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COLLISION AVOIDANCE MANEUVER PLANNING TOOL

SALVATORE ALFANO

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COLLISION AVOIDANCE MANEUVER PLANNING TOOL

Salvatore Alfano*

Satellite Collision Avoidance Maneuver (CAM) planning must take into account many factors. The risk of collision must be weighed against the risk of performing a maneuver. Fuel usage may shorten the satellite's operational lifetime. Mission degradation must also be considered if the satellite departs from its designed orbit. Return to nominal orbit may be an additional factor. This work describes the development and use of a MATLAB analysis tool that can perform parametric studies of single-axis and dual-axes maneuvers. The tool reads the object pair's positions, velocities, covariances, and physical sizes from Satellite Tool Kit (STK) and allows the user to modify the covariances and physical object sizes. MATLAB then creates collision-probability contour plots for a range of user-specified maneuver times and velocity changes. To reduce risk to an acceptable level, the user selects from a family of possible maneuvers. The candidate maneuver is then fed back into STK where further analysis can be performed to address other maneuver concerns.

IINTRODUCTION

There is a growing body of work that addresses collision probability computations for neighboring space objects[1-12] and some literature that examines the associated accuracy requirements[13-14]. Typically, a determination is made when a secondary object transgresses a user-defined safety zone. The positional uncertainties are represented by three-dimensional Gaussian probability densities. At a given time, these densities take the form of covariance matrices and can be obtained from the owner-operators or independent surveillance sources such as the US Space Object Catalog (Special Perturbations). Positions and covariances are propagated to the time of closest approach, relative motion is assumed nearly linear[12], and the positional covariances are assumed constant and uncorrelated for the encounter. Visually, the encounter region looks like a straight tube (collision tube) in three-dimensional space.

* Salvatore Alfano, Technical Program Manager, Center for Space Standards and Innovation
7150 Campus Drive, Suite 260, Colorado Springs, CO 80920

Space object collision avoidance (COLA) is usually conducted with the objects modeled as spheres. The combined covariance size, shape, and orientation are coupled with physical object sizes to determine collision potential. At the point of closest approach, each object's positional uncertainty is combined and their radii summed. For linear relative motion, the resultant is projected onto a plane perpendicular to the relative velocity where the collision probability is calculated[9]. The projection reduces the probability formulation to a double integral that can be further simplified to a single integral through use of error functions.

If the collision probability exceeds a user-defined threshold, there are several courses of action that can be taken by the primary satellite operator. If the secondary satellite can be actively controlled, its owner/operator should be contacted to determine if a maneuver is scheduled prior to the Time of Closest Approach (TCA). If time permits, better and/or more-recent surveillance data should be obtained to reassess the collision probability. The tool described in this paper determines the effectiveness of performing a range of Collision Avoidance Maneuvers. A maneuver might involve simply changing the primary satellite's attitude to minimize its cross-section in the encounter plane thereby reducing collision probability. As a final resort, an evasive maneuver (orbit change) might be warranted.

A MATLAB tool has been developed to assess changes in cross-sectional area as well as the timing and velocity change for evasive maneuvering. This analysis tool performs parametric studies of single-axis and dual-axes maneuvers. The tool is designed to evaluate the conjuncting objects by reading in their positions, velocities, covariances and physical sizes from the Satellite Tool Kit (STK) Advanced Close Approach Tool (AdvCAT) at TCA. The user is given the ability to modify the object sizes and covariances to determine if an attitude change and/or more accurate positional data is/are sufficient to reduce collision probability to an acceptable level. Given a maximum permissible velocity change (ΔV) and a range of permissible maneuver times, MATLAB maps out contours relating ΔV and maneuver time to collision probability for single-axis maneuvering. By picking a specific time, similar contours are produced for dual-axes maneuvering. The chosen maneuver parameters can then be fed back into STK to examine other concerns such as mission degradation or inadvertently increasing collision probability with respect to a different satellite.

