DETERMINATION OF CLOSE APPROACHES FOR CONSTELLATIONS OF SATELLITES

James Woodburn, Chief Orbital Scientist, Analytical Graphics, Inc.
Donald Dichmann, Senior Development Specialist, Analytical Graphics, Inc.

Abstract

The deployment of large constellations of satellites both increases the potential for close approaches between orbiting objects and increases the importance of predicting potential collisions between objects. Improvements and extensions to existing close approach algorithms are presented for application to constellations of satellites. Many of the constellation configurations currently under consideration consist of a distinct set of orbital planes each containing a group of phased satellites in circular orbits. Similarities in the orbits of the satellites in the constellation are utilized to improve the efficiency of close approach predictions by as much as 62 percent. The associated computation times are projected to be reasonable for even the largest of the currently proposed constellations.

Introduction

The probability of collisions between orbiting objects is increasing with the launch of each new satellite. The first confirmed collision between two objects in orbit about the Earth, the French satellite CERISE and a piece of debris from a rocket body, occurred in July 1996.[1] In light of this, the daily maintenance and operations of large constellations of satellites will probably include the determination of potential future collisions between the satellites that comprise the constellation and other objects in orbit. These potential collisions are often referred to as close approaches. Because each orbiting body occupies a volume in space, and because there is uncertainty associated with the ephemeris of each object, exact intersections of the trajectories are not the only events of interest. It is more realistic to define an exclusion zone about each satellite in the constellation such that passage of another object through the exclusion zone is considered to represent an unacceptable risk to the satellite. Close approaches are defined as the periods of time when an object is within the exclusion zone of a satellite in the constellation. Once future close approaches have been identified, contingency plans can be enacted to reduce the probability of collision.

The basic problem is to predict when a satellite of interest, the primary satellite, will have an unacceptably high risk of collision with any another Earth orbiting object. The source of epemeris information for objects in Earth orbit is usually the United States Space Command (USSPACECOM) satellite catalogue which currently contains over 8000 objects.[1] The simplest method of close approach prediction is to step along the trajectories of the primary and each candidate object, computing the distance between the objects at each time step, and detect crossings through the boundary of the exclusion zone. The drawback of this simple technique is the large computational burden that it imposes. To lighten the computational load associated with this approach, most methods for determining close approaches between satellites include filters which are used to eliminate candidate objects from consideration if the range between the two satellites cannot be less than the radius of the exclusion zone.

Hoots et al.[2] designed a series of three filters through which candidate objects have to pass before a final determination of the close approach distance is made. Two of the filters are purely geometrical and one uses the known properties of the orbital motion of the two objects. These filters serve to “weed out” the majority of the objects in the catalogue and greatly reduce the number of computations needed. After the application of the filters, the trajectories of the remaining candidate objects are sampled to determine the actual close approach periods. The exclusion zone is modeled as a sphere centered at the primary satellite. Alfano and Negron[3] developed a technique for modeling the distance between two objects using localized cubic polynomials. In this approach, the geometrical filters developed by Hoots et al.[2] are still applied, but the final filter is removed and the trajectories of the vehicles are sampled at large time steps (up to 10 minutes) to create waveforms describing either the relative distance[3] or range rate[4] between the satellites. This waveform provides a model from which estimates of the time of closest approach and the entrance and exit times for crossing an exclusion zone boundary are made. The work of Alfano and Negron allows the exclusion zone boundary to be modeled as an ellipsoid centered at the primary satellite to account for uncertainties in the along-track position of the objects being greater than the uncertainty in the cross-track and radial directions. Other authors have approached restricted versions of the problem considering only the distance between the orbital paths[5] or only circular orbits[6].

