incorporate formulas into a report to enhance analysis of the reported data. The graphing package provides an integrated but hide-able toolbar and right-click menus to optimize graph viewing. Real time, dynamic graphs are also presented.

10 ODTK Engine

The ODTK engine is an object-oriented package that encompasses all the ODTK astrodynamics functionality and data encapsulated in C++ classes and function libraries. All objects visible in the Object Browser are implemented as C++ classes. The ability to modify the data and control the behavior of these objects is provided through the use of ATP attributes. The Filter object consists of many sub-objects. The most important of these is the State Estimate object, which encompasses the filter state and covariance matrices.

Standard Template Library (STL) containers are used extensively in the engine to enable run-time definition of various object lists. Most significantly the State Estimate object consists of STL containers, which provide for arbitrary user definition of the number of orbits for multiple simultaneous orbit determination.

Several scripting points have been exposed in the engine where customized processing can be combined to update exposed objects and their attributes. These scripts can be used to automate several key functions including observation loading, reporting, satellite maneuver input, ephemeris generation and smoothing.

ODTK reuses several STK astrodynamics and utility libraries to maximize process commonality and simplify interfacing with STK.

11 ODTK GUI

The ODTK GUI is built on top of the ATP. The ODTK GUI includes the necessary components to construct, maintain and perform ODTK tasks. Similar to STK, the controls and data viewable and editable by the user are organized into an object-oriented architecture. The classes of objects and their hierarchy relationships are pictured on the figure below. ODTK also provides several GUI wizards which streamline the creation of scenarios for typical OD tasks including: Filtering, Smoothing, Simulation, Initial Orbit Determination and Least Squares Estimation.

12 Summary

AGI has implemented and integrated modern software components to provide a high-value, reliable and expandable solution for customers. Future additions to the product can be more easily incorporated based

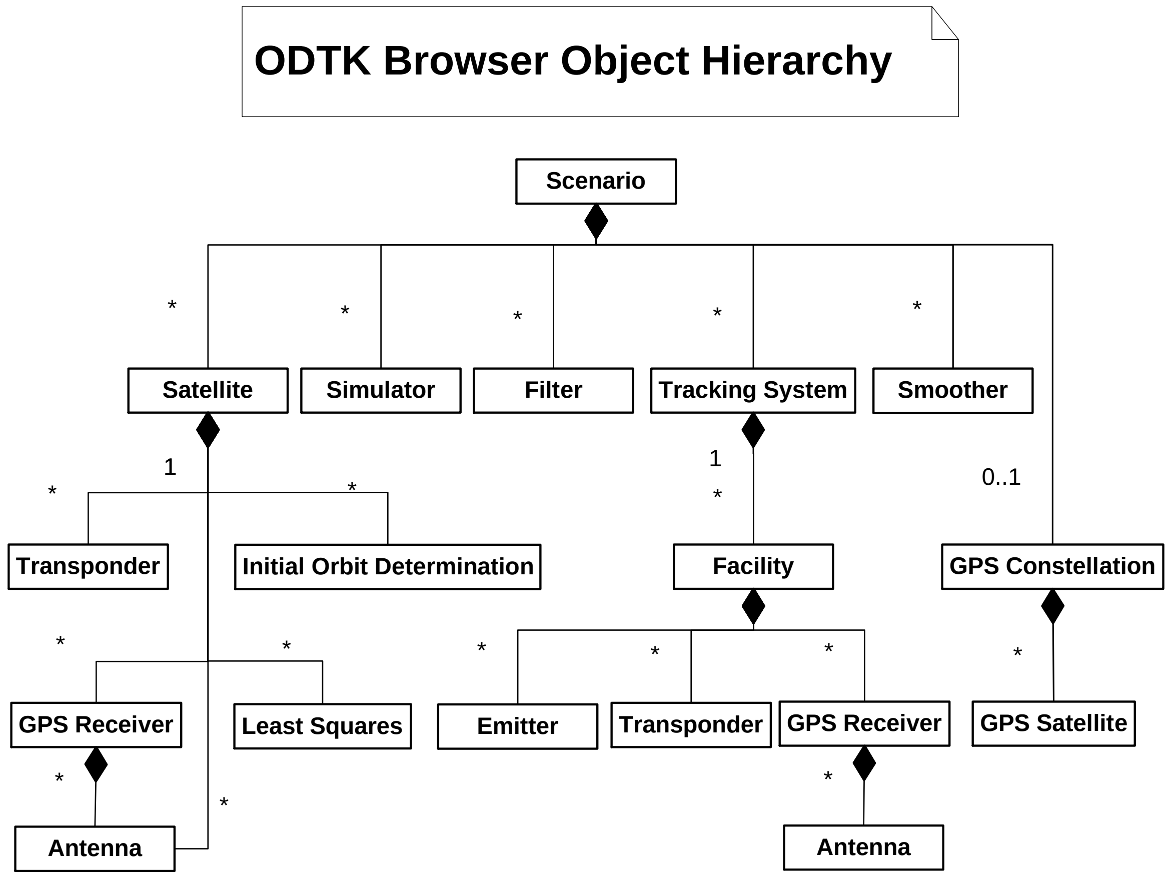

Figure 2: ODTK Browser Object Hierarchy

on this well-structured, layered architecture with minimal perturbations.

ODTK is an *optimal* orbit determination capability, and optimality enables superior performance in real-time operations, post-pass operations, and analysis via simulation and covariance.

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