COLLISION PROBABILITY COMPUTATION REVIEW

There are many assumptions that reduce the problem's complexity. The physical objects are treated as spheres, thus eliminating the need for attitude information (Fig. 1). Their relative motion is considered linear for the encounter by assuming the effect of relative acceleration is dwarfed by that of the velocity. The positional errors are assumed to be zero-mean, Gaussian, uncorrelated, and constant for the encounter. The relative velocity at the point of closest approach is deemed sufficiently large to ensure a brief encounter time and static covariance. The encounter region is defined when one object is within a standard deviation (σ) combined covariance ellipsoid shell scaled by a factor of n. This user-defined, three-dimensional, n-σ shell is centered on the primary object; n is typically in the range of 3 to 8 to accommodate conjunction possibilities ranging from 97.070911% to 99.999999%.

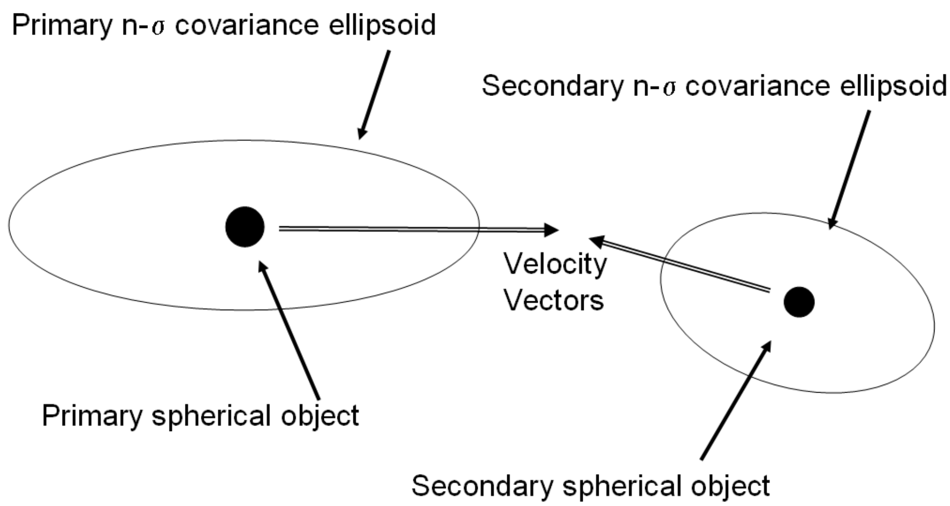

Fig. 1 Conjunction Encounter Geometry

Because the covariances are expected to be uncorrelated, they are simply summed to form one, large, combined, covariance ellipsoid that is centered at the primary object (Fig. 2). The secondary object passes quickly through this ellipsoid creating a tube-shaped path. A conjunction occurs if the secondary sphere touches the primary sphere, i.e. when the distance between the two projected object centers is less than the sum of their radii. The radius of this collision tube is enlarged to accommodate all possibilities of the secondary touching the primary by combining the radii of both objects.

