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ORBIT DETERMINATION DURING HIGH THRUST AND LOW THRUST MANEUVERS

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ORBIT DETERMINATION DURING HIGH THRUST AND LOW THRUST MANEUVERS

Richard S. Hujsak[1]

The recent launch of ANIK-F2 provides a preview of future GEO satellite operations, exploiting low thrust technology to improve fuel efficiency, thereby raising payload mass, and providing more precise orbit position control. As satellite operators abandon impulsive thrusting in favor of long duration low thrust maneuvers, various agencies, especially those providing close-approach predictions, will face a challenging orbit determination problem.

This paper demonstrates the capability of Orbit Determination Tool Kit[2], a sequential filter, to fit the tracking data and predict the ephemeris in the presence of known and unknown long-duration low-thrust events. If the commanded maneuver is known, then orbit accuracies on the order of 100 meters or less are possible, with prediction accuracy suitable for space safety analysis. If the commanded maneuvers are not known, the filter can solve for the thrust magnitude, detect pauses in thrusting, and generally be used to reverse engineer the satellite operation.

This paper also demonstrates the capability of this sequential filter to process through high thrust events, using the ANIK-F2 perigee raising maneuvers as an example. In this analysis the commanded maneuver is always assumed to be known and dedicated tracking is collected during thrusting. The results demonstrate a capability to maintain orbit errors under 10 km throughout a multi-burn scenario, with thrust events lasting an hour or more. As a side-benefit, the filter's predicted covariance can be used to schedule tracking assets to minimize both the orbit error and the load on the tracking stations.

If the filter can perform orbit determination through low thrust and high thrust events, then it can support the entire spectrum of thrust events.

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[2] Orbit Determination Tool Kit, OD Tool Kit, and ODTK are trademarks of Analytical Graphics, Inc.

INTRODUCTION

This analysis is motivated by the recent launch of ANIK-F2, which employed a combination of thrusting methods to raise perigee and then circularize in geosynchronous orbit. A series of bipropellant motor firings across multiple apogee passes were used to raise perigee and change inclination. The result was a zero inclination, eccentric orbit with a period of one day. The orbit was then circularized by employing a low thrust Xenon Ion Propulsion System (XIPS) almost continuously over 18 days. This approach to geosynchronous (GEO) insertion minimizes the fuel used and maximizes the payload mass per launch dollar. Once circularized and handed over by Boeing to Telesat Canada, the ANIK F2 mission continues to employ the XIPS thruster to maintain the satellite well within the assigned orbit slot, with daily thrusting for the life of the satellite, nominally as four distinct 2-hour burns. The ANIK-F2 mission poses some interesting orbit determination challenges, and given the benefits of greater payload mass, fuel efficiency, and tighter positioning control it is a harbinger of future operations in GEO.

In 1995 Spitzer (Ref 1) presented the thrusting strategy subsequently employed by Boeing during the July 2004 launch of ANIK-F2. Spitzer was interested in optimizing the fuel expenditure for GEO launches and demonstrated substantial savings in employing thrusters with I_{SP} on the order of 4000 sec. With the success of the ANIK-F2 launch we can expect more satellites to use similar GEO transfer and daily operating strategies.

While the Boeing GEO transfer strategy is optimal for the satellite operator, it does pose problems for other satellite operators and tracking agencies. Similarly, daily XIPS thrusting to maintain orbit position is optimal for the satellite operator, but makes orbit determination (OD) and orbit prediction difficult for other agencies. These difficulties currently impact efforts to provide space safety analyses for the GEO belt.

This analysis seeks to address orbit determination in the presence of such maneuvers, employing a sequential filter. For the GEO transfer problem, our analysis is limited to the capabilities an operator may enjoy, having knowledge of the commanded maneuvers. For the XIPS-circularization maneuver our analysis also addresses the case where the commanded maneuver is not known.

This analysis illustrates that a sequential OD filter can continually estimate the orbit and thrust parameters in the presence of perigee-raising and during XIPS circularization maneuvers. (We predict that similar results will be obtained for routine day-to-day XIPS thruster operations.) This analysis also provides some sensitivity to the level of tracking required to maintain an accurate orbit under these conditions.

The methods we employ are particularly relevant to Analytical Graphic's Orbit Determination Tool Kit[3], which employs a sequential filter for orbit determination. Different results may be obtained with other OD products.

The discussion consists of four topics. Two brief sections present the history of thrust events and describe our analysis testbed, followed by two sections that analyze high thrust and low thrust scenarios. The fit accuracy achieved when the commanded thrust is known is quite good, with errors less than 10 km for high thrust maneuvers and a 100 meters for low thrust maneuvers. For low thrust maneuvers it appears possible to use the sequential filter to first bound the thrust magnitude and ultimately solve for thrust components, leading to orbit errors in the presence of long durations thrusting that is generally under one km.

TLE HISTORY OF APOGEE AND PERIGEE EVENTS

The TLE history for ANIK-F2 (Figure 1) indicates approximately five days of high-e transfer orbit followed by four perigee raising burns and a possible fifth burn that simultaneously raises perigee and lowers apogee, all before Aug 1. This sets up the XIPS circularization maneuver starting on Aug 9 that takes 18 days. The perigee history generated by these TLE's is given in Table 1, below.

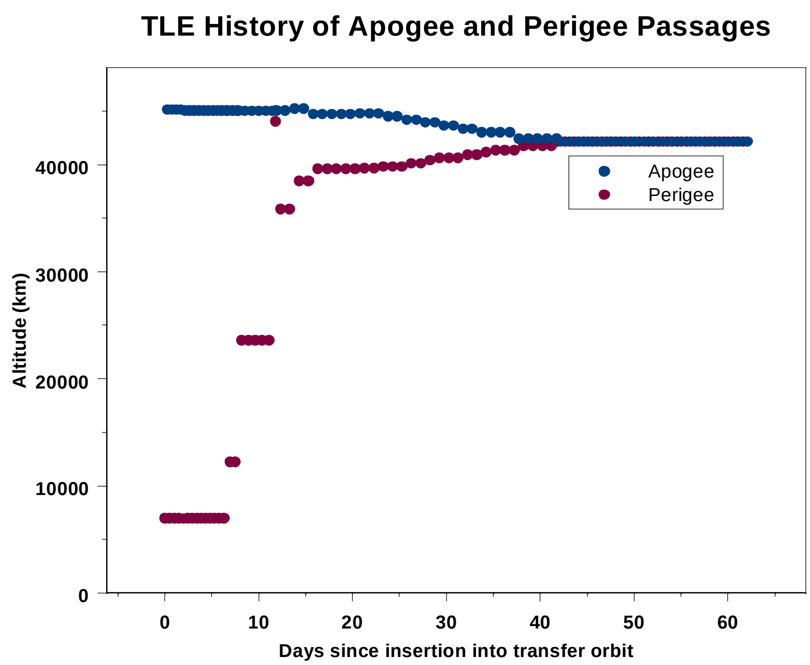

Figure 1 TLE History

[3] Orbit Determination Tool Kit, OD Tool Kit, and ODTK are trademarks of Analytical Graphics, Inc.

Date	Time UTCG	Perigee(km)
19-Jul-04	11:24:55 PM	6956
20-Jul-04	11:00:56 AM	6956
20-Jul-04	10:36:57 PM	6958
21-Jul-04	10:12:58 AM	6959
21-Jul-04	9:48:59 PM	6960
22-Jul-04	9:25:01 AM	6961
22-Jul-04	9:01:02 PM	6962
23-Jul-04	8:37:04 AM	6963
23-Jul-04	8:13:05 PM	6964
24-Jul-04	7:49:06 AM	6964
24-Jul-04	11:59:27 PM	12236
25-Jul-04	12:37:12 PM	25727
26-Jul-04	4:56:24 AM	23612
26-Jul-04	10:31:09 PM	23611
27-Jul-04	4:05:54 PM	23610
28-Jul-04	12:17:28 PM	35635
29-Jul-04	11:10:53 AM	35948
30-Jul-04	9:21:00 AM	35868
31-Jul-04	7:51:02 AM	35871
1-Aug-04	8:17:45 AM	38474

Table 1 Perigee Altitude due to Bipropellant Thrusting

ANALYSIS TESTBED

OD Tool Kit and STK[4] are used to plan and simulate all thrusting events, schedule tracking passes, simulate tracking data, estimate various parameters, including orbit, bias, and thrust states, compare the solutions to "truth", and generate the graphs presented in this report.