In this paper, we describe an algorithm for the detection of close approaches based upon Hoots et al.[2] with extensions for constellations of satellites. These extensions improve the efficiency of the close approach algorithms when applied to most constellations of satellites under current consideration. The overall number of computations can be reduced by taking advantage of similarities in the orbits of the satellites in the constellation. Most constellations under current consideration, for example, are comprised of satellites in circular orbits in a distinct set of orbital planes. This information can be used to reduce the need for application of some of the filters to once for the entire constellation or once per orbital plane. While the exclusion zone is modeled as a sphere centered at the primary satellite, an alternative definition of the exclusion zone could be implemented.

Description of Filters

The close approach filters applied to the constellation problem are those described by Hoots et al.[2] with some significant modifications to the orbit path filter. The application of each of these filters requires the specification of the time interval over which the close approach analysis will be performed and assumes that all objects are in closed orbits. A candidate object which may still have a possibility for a close approach with the primary object after a filter has been applied is said to have passed that filter. The purpose of these filters is to eliminate candidate objects which cannot come closer than the minimum allowed separation distance to the primary satellite.

Apogee-Perigee Filter

The apogee-perigee filter is used to eliminate candidate objects which do not come within the minimum allowed separation distance, D, of having an overlap in altitude with the primary object. Candidate objects are eliminated only if they fail this filter at both ends of the time period being considered. For constellations where all of the satellites share the same basic range of altitude, this filter need only be applied once. The apogee and perigee for the filter are set to be the extreme values as sampled from the entire constellation.

This filter can be applied using a different definition of the exclusion zone surface if the radial dimension of the surface is bounded.

Orbit Path Filter

The orbit path filter is used to eliminate candidate objects whose orbital paths, independent of the location of the satellite, do not come within the minimum allowed separation distance of the primary object. For the case of two circular orbits, in the absence of perturbations, an analytical solution exists. In this case, the minimum distance occurs along the line of intersection, the relative line of nodes, of the two orbital planes. This solution was used by Hoots et al.[2] as the starting point for a Newton iteration scheme to solve the more general problem where the orbit paths are elliptical. For cases where either orbit has moderate eccentricity, however, the Newton method usually requires an initial guess which is closer to the final solution than the points along the relative node in order to converge.

A new algorithm has been developed for the orbit path filter which solves the problem in a slightly different way. The previous approach solved for the minimum distance between the two orbits and then compared that result to the minimum allowed distance. The new method first determines if the distance between the orbits can be less than the minimum allowed distance, and only solves for the minimum distance in a small subset of the cases. The geometry of the two orbit planes is shown in Figure 1.

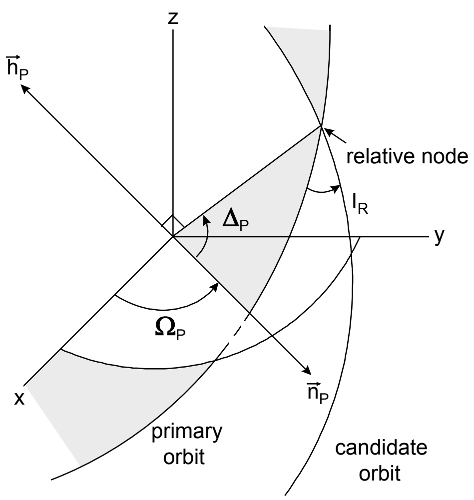

Figure 1. Orbit path geometry

The two orbit planes have a relative inclination, I_R, which may be defined using the cross product of the two orbit normal unit vectors as

$$\sin I_R = |\vec{h}_P \times \vec{h}_C|, \qquad (1)$$

where $\vec{h}_P$ is the normal to the orbit plane of the primary object and $\vec{h}_C$ is the normal to the orbit plane of the candidate object. Based on the relative inclination, the minimum allowed separation distance, D, and the radius of the orbit at the relative node, it is possible to determine the maximum distance from the relative node that a satellite could be and still be within the minimum allowed distance. The distance between one object and the orbit plane of the other object may be written as

$$\pm D = r \sin I_R \sin u_R , \qquad (2)$$

where u_R is the argument of latitude relative to the intersection of the two orbit planes and r is the orbit radius. Figure 2 illustrates the relationship between the relative inclination, the relative argument of latitude and the distance between the orbit paths.