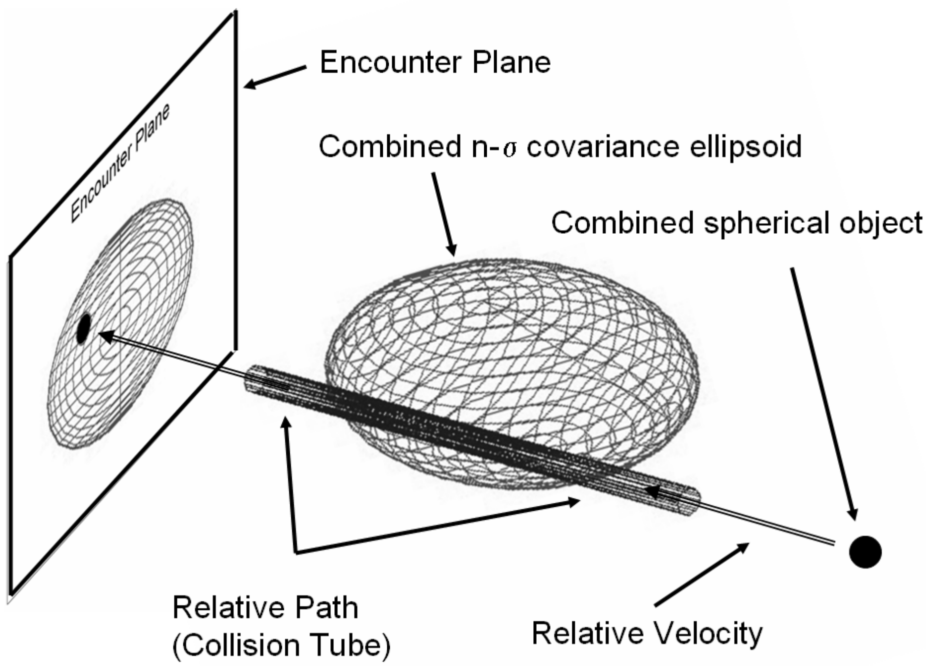

Fig. 2 Conjunction Encounter Visualization and Reduction

A plane perpendicular to the relative velocity vector is formed and the combined object and covariance ellipsoid are projected onto this encounter plane. As stated previously, the encounter region is defined by an n-σ shell determined by the user to sufficiently account for conjunction possibilities. Within that shell the tube is straight and rapidly traversed, allowing a decoupling of the dimension associated with the tube path (i.e. relative velocity). The tube becomes a circle on the projected encounter plane. Likewise, the covariance ellipsoid becomes an ellipse (Fig. 3).

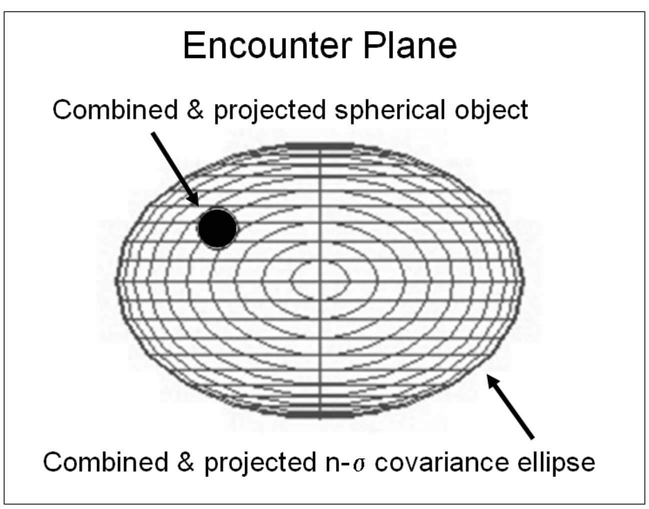

Fig. 3 Projection onto the Encounter Plane

The relative velocity vector (decoupled dimension) is associated with the Time of Closest Approach (TCA). The conjunction assessment here is concerned with cumulative probability over the time it takes to span the n-σ shell, not an instantaneous probability at a specific time within the shell. Along this dimension, integration of the probability density across the shell produces a number very near unity, meaning the close approach will occur at some time within the shell with near absolute certainty. Thus the cumulative collision probability is reduced to a two-dimensional problem in the encounter plane that is then multiplied by the decoupled dimension's probability. By rounding the latter probability to one, it is eliminated from further calculations.

The resulting two-dimensional probability equation in the encounter plane is given as

$$P = \frac{1}{2\cdot\pi\cdot\sigma x\cdot\sigma y}\cdot\int_{-OBJ}^{OBJ}\int_{-\sqrt{OBJ^2-(x)^2}}^{\sqrt{OBJ^2-(x)^2}} \exp\left[\left(\frac{-1}{2}\right)\cdot\left[\left(\frac{x+xm}{\sigma x}\right)^2+\left(\frac{y+ym}{\sigma y}\right)^2\right]\right] dy\, dx \tag{1}$$

where OBJ is the combined object radius, x lies along the minor axis, y lies along the major axis, xm and ym are the respective components of the projected miss distance, and σx and σy are the corresponding standard deviations. For the formulation that follows, the aspect ratio AR is incorporated as a multiple of the minor axis standard deviation (AR $\geq$ 1) and equation (1) is expressed as

$$P = \frac{1}{2\cdot\pi\cdot\sigma x^2\cdot AR}\cdot\int_{-OBJ}^{OBJ}\int_{-\sqrt{OBJ^2-(x)^2}}^{\sqrt{OBJ^2-(x)^2}} \exp\left[\left(\frac{-1}{2}\right)\cdot\left[\left(\frac{x+xm}{\sigma x}\right)^2+\left(\frac{y+ym}{\sigma x\cdot AR}\right)^2\right]\right] dy\, dx \quad . \tag{2}$$

MAXIMUM PROBABILITY FORMULATION REVIEW

This formulation determines the worst-case conjunction scenario by finding the combined Gaussian probability density that maximizes collision probability. The only parameters required are distance (dist) of closest approach, the radius of the combined object (OBJ), and the ratio of major-to-minor projected covariance ellipse axes (AR). The major axis of the combined covariance ellipse is aligned with the relative position vector (at the point of closest approach) such that it passes through the center of the combined object. The projected, combined object is assumed circular with its probability mass distributed symmetrically about the major axis. This means that only a single axis length needs to be examined to maximize the probability, the other being determined from the aspect ratio.