The TLE history indicates four or five maneuvers raising perigee and resulting in a geosynchronous period with substantial apogee-perigee differential and the satellite at 120° W longitude. We emulate the process with three finite burns, centered on apogee, resulting in approximately the same apogee-perigee differential and an orbit at correct longitude.

Simulation of XIPS Circularization maneuvers required writing a force model plug-in for OD Tool Kit to compute an inertially fixed thrust direction that would lower apogee and raise perigee. Thrust direction was recomputed for each of the seven XIPS Circularization burns. Once these thrust directions were established the plug-in was discarded and the usual input controls were used to simulate the maneuvers.

Deviates were applied in all cases to the initial orbit, tracking measurement biases and white noise, transponder bias, and thrust direction and magnitude. Chemical thrusters

[4] Satellite Tool Kit and STK are registered trademarks of Analytical Graphics, Inc.

were assigned 3.0 % time-varying error r.m.s. in magnitude and 0.02° degree error r.m.s. in direction. XIPS thrusters were assigned 0.1 % time-varying error r.m.s. in magnitude and 0.02° degree error r.m.s. in direction, consistent with historical performance for XIPS (Ref 2). Range data error deviates were computed from a bias uncertainty of 5 meters and a noise sigma of 1 meter.

ORBIT DETERMINATION DURING PERIGEE RAISING

Perigee Raising Maneuvers

Our maneuver plan achieves approximately the same altitude and longitude sub-point in three maneuvers (Table 2) as the actual flight achieved in four. Although our plan may be optimistic, it is sufficiently close to the orbit history to characterize the OD problem. We assumed a specific impulse of 350 sec and a thrust magnitude of 490 nt. We started with a mass of 5452 kg.

Start	Stop	Duration
24 Jul 2004 13:00	24 Jul 2004 14:10	70 min
25 Jul 2004 15:46	25 Jul 2004 17:06	80 min
27 Jul 2004 21:10	27 Jul 2004 22:01	51 min

Table 2 Simulated Perigee-Raising Maneuvers

Orbit Determination Using Known Maneuvers

The classical orbit determination method of dealing with large maneuvers is to restart the OD process following the maneuver. However it is possible to continue the orbit determination process through the maneuver, provided that the commanded maneuver is known and tracking is conducted during the maneuver. The OD process is still challenged by the fact that the actual maneuver is different from the commanded maneuver in both magnitude and direction. The motivation for tracking through the maneuver rather than restarting the OD process is to reduce the timeline required to prepare for subsequent maneuvers. The question is: what are the tracking requirements to continuously filter through these three maneuver events?

The following series of tests parametrically varies the number of tracking passes and number of tracking stations, beginning with an absurdly high tracking rate and systematically decreasing the rate. Every case has a radar site tracking the satellite continuously during the maneuvers. In every case the filter state space includes 6 orbit parameters, a solar pressure parameter, 9 thrust states (3 components x 3 events), and tracking station bias states). This description refers to other tracking passes.

Case ID	No. Stn	No. Obs	No. Tracks	Description
ALL	5	24,642	22	All stations horizon-to-horizon
4 STN	4	566	31	One 10 min track per pass per station
3 STN	3	479	23	One 10 min track per pass per station
2 STN	2	490	24	One 10 min track per pass per station
2 STNB	2	513	17	One 10 min track per orbit, add 30 minutes extended tracking after maneuvers
Optimized	3	513	17	Optimize 10 min track schedule using covariance analysis

Figure 2 illustrates how accuracy can degrade as tasking levels are reduced. These values are derived from the orbit error covariance generated by the OD Tool Kit filter. The largest spike is due to a gap in tracking following the maneuver.

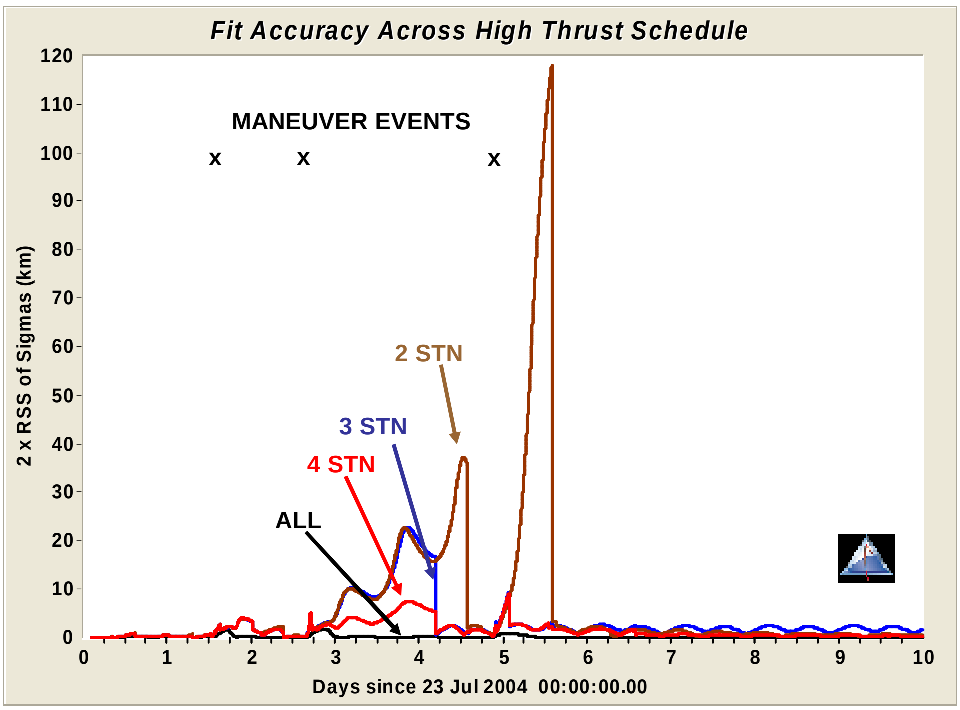

Figure 2 OD Sensitivity to Tracking Across Maneuvers

Figure 3 examines how a few simple changes in the tracking schedule can substantially reduce the orbit error. The first improvement (2 STNB) simply extends the dedicated tracking during the maneuver for an additional 30 minutes. That additional tracking is sufficient to remove 100 km of orbit error. The second improvement (OPTIMIZED) is to change the schedule of tracking to sample the orbit while the covariance is growing following the maneuver. As a result the orbit accuracy can be maintained within 10 km throughout the 10-day scenario.

It is quite likely that the tracking levels can be further reduced without impacting accuracy.

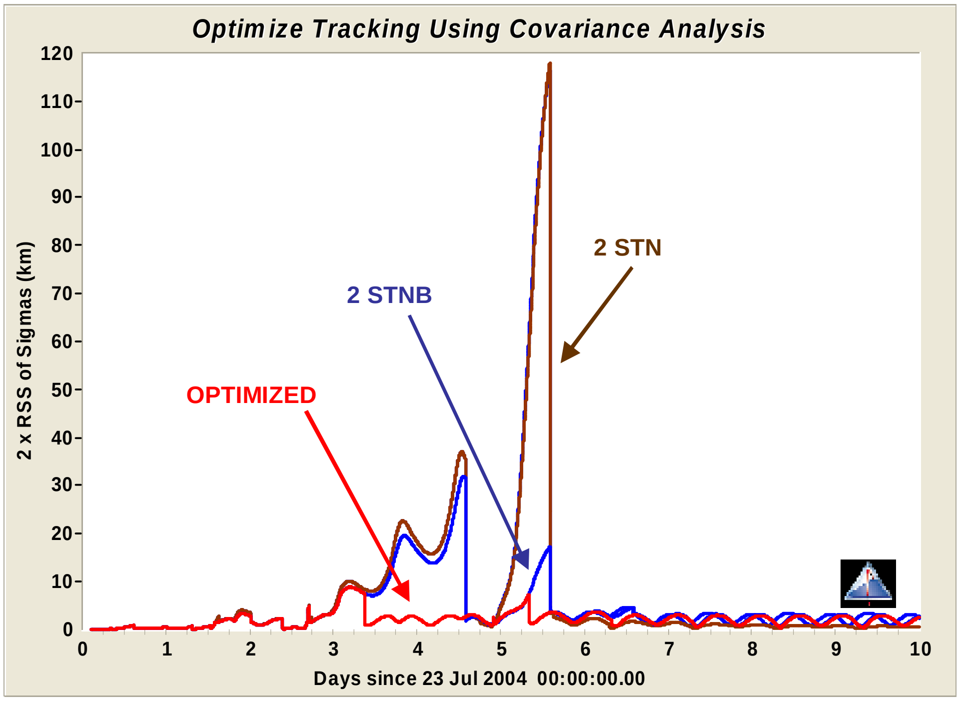

Figure 3 Importance of Optimizing Tracking

The lessons learned in this analysis are:

1. A sequential filter can provide very good orbit accuracy during high thrust finite maneuvers, provided that the commanded maneuver is known and dedicated tracking is available during the maneuver.
2. Tracking resources can be minimized during non-maneuver timelines without sacrificing accuracy if the tracking is scheduled to minimize the orbit error covariance (which assumes the OD process produces a realistic covariance).