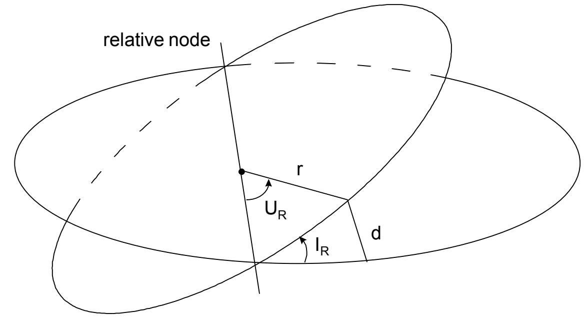

Figure 2. Relative argument of latitude

The orbit radius is expressed as

$$r = \frac{a(1-e^2)}{1 + e\cos\left(u_R - (\omega - \Delta)\right)} \qquad (3)$$

where a is the semi-major axis length, e is the eccentricity, ω is the argument of perigee, Δ is the argument of the relative node and u_R is defined by the relationship

$$u_R = \nu + \omega - \Delta , \qquad (4)$$

where ν is the true anomaly. Equation (3) may then be substituted into Equation (2) and solved for u_R to yield

$$A \sin u_R + B \cos u_R = 1 , \qquad (5)$$

where

$$A = \frac{a(1-e^2)}{\pm D} \sin I_R + e \sin(\omega - \Delta) , \qquad (6)$$

and

$$B = e \cos(\omega - \Delta) . \qquad (7)$$

When the substitutions

$$C = \sqrt{A^2 + B^2} , \qquad (8)$$

$$A = C \cos\delta , \qquad (9)$$

$$B = C \sin\delta , \qquad (10)$$

are made in Equation (5), then a simple trigonometric identity yields

$$\sin(u_R + \delta\ ) = \frac{1}{C}. \tag{11}$$

Equation (11) has two solutions for u_R,

$$u_R = -\delta + \sin^{-1}\left(\frac{1}{C}\right), \tag{12}$$

$$u_R = -\delta + \pi - \sin^{-1}\left(\frac{1}{C}\right). \tag{13}$$

Equations (12)-(13) are each solved with both possible signs in Equation(6) to yield a total of four solutions, two of which bound each crossing of the relative node. If the quantity $1/C$ is greater than one, then the orbit paths never reach a cross-track distance of D and the two orbit paths are considered to be coplanar. In a coplanar case, the orbit path filter and time filter are skipped.

Any close approach between the two objects must occur in the range of values of u_R defined by Equations (12)-(13). The next step in the filter is to determine the minimum distance between the two orbits inside the allowed range of the relative argument of latitude for one of the orbits. The minimum distance is determined by computing the distances from the interval endpoints and the relative node on the candidate orbit to the primary orbit.

The distance between a point and an ellipse may be computed using the following algorithm. First, the position of the candidate object is projected into the plane of the primary orbit as

$$\vec{r}_{PQW} = T\vec{r} \tag{14}$$

where T is the transformation matrix between the inertial coordinate system and the orbit plane coordinate system with the $\vec{P}$ axis pointing towards perigee and the $\vec{W}$ axis pointing along the orbital angular momentum vector. The projection of the candidate satellite position vector into the orbit plane of the primary satellite is shown in Figure 3.