Clearly, if the combined object footprint contains the covariance ellipsoid center, the minor axis' standard deviation can be chosen to drive the maximum probability to one. For spherical objects this occurs when the predicted miss distance is less than the combined object size (dist < OBJ). This is the limiting case and need not be addressed; it is inferred that a decision maker faced with such a predicted "direct hit" would not need a probability calculation. The method described here only applies when the combined object does not encompass the covariance center (dist $\geq$ OBJ). Given the combined object radius and distance from center, the minor axis size can be determined by maximizing a two-dimensional probability expression. Once determined, the worst-case collision probability is calculated.

In the encounter plane, the xm and ym components are varied as a function of the fixed relative distance (dist) and the angle θ (Fig. 4).

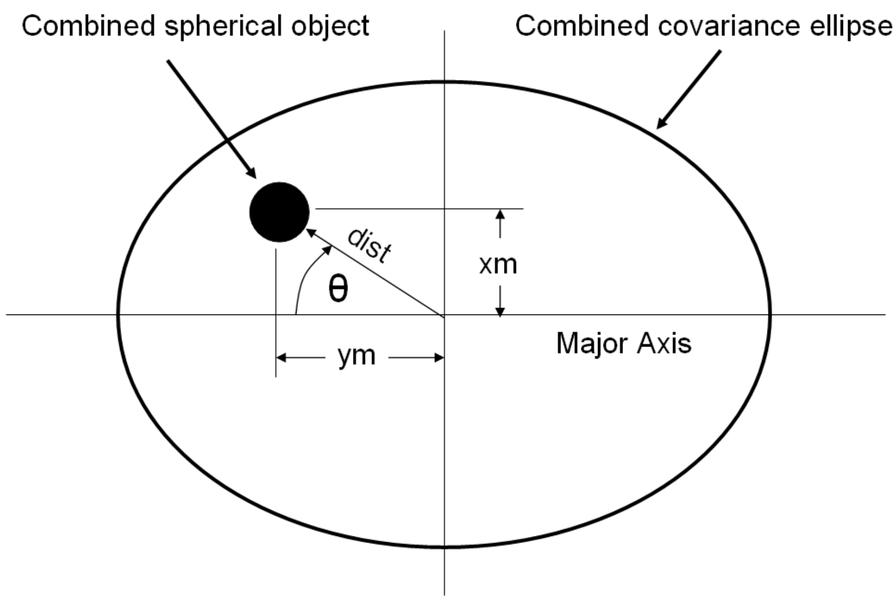

Fig. 4 Projected position relative to θ angle

This is the first step in determining the orientation of the distance vector with respect to the covariance axes to produce the greatest probability. Equation (2) becomes

$$P = \frac{1}{2\cdot\pi\cdot\sigma x^2\cdot AR}\cdot\int_{-OBJ}^{OBJ}\int_{-\sqrt{OBJ^2-(x)^2}}^{\sqrt{OBJ^2-(x)^2}} \exp\left[\left(\frac{-1}{2}\right)\cdot\left[\left(\frac{x + dist\cdot\sin(\theta)}{\sigma x}\right)^2 + \left(\frac{y + dist\cdot\cos(\theta)}{\sigma x\cdot AR}\right)^2\right]\right] dy\, dx \quad .(3)$$

The derivative with respect to θ is then set equal to zero to find the occurrences of maximum probability. The derivative equals zero whenever θ is an integer multiple of $\pi/2$. The probability is at a maximum whenever θ is an integer multiple of π. This means that the maximum probability occurs when the relative distance is along the major axis (xm = 0, ym = dist)

$$P = \frac{1}{2\cdot\pi\cdot\sigma x^2\cdot AR}\cdot\int_{-OBJ}^{OBJ}\int_{-\sqrt{OBJ^2-(x)^2}}^{\sqrt{OBJ^2-(x)^2}} \exp\left[\left(\frac{-1}{2}\right)\cdot\left[\left(\frac{x}{\sigma x}\right)^2 + \left(\frac{y + dist}{\sigma x\cdot AR}\right)^2\right]\right] dy\, dx \quad . \qquad (4)$$