ORBIT DETERMINATION DURING XIPS CIRCULARIZATION

XIPS Circularization Maneuvers

The XIPS circularization schedule provided by Tony Grise (Ref 2) of Telesat Canada is as follows:

Start (GMT)	Stop (GMT)	Duration
9 Aug 22:27	11 Aug 10:16	35.8 hrs
11 Aug 13:16	13 Aug 10:10	44.9 hrs
13 Aug 13:10	17 Aug 13:54	96.7 hrs
18 Aug 14:58	22 Aug 10:07	91.2 hrs
22 Aug 13:07	25 Aug 00:21	59.2 hrs
25 Aug 03:49	26 Aug 14:03	34.2 hrs
27 Aug 10:20	27 Aug 17:20	7.0 hrs

Table 3 ANIK-F2 Actual XIPS Circularization Maneuvers

The satellite was placed in a fixed inertial attitude and a pair of XIPS thrusters were used to provide a net thrust magnitude of 0.333 NT. Satellite mass at the start of the burn was 3820 kg, so initial acceleration magnitude was 0.000087 m/sec^2. Grise also provided satellite position and velocity before the first maneuver and four other vectors at various times during the sequence. After the last maneuver, apogee-perigee separation was 11 km. (Grise did not provide the actual thrust vectors.)

Our reconstruction started with the same initial conditions and used all of the same start and stop times, however our simulation did not require 7 hours for the last burn. We achieved an 11 km apogee-perigee separation after 30 minutes into the seventh burn. Our simulation is not an exact reconstruction, but is within 2% of the total burn time, which is sufficiently close to the actual maneuver sequence to be representative of this class of thrusting problems.

Orbit Fit Using Known Maneuver

We classify “radar” as active radio frequency trackers, whether or not there is an active transponder, to include satellite operator tracking (SGLS) and skin-track (SSN) radars. In our analysis we use AFSCN locations as a good quality generic ranging system, with white noise of 0.5 meters noise and an unknown (slowly) time-varying bias of 5 m one sigma.

OD Tool Kit was exercised with 6 states for the orbit, 21 states for thrust errors A(7 events x 3 components per event, a solar pressure coefficient, and tracking bias states.

Assume that the maneuver schedule is known and the uncertainty between the schedule and the actual maneuvers is 0.1 % in thrust magnitude and 0.05 deg in direction. Then the question is how well various tracking systems and various tracking schedules will perform in the orbit determination process. The following cases are evaluated:

Legend	Resources
Continuous	HULA, COOK, BOSS dedicated to the mission 24 hours x 20 days (baseline)
8 Hr x 3	HULA, COOK, BOSS 3 tracks each per day equally spaced in time
18 Hr x 3	HULA, COOK, BOSS 1 tracks each per 18 hours, equally spaced in time
24 Hr x 3	HULA, COOK, BOSS 1 tracks each per 24 hours, equally spaced in time
COOK Only	COOK 3 tracks each per day
BOSS Only	BOSS 3 tracks each per day
HULA Only	HULA 3 tracks each per day

Table 4 Parametric Tracking for XIPS Circularization

Except for continuous coverage cases, all tracks are 10 minutes in duration. The continuous tracking case is only intended to provide an asymptotic limit to achievable accuracy against which all other cases can be measured.

Given the planned maneuver and continuous coverage from 3 radars, it is possible to maintain a continual orbit fit accuracy on the order of 40 m during periods of continuous thrust. For all practical purposes this is the same as the achievable orbit accuracy for a satellite in free-fall. Figure 4 depicts the degradation in accuracy as the three trackers provide fewer tracks per day. The case where each tracker revisits the target every 8 hours is almost as accurate as the continuous track case. Backing off to one track every 18 or 24 hours generates more error during thrusting, but maintains orbit error (RSS of sigmas) within 250 meter. We conclude that multiple radars with good geometric coverage (see Figure 6) can give very good fit accuracy when the planned maneuver is provided.

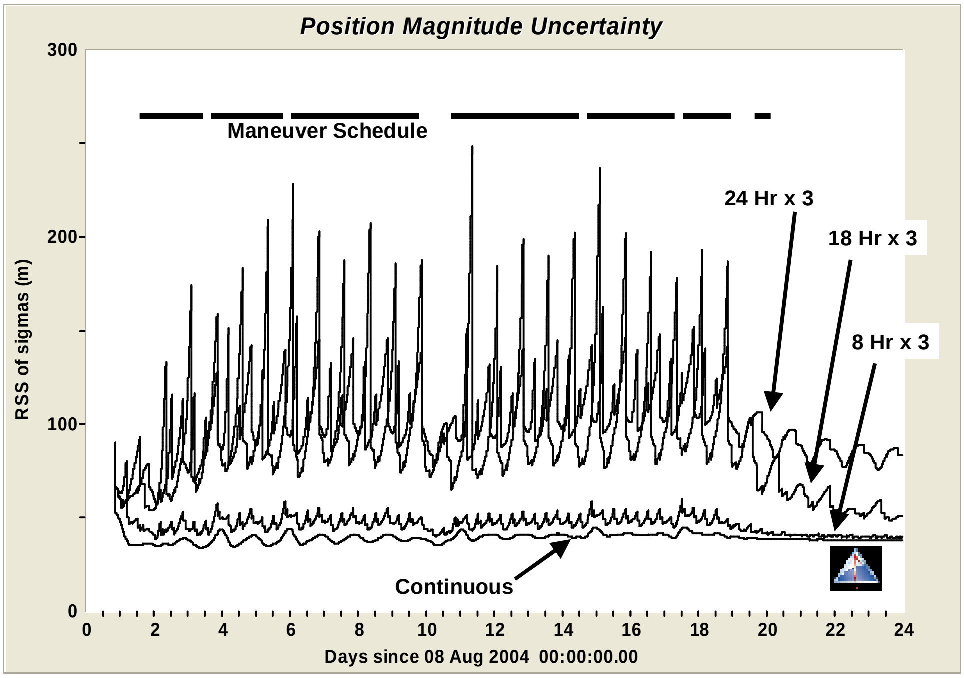

Figure 4 Orbit Fit Accuracy with 3 Radars

In Figure 5 the performance of single site tracking is added, with each radar site providing 3 tracks per day. The relative accuracy for the three sites follows the general rules of thumb for tracking GEO satellites; the best performance is from BOSS, which has the greatest displacement in both latitude and longitude from ANIK, while the poorest performance is for COOK, with classical observability issues at the same longitude as ANIK (see Figure 6).