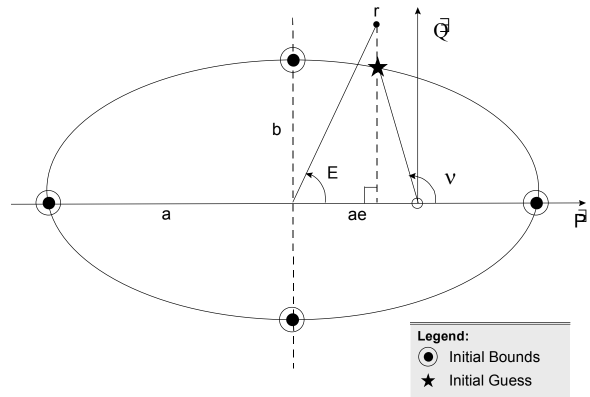

Figure 3. Projection of position vector into the orbital plane

The eccentric anomaly associated with a vector from the center of the ellipse through the point is then computed from the in-plane component of $\vec{r}_{PQW}$ as

$$E = \tan^{-1}\left[\frac{r_Q}{r_{P*}}\right], \tag{15}$$

where

$$r_{P*} = ae + r_P. \tag{16}$$

Since no minimization can be done on the out of plane distance $|r_W|$, the quantity to be minimized is the distance from the in-plane projection of the point to the orbit path. This distance can be written as

$$d(E) = \sqrt{\left(a\cos E - r_{P*}\right)^2 + \left(b\sin E - r_Q\right)^2} \tag{17}$$

where b is the semi-minor axis length, $b = a\sqrt{1-e^2}$. We use the secant method to minimize the function $\frac{1}{2}d^2(E)$ by driving the value of its slope to zero. The initial bounds for the secant method are given by the bounds of the quadrant in which the projected point lies. The slope of the distance function with respect to the eccentric anomaly is given by

$$\begin{aligned} d\,d' = a r_{P*}\sin E - b r_Q \cos E - \\ a^2 e^2 \cos E \sin E. \end{aligned} \tag{18}$$

The secant method locates the eccentric anomaly where the distance between the point and the ellipse is a minimum,

$$D_{\min} = \sqrt{\left(d(E)\right)_{\min}^{2} + r_{W}^{2}} \; . \qquad (19)$$

If the distance at the relative node is less than the distance at both ends of the interval of the relative argument of latitude, found by solving Equation (2), and the distance at the relative node is greater than the minimum allowed separation distance, then we solve for the true minimum using a parabolic approximation method. Otherwise the minimum sample is considered to be the minimum distance. If the minimum distance between the orbit paths is less than the minimum allowed separation distance, then the candidate object passes the orbit path filter. To account for the changing geometry due to the precession of the orbit plane and the argument of perigee, we perform this filter at the ends as well as at the midpoint of the time interval. If the candidate object passes the filter at any of these evaluations, it is retained.

In application of the orbit path filter to constellations of satellites, a computational savings may be possible if the satellites within an orbital plane have very nearly the same semi-major axis length, eccentricity and argument of perigee. In this case, it is possible to run the orbit path filter once and apply the results to all of the satellites in the plane. To account for the fact that the satellites will not be in identical orbits, we can increase the minimum allowable distance for this filter by the maximum distance between the orbit of the first satellite in the plane and the orbits of the other satellites in the plane. This value can be determined by stepping around the orbits in the plane, other than that of the first satellite, and computing the distance to the orbit of the first satellite at each step. The maximum distance, S, to the orbit of the first satellite is computed for each subsequent satellite in the plane and the maximum of these is added to the value of D used in Equation (2),

$$D_{plane} = D + S_{\max} \; . \qquad (20)$$

This filter can also be applied to different definitions of the exclusion zone if the radial and cross-track dimensions of the exclusion zone are bounded. In this case, the square root of the sum of the squares of the maximum radial and cross-track dimensions of the exclusion surface can be used as the minimum allowed separation distance.

Time Filter

In order for a close approach to occur, not only must the paths of the primary and candidate objects come sufficiently close, but the two objects must simultaneously be within the intervals defined by the orbit path filter. The purpose of the time filter is to compute the time intervals for each satellite when it is within the minimum allowed separation distance of the trajectory of the other object. The two sets of intervals are then searched for overlaps. Candidate satellites pass this filter if one or more overlaps exist. Furthermore, only the overlap intervals are retained for use in the final filter.