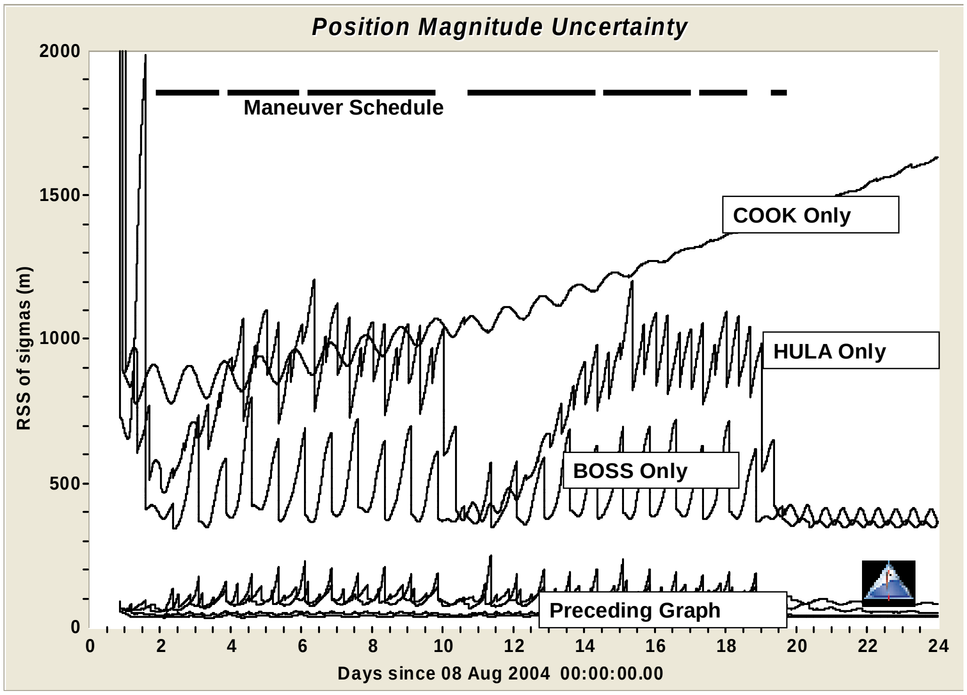

Figure 5 Orbit Fit Accuracy For Single Radar Sites

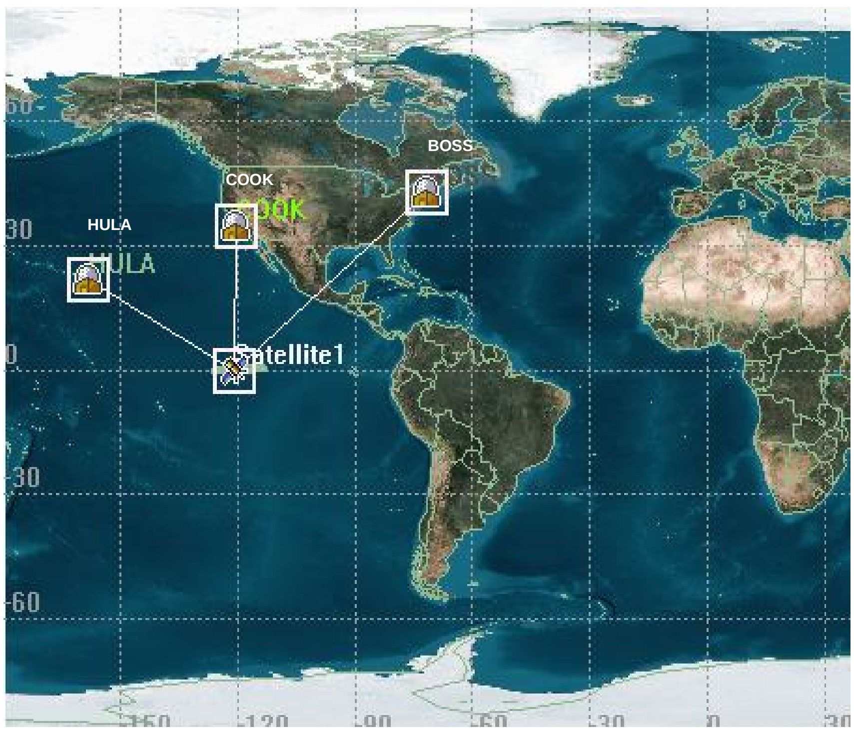

Figure 6 Tracking Geometry for Radar

Orbit Prediction for Known Maneuver

Given the planned maneuver it is possible to use minimal tracking to support space safety analysis. The following predictions (Figure 7) reflect two cases (Table 1), with tracking terminated in the midst of a long maneuver and a 5-day prediction is generated. Whether there are 9 total tracks per day or 3 total tracks per day the prediction error is comparable.

Legend	Resources
8 Hr x 3	HULA, COOK, BOSS 3 tracks each per day equally spaced in time
24 Hr x 3	HULA, COOK, BOSS 1 tracks each per 24 hours, equally spaced in time

Table 5 Parametric Tracking Schedules

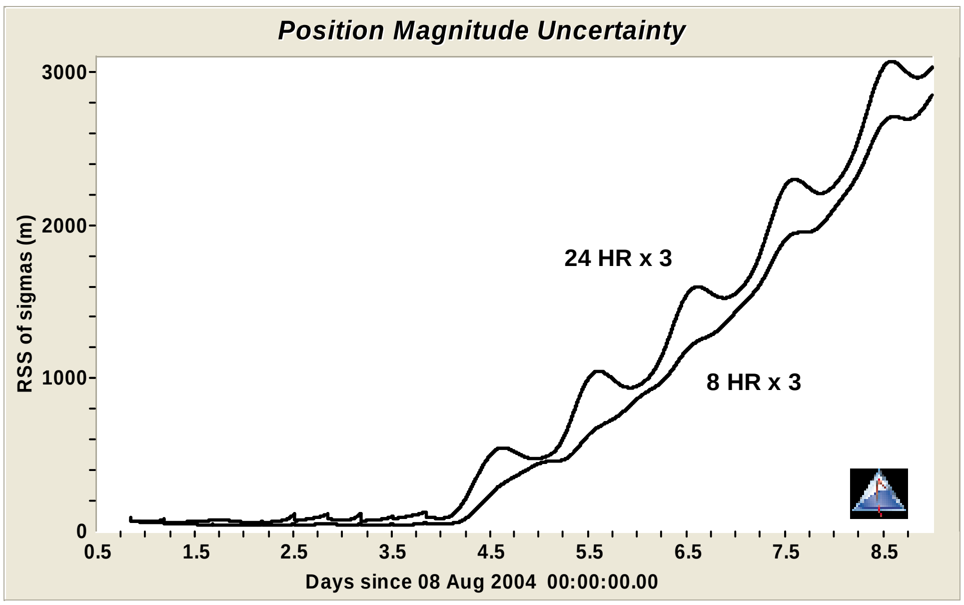

Figure 7 Prediction Error for Known Maneuvers

Orbit Accuracy for Unknown Maneuver

Detecting the Unknown Maneuver

If the satellite operator does not provide predicted maneuver information, then detection of a maneuver event is via large measurement residuals. The following residuals are due to processing tracking data (for “8 Hr x 3” case) through the first maneuver and well into the second maneuver, all without knowledge of the operator planned event:

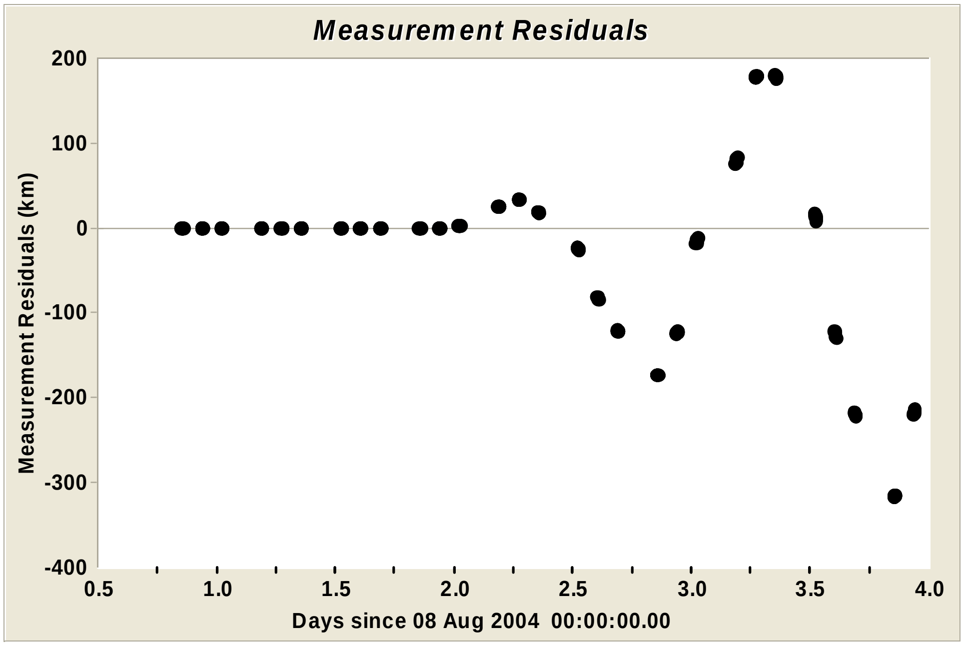

Figure 8 Residuals Indicating a Maneuver Event

Given detection of a maneuver event, an orbit analyst would seek to model the maneuver or at least fit the tracking data to support his mission. A classic approach is to force acceptance of all residuals, which would allow the "orbit" to "fit" the data, but would provide no information to support predictions. If the thrust magnitude and direction could be deciphered, then prediction accuracy would improve dramatically. The following is an engineering approach to that problem.