The time intervals are computed for each satellite based on the ranges of the relative argument of latitude computed in the orbit path filter and the related intervals for the primary satellite. These intervals are converted to intervals in mean anomaly for each satellite. The orbital elements of the satellite at the midpoint of the consideration interval are then used to determine the times when each satellite is within the closest mean anomaly interval around each of the two nodes. The satellites will return to this geometry on a periodic basis. Since the orbits are affected by perturbations, however, we cannot simply add multiples of the orbit period to generate the entire set of time intervals. The period of interest is the amount of time required to return to the same argument of relative latitude. This period is dependent upon both the mean motion of the satellites and the motion of the relative node,

$$\mathrm{T}_{R} = \frac{2\pi}{\dot{M} + \dot{\omega} - \dot{\Delta}} . \qquad (21)$$

The secular rates of change of the argument of perigee, ω, the right ascension of the ascending node, Ω, and the mean anomaly, M, are modeled as[7]

$$\dot{\omega} = \frac{3}{2}\frac{J_2 a_e^2}{a^2\left(1-e^2\right)^2}\bar{n}\left(5\cos^2 I - 1\right) , \qquad (22)$$

$$\dot{\Omega} = -\frac{3}{2}\frac{J_2 a_e^2}{a^2\left(1-e^2\right)^2}\bar{n}\cos I \qquad (23)$$

$$\dot{M} = \bar{n} + \dot{\bar{n}}t\,, \qquad (24)$$

where

$$\bar{n} = n_0\left[1+\frac{3}{4}\frac{J_2 a_e^2}{a^2\left(1-e^2\right)^2}\left(3\cos^2 I - 1\right)\right], \qquad (25)$$

a_e is the equatorial radius of the Earth, J_2 is the second zonal harmonic of the gravitational potential, n_0 is the unperturbed mean motion, $\dot{\bar{n}}$ is the rate of change of the mean motion and t is the time past the epoch of the element set. The rate of change of the perturbed mean motion is computed based on the change in the perturbed mean motion over the span of the interval of consideration.

Hoots et al.[2] give the rate of change of the relative node to be

$$\dot{\Delta}_P = \frac{\sin I_C}{\sin I_R}\cos\Delta_C\left(\dot{\Omega}_P - \dot{\Omega}_C\right)\,, \qquad (26)$$

$$\dot{\Delta}_C = \frac{\sin I_P}{\sin I_R}\cos\Delta_P\left(\dot{\Omega}_P - \dot{\Omega}_C\right)\,, \qquad (27)$$

and a useful expression for computing successive relative periods after k revolutions,

$$\mathrm{T}_{R_K} = \mathrm{T}_{R\_DF}\left[1 - k\frac{2\pi}{\bar{n}}\frac{\dot{\bar{n}}}{\bar{n}}\right], \qquad (28)$$

where T_{R_DF} is the relative period in the absence of drag effects ($\dot{\bar{n}} = 0$).

The time intervals computed from the time filter represent the periods of time when one satellite is within the minimum allowed distance of the orbit path of the other satellite. Since the time filter is simply converting the ranges of relative argument of latitude produced by the orbit filter into time intervals, the application of the time filter to a entire plane of the constellation is subject to the same requirements as the orbit path filter. In this case, the intervals computed for the candidate object can be shared by all of the satellites in the plane. The intervals for each satellite in the plane must still be computed separately as do the resulting overlaps with the intervals for the candidate satellite.

The time filter can be applied to different definitions of the exclusion zone subject to the same conditions as the orbit path filter.