Solving for the Unknown Maneuver

In OD Tool Kit there is a capability to specify two inputs, one being the residual rejection criteria and the other a level of random process noise, introduced as a small delta-V uncertainty in each thrust direction. If we force acceptance of all tracking data and experiment with the random process noise magnitude we can bound the thrust magnitude. Then we can take advantage of the capability to estimate thrust magnitude and direction. The process is as follows:

1. Force acceptance of all residuals and postulate a small random process noise
2. Inspect the magnitude of the post-fit residuals
3. Inspect the ratio of residuals to the predicted residual root variance
4. Iterate on magnitude of random process noise and repeat steps 1,2,&3
5. The "best" setting for random process noise results in the smallest residuals and most residuals between $\pm$ 3 sigma.
6. The "best" random noise setting bounds the likely thrust level. Use this information to add thruster states to state space and solve for thrust direction and magnitude
7. Use the solve-for thrust values to improve prediction accuracy

8. Use estimated thrust parameters to determine most likely thrust levels and thrust start/stop times.

Brute Force Fit to Tracking Data

As an example, consider the ANIK-F2 XIPS circularization maneuver. Use a filter state space with 6 orbit states, a solar pressure coefficient state, and measurement bias states, but no thrust states. Forcing acceptance of all residuals, with zero random process noise yields the residuals on the order of 300 km error (see Figure 9). (The very large residuals are the first points in each track, and due to forced acceptance, the remaining residuals in the track appear to be near zero on this scale.)

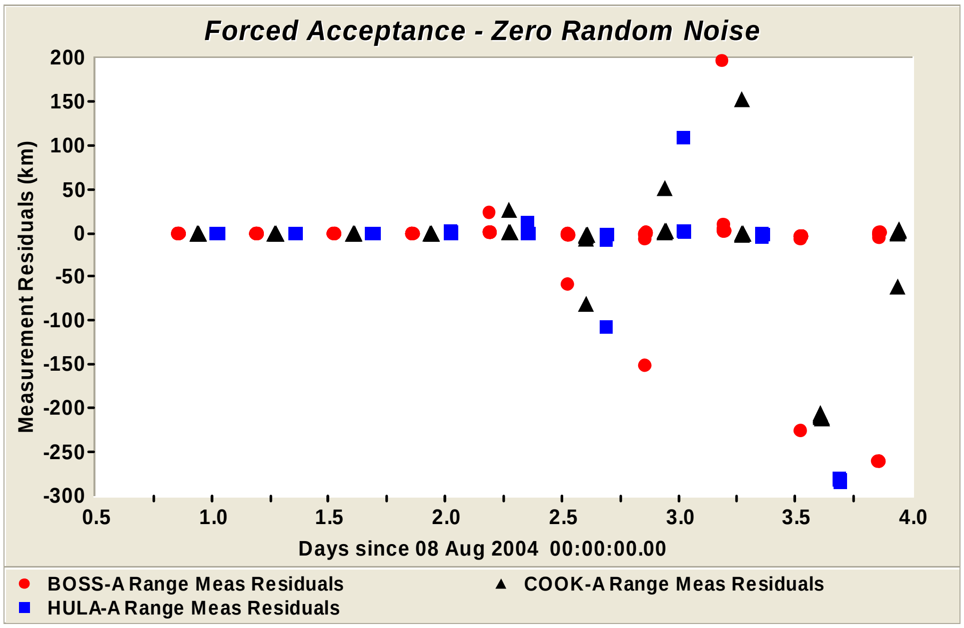

Figure 9 Residuals due to Forced Acceptance

Zero random noise results in enormous orbit errors (see Figure 10). Recall that tracking in this case stops at Aug 12 00:00:00, so the last 5 days reflect the orbit prediction accuracy. Note that even though the tracking data is force-fit, the orbit error during the fit is quite large (up to 2000 km), and prediction error grows from the time of maneuver and not from the end of the tracking data.

Next we search for a process noise level that provides a reasonable ratio of residual to covariance. The state space remains unchanged from the forced fit approach.

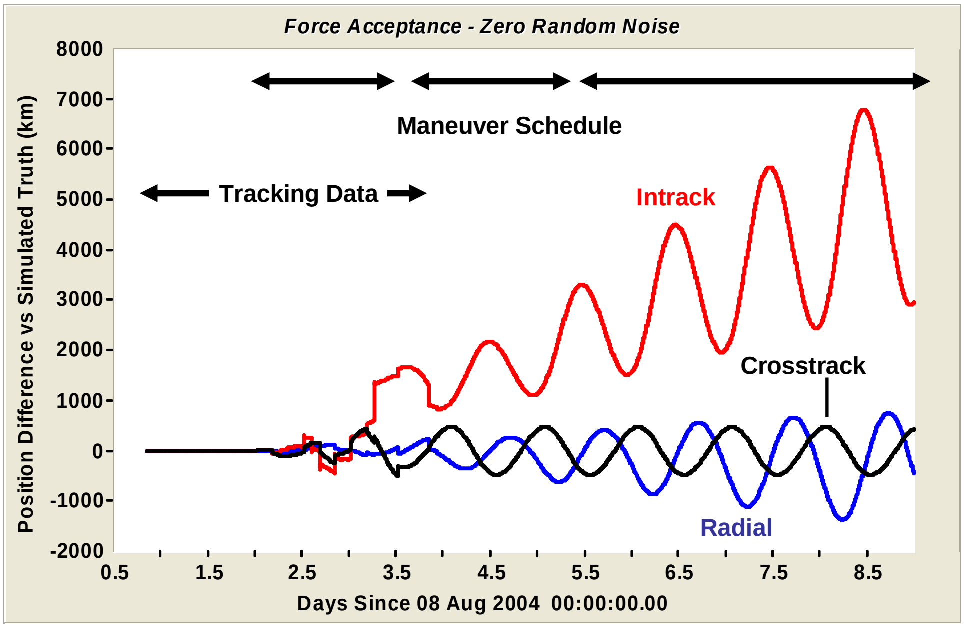

Figure 10 Orbit Error due to Forced Acceptance

Omitting the details of an iterative search, we found that setting the random process noise to 0.6 cm/sec in each component, to be applied at one minute intervals generates much smaller residuals (Figure 11) better prediction accuracy (Figure 12) and a better orbit accuracy through the end of the tracking data (Figure 13). We also found that the crosstrack random process noise could be set to zero without significant degradation, implying that there is negligible crosstrack thrusting.

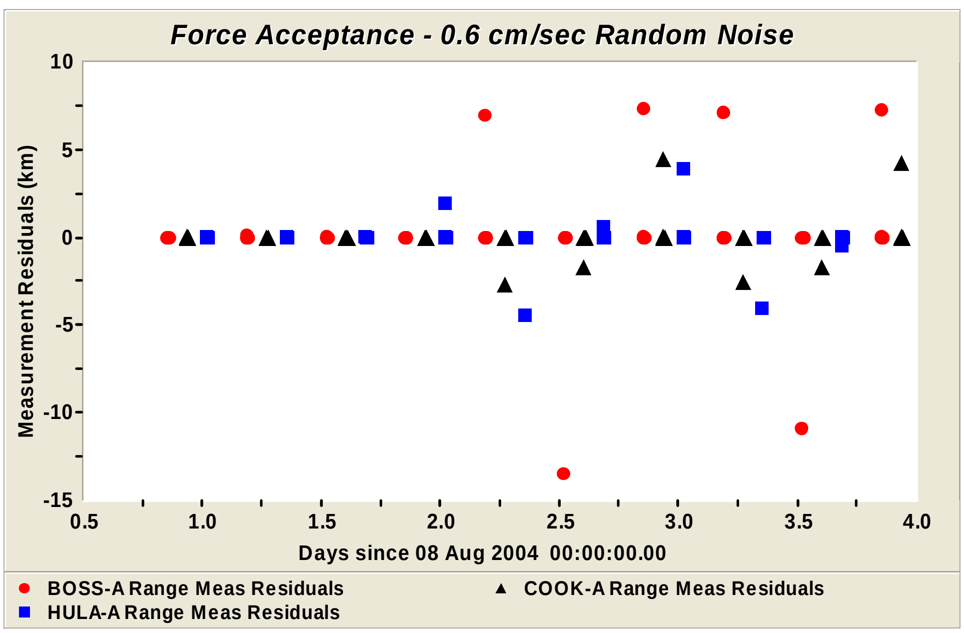

Figure 11 Residuals using Random Noise

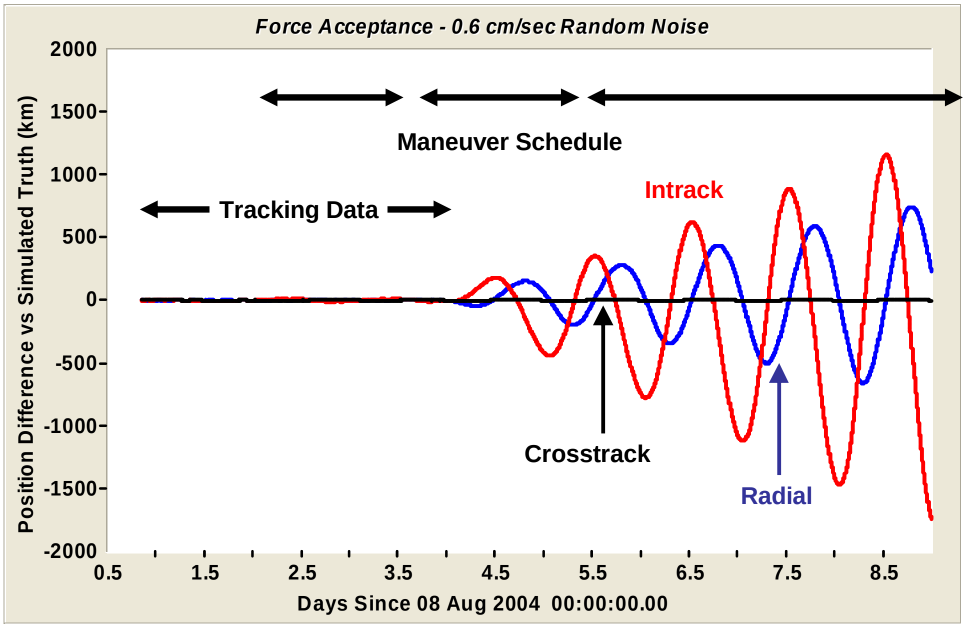

Figure 12 Orbit Error using Random Noise

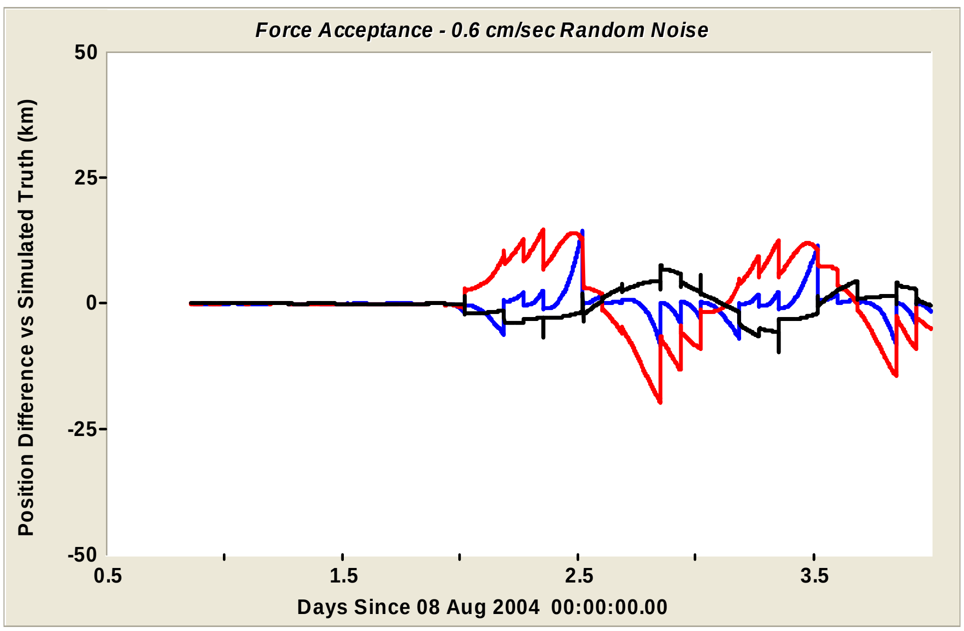

Figure 13 Orbit Error using Random Noise - Over Fit Span

Estimating Thrust Components

The next step is to use the filter to solve for the thrust. Given that a random process noise of 0.6 cm/sec resulted in a good fit to the data, the thrust acceleration uncertainty should be less than 0.01 cm/sec^2. Since there is apparently negligible thrusting in the crosstrack direction (or Z direction for inertially fixed thrust directions), we only estimate two in-plane thrust components. Three attempts are made to solve for thrust components, solving for values in Gaussian, Frenet, and inertial coordinates in addition to the states for orbit, solar pressure and tracking biases. The estimated values appear relatively constant in inertial coordinates and, a pleasant surprise, the estimate goes abruptly to zero approximately when the actual thrust goes to zero (Figure 14). Recall that the simulated initial thrust acceleration magnitude was 0.000087 m/sec^2 and note that the two estimated components are -0.000065 m/sec^2 and -0.000057 m/sec^2, for a magnitude of 0.000086 m/sec^2.

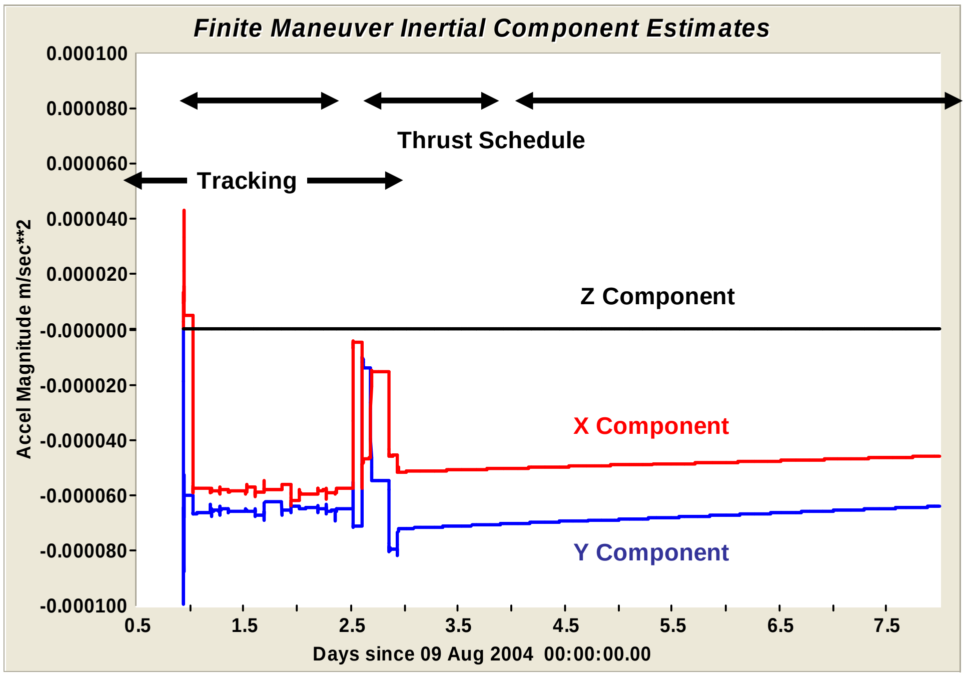

Figure 14 Estimated and Predicted Thrust Components

Having estimated thrust components, the residuals are much smaller (Figure 15) and the fit accuracy (Figure 17) and prediction accuracy (Figure 16) are improved.