Boundary Crossing Filter

The final filter in the determination of close approaches is the boundary crossing filter. The purpose of this filter is to detect crossings of the exclusion zone boundary during the intervals remaining after the time filter. Since we are considering the boundary to be a sphere, boundary crossing are analogous to the range between the two satellites crossing the threshold defined by the minimum allowed separation distance. In this filter, the range between the satellites is computed during the intervals computed during the time filter. These intervals are typically very short. For cases where the interval duration is less than 10 minutes, we compute the range at the ends and midpoint of the interval. If the range at the midpoint of the interval is less than the range at both ends, then we solve for the minimum using a parabolic approximation method. Otherwise the minimum range is simply the smallest of the sampled ranges. If the minimum range is less than the specified minimum allowed separation distance, then a close approach has been found and the exact crossing times are determined. If the time interval is greater than 10 minutes in duration, then the range is sampled at intervals of 10 minutes and the minimum range determined. This is usually only necessary in the case of coplanar orbits. There are no efficiency improvements to the boundary crossing filter during the computations of close approaches for constellations of satellites.

Coplanar Orbits

If a candidate satellite is detected to be coplanar with the primary satellite in the orbit path filter, the orbit path filter and time filters are skipped and the candidate satellite is subjected to the boundary crossing filter over the entire interval of consideration.

Results

The IRIDIUM™ satellite constellation was selected for study because the number of satellites in orbit at the time of this writing (34) is fairly large and because the altitude regime allows for a large number of candidate objects for close approaches. Two issues were of interest in this study: the ability of the constellation algorithms to report the same close approaches as are reported when the satellites are processed individually and improvements in processing efficiency. A baseline set of close approaches were determined by processing the satellites in the constellation individually for the period of 10 November 1997 00:00:00 to 12 November 1997 00:00:00 (GMT). The satellites considered to be part of the IRIDIUM™ constellation are listed in Table 1 along with an orbit plane designation created for this analysis and the number of approaches closer than 10 Km experienced by each of the satellites in the constellation over the two day time period.

Table 1. Iridium constellation

SSC Number	Plane	# Close Approaches
24792	A	6
24793	A	8
24794	A	3
24795	A	3
24796	A	5
24836	B	6
24837	B	6
24838	B	4
24839	B	11
24840	B	7
24841	B	5
24842	B	5
24869	C	12
24870	C	4
24871	C	6
24872	C	9
24873	C	4
24903	D	4
24904	D	6
24905	D	5
24906	D	6
24907	D	3
24944	E	3
24945	E	8
24946	E	5
24947	E	2
24948	E	6
24949	E	6
24950	E	4
24965	F	6
24966	F	9
24967	F	6
24968	F	8
24969	F	4
Total	6	229

The same analysis was performed three more times using the constellation-specific enhancements to the filters. In the first case, the apogee-perigee filter was applied once for the entire constellation. The orbit path, time and boundary crossing filters were then applied individually for each remaining candidate satellite. In the second case, the apogee-perigee filter was applied once for the entire constellation and the orbit path filter was applied once per plane. The time and boundary crossing filters were then applied individually for each remaining candidate satellite. In the third case, the apogee-perigee filter was applied once for the entire constellation and the orbit path and time filters were applied once per plane. The boundary crossing filter was then applied individually for each remaining candidate satellite. In all three cases the same set of close approaches was found as in the baseline case. The effect of sharing the results of the various filters in the close approach processing for a constellation is shown in Table 2. The processing times shown in Table 2 were generated on a Silicon Graphics O2 workstation with a 150 MHz processor.

Table 2. Effects of sharing filters in close approach processing

Sharing Configuration	Average # Candidates Passing Filter			Close Approaches	Processing Time (sec)	Percent Improvement
	Apogee-Perigee	Orbit Path	Time			
None	2228	852	384	229	270	0%
Apogee-Perigee	2905	866	393	229	136	50%
Orbit Path	2905	866	456	229	113	17%
Time	2905	866	457	229	102	10%

Practical Considerations

Actual close approaches will not occur exactly as predicted by the algorithm presented here. The element sets from which the close approaches are computed have an associated uncertainty at the epoch of the element set. The orbit prediction model used to generate ephemeris from the original element sets is imperfect. The combination of these two error sources leads to predictions of position and velocity of the satellites which in general degrade in accuracy over time. Since close approaches are typically determined using a relatively small separation distance, 10 Km in our example, it is not long before the uncertainties in the ephemeris greatly reduce the reliability of predicted close approaches. Jenkins and Schumacher[1] report that the reliability of predicted close approaches 7 days out only have about a 50% chance of actual occurrence.