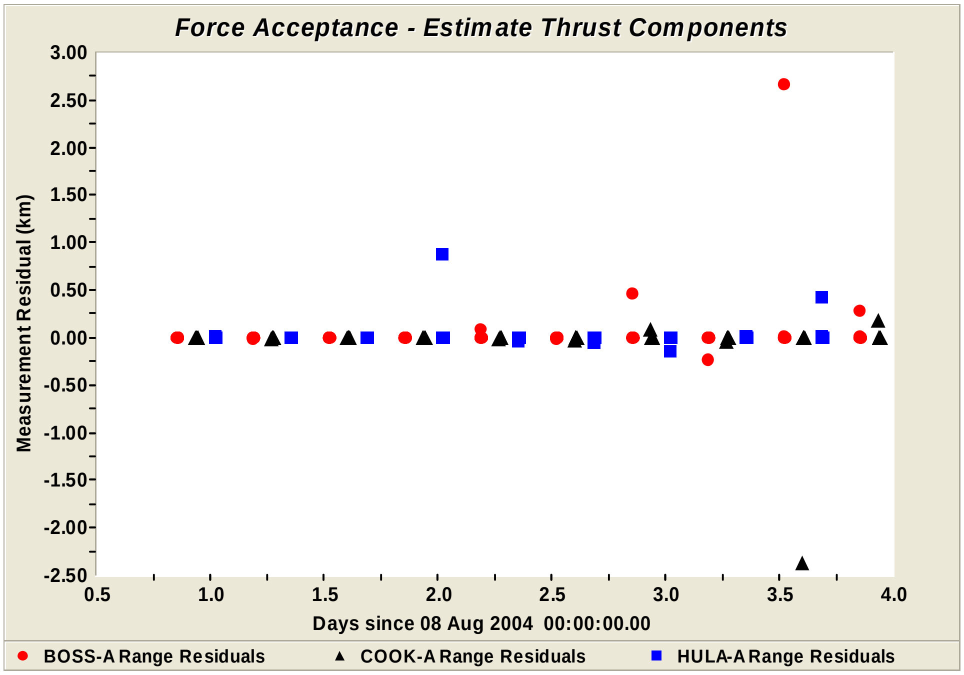

Figure 15 Residuals due to Estimating Thruster Components

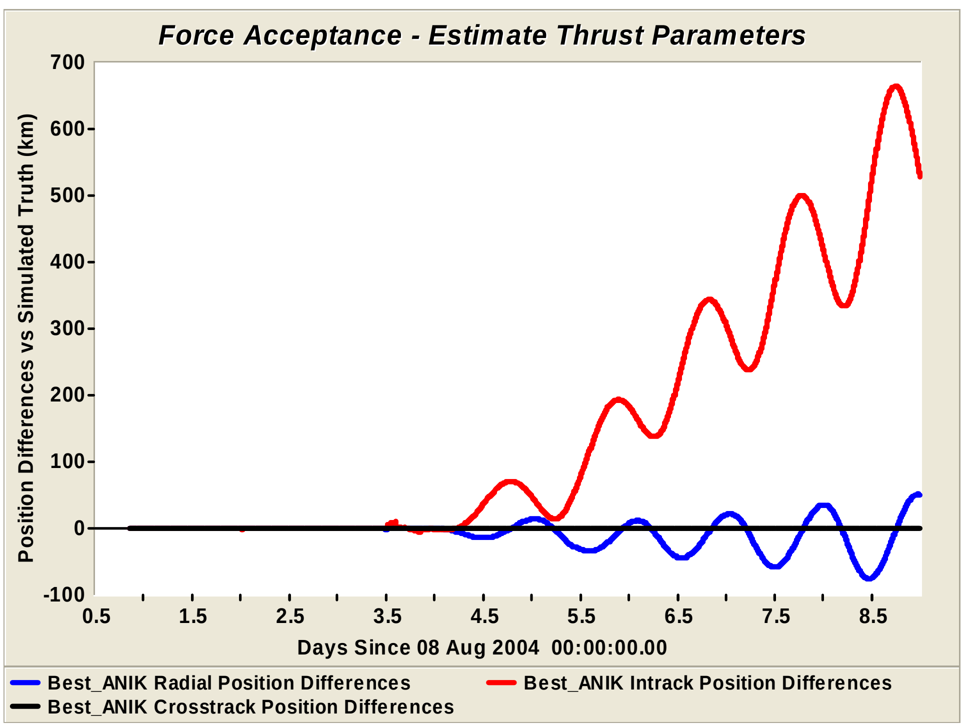

Figure 16 Orbit Error during Fit and Prediction

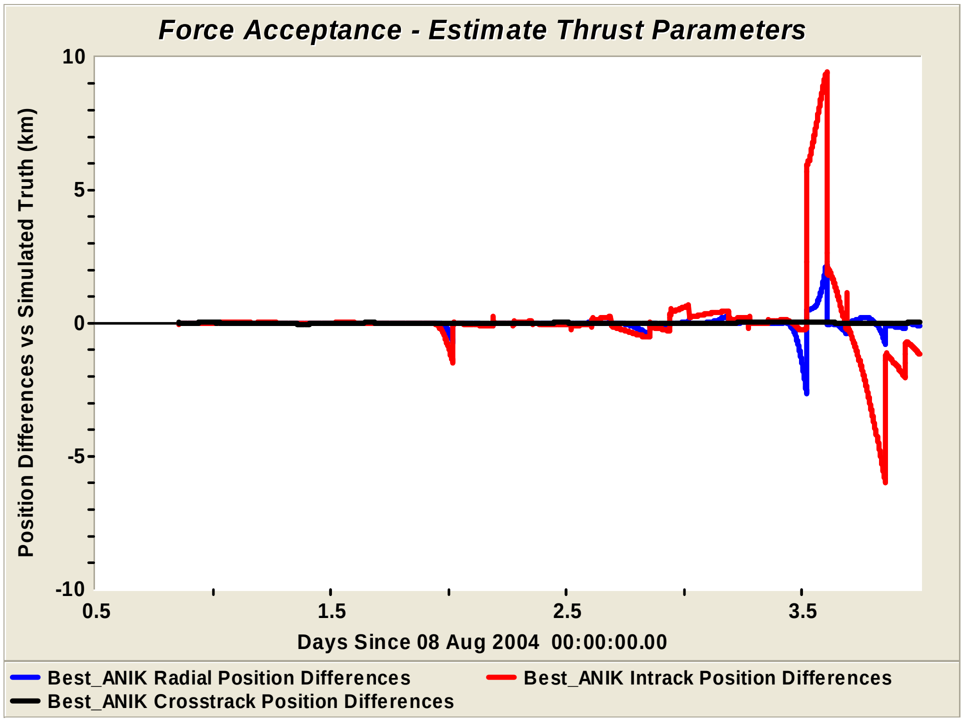

Figure 17 Details of Fit Accuracy

For a final analysis we processed through the entire 18 days of thrusting. Since the actual thrust levels are not known and various thrust stop and start times are not known, it is still necessary to force acceptance of the tracking data. Filter state space includes orbit states, solar pressure states, bias states and three components of thrust. In Figure 18 the large residuals are on the order of hundreds of meters, whereas they were previously measured in kilometers. As before, the large residuals are all the first points in their respective tracks, and the rest of the residuals in each track are small. Orbit accuracy, measured versus the simulated truth (Figure 19) reflects the fact that we are sequentially estimating a single constant thrust, and when there are breaks in thrusting our orbit errors grow. Where thrusting is estimated and does occur, orbit errors are substantially less than one kilometer. The filter does detect the breaks in thrusting (Figure 20), which may allow better accuracy over the fit to be obtained through further iteration.

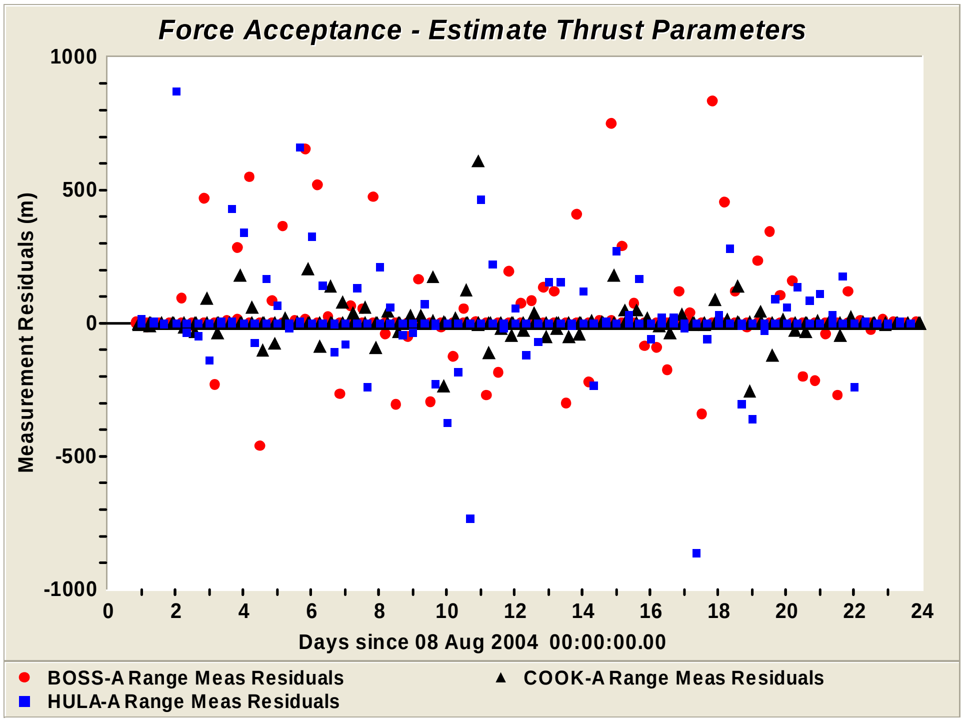

Figure 18 Residuals when Estimating Thrust

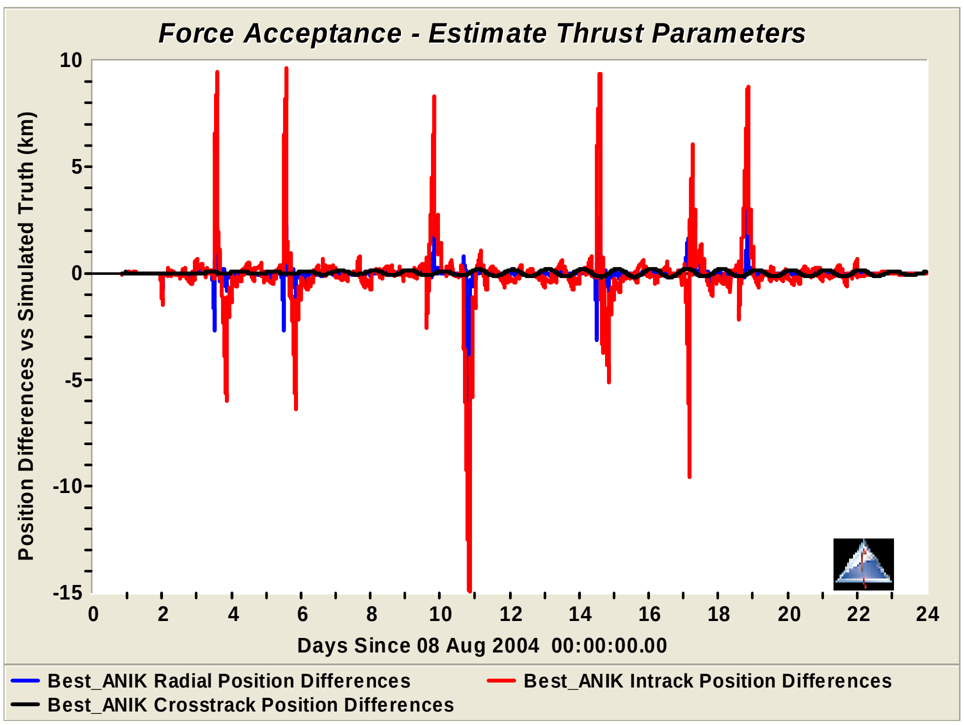

Figure 19 Fit Error over Entire XIPS Circularization

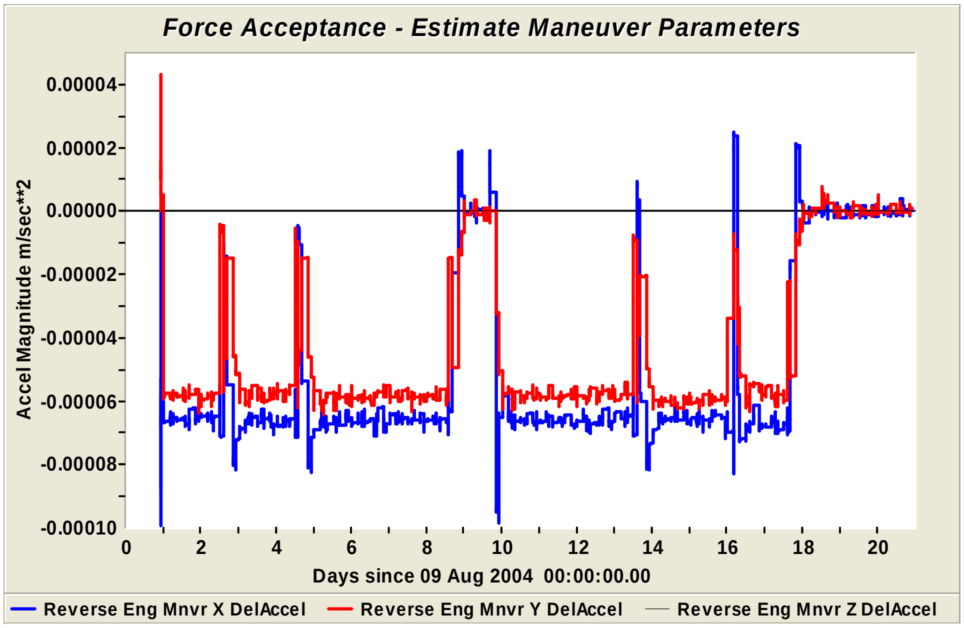

Figure 20 Filter Estimates of Thrust Components

The key results for scenarios of this type are

1. If the commanded maneuver is known a sequential filter can be used to obtain post-fit orbit accuracies (Figure 7) on the order of a 100 meters and 5-day predictions with errors less than 6 km (r.s.s.).
2. If the commanded maneuvers are not known, the sequential filter can be used in an iterative fashion to force acceptance of residuals, to find a bounding random process noise, to determine a frame (Gaussian or Inertial) where thrust is "constant", to form a refined fit by estimating thrust components, and to identify discontinuities in thrust.

CONCLUDING REMARKS

The sequential filter, as employed in the Orbit Determination Tool Kit, is highly capable in the presence of high thrust and low thrust maneuvers. It makes it possible to track through multiple hour-long perigee-raising thrusting events and maintain orbit accuracy better than 10 km. Since the filter provides a realistic covariance it is possible to optimize the tracking schedule to minimize the tracking load while improving orbit accuracy. This capability should allow the satellite operator to minimize the time between successive maneuvers, and to provide a high quality ephemeris for use in space safety analyses.

The sequential filter has even more capabilities when long duration low thrust maneuvers are employed. Satellite operators should be able to maintain orbit knowledge to within 100 meters throughout all maneuver events. If the maneuver parameters are unknown, it is possible for an analyst to solve for thrust parameters and reverse engineer the actual events, while maintaining orbit accuracy to better than 1 km through such events.

ACKNOWLEDGEMENTS

Tony Grise of Telesat Canada has been extremely helpful in providing insight into ANIK-F2 operations under Telesat control. Tony has been using a Kalman Filter of his own design for more than two decades and claims great stability in ANIK-F2 operations. Tony has been advocating that more satellite programs use this approach for OD, particularly for GEO satellites where thrusting is a fact of life. Tony has also been predicting that low thrust technology is going to be the standard for GEO operations for the foreseeable future.

REFERENCES

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