To achieve accurate results using the described filters, it is necessary to add a pad to the minimum allowed separation distance regardless of whether a single satellite or constellation is being processed. In the case of the apogee-perigee filter and the orbit path filter, the trajectories of the satellites are assumed to be ellipses. Only secular changes in the size and shape of the ellipses is modeled by the filters. The addition of a pad to the minimum allowed separation distance for these filters accounts for the osculation of the orbital elements that is not being modeled as part of the filters. For the example presented here, a 20 Km pad was added. The time filter also requires the addition of pads on the computed intervals due to the fact that the mean motion of the satellites computed from Equation(25) will not be exact for each revolution. To account for this a pad of 10 seconds plus one second per revolution was added to the time intervals for each satellite before the overlaps were computed.

Conclusions

An efficient algorithm for determining close approaches to the members of a constellation of satellites has been presented. The algorithm is based on a set of filters which are typically applied in the determination of close approaches for single satellites. A significant improvement in the orbit path filter has been made which eliminates the problems experienced by the existing algorithm with orbits of moderate eccentricity. A total computational savings of 62% was achieved for the case study of the IRIDIUM^{TM} constellation over a two day time period. The most significant improvement, and the easiest to implement, is the sharing of the results of the apogee-perigee filter over the entire constellation. This improvement alone provides a computational savings of approximately 50%. The processing time of well under two minutes for 34 satellites can be extrapolated to infer that the processing times for even the largest constellations (Teledesic has 288 satellites in the currently proposed configuration) will be under 30 minutes.

Recommendations for Future Work

A more robust method of determining the appropriate pads to be used in the filtering process would be useful. Currently, conservative padding is applied to avoid missing close approaches. If smaller pads could be used, the efficiency of the algorithm would be enhanced. An additional filter which removes coplanar members of the constellation should be applied. This filter would eliminate the need to process coplanar objects for which close approaches are not possible. This could be important in reducing the overall processing time for large constellations, since coplanar objects are the most computationally intensive to investigate.

References

[1] JENKINS, E.L. and SCHUMACHER, P.W. "Predicting Conjunctions with Trackable Space Debris: Some Recent Experiences", Paper No. 97-014, 20th Annual AAS Guidance and Control Conference, Breckenridge, Colorado, February 5-9, 1997.

[2] HOOTS, F.R., CRAWFORD, L.L. and ROEHRICH, R.L. "An Analytic Method to Determine Future Close Approaches Between Satellites", *Celestial Mechanics*, Vol. 33, 1984, pp. 143-158.

[3] ALFANO, S. and NEGRON, D., JR. "Determining Satellite Close Approaches", *The Journal of the Astronautical Sciences*, Vol. 41, No. 2, April-June 1993, pp. 217-225.

[4] ALFANO, S. "Determining Satellite Close Approaches, Part II", *The Journal of the Astronautical Sciences*, Vol. 42, No. 2, April-June 1994, pp. 143-152.

[5] DYBCZNSKI, P.A., JOPEK, T.J. and SERAFIN, R.A. "On the Minimum Distance Between Two Keplerian Orbits with a Common Focus", *Celestial Mechanics*, Vol. 38, 1986, pp. 345-356.

[6] BEERER, J. and BAUER, T. "Determination of Closest Approach and Duration of Encounter for Two Satellites in Circular Non-Coplanar Orbits", Paper No. 83-336, AAS/AIAA Astrodynamics Conference, Lake Placid, New York, August 1983.

[7] ROY, A.E., *Orbital Motion*, 2nd Edition, Adam Hilger Ltd, Bristol 1982.