The constant term is then brought outside the integral

$$P = \frac{\exp\left[\left(\frac{-1}{2}\right)\cdot\frac{dist^2}{\sigma x^2\cdot AR^2}\right]}{2\cdot\pi\cdot\sigma x^2\cdot AR}\cdot\int_{-OBJ}^{OBJ}\int_{-\sqrt{OBJ^2-(x)^2}}^{\sqrt{OBJ^2-(x)^2}} \exp\left[\left(\frac{-1}{2}\right)\cdot\left[\left(\frac{x}{\sigma x}\right)^2 + \frac{y^2 + 2\cdot y\cdot dist}{\sigma x^2\cdot AR^2}\right]\right] dy\, dx \qquad (5)$$

To determine the minor axis standard deviation that maximizes the probability, the derivative of the above equation is taken with respect to σx and set to zero. An exact analytical solution does not exist, so a numerical search must be performed or an approximate expression used.

PROBABILITY DILUTION REVIEW

For fixed object sizes and miss distance, the σx that produces the maximum probability (Pmax) defines the dilution region boundary as shown in Fig 5. To the left of the vertical line, greater positional accuracy (smaller σx) decreases collision probability. To the right of the vertical line, lesser positional accuracy (greater σx) also decreases collision probability. Both good and poor quality data can produce the same probability (10^{-6} is given as an example in Fig 5). Although both calculations are mathematically correct, only the former is operationally meaningful.

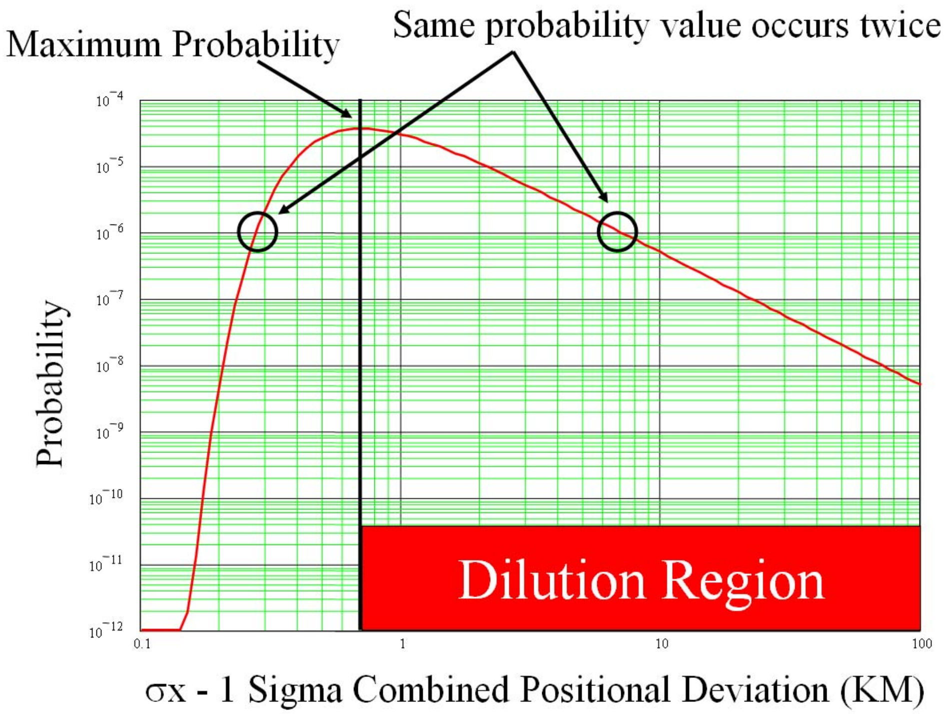

Fig. 5 Dilution Region Defined for Notional Encounter

The probability dilution region is that region where the standard deviation of the combined covariance minor axis (σx) exceeds that which yields Pmax. If operating outside this dilution region (left of vertical line) it is reasonable to associate low probability with low risk. If operating within the dilution region, then the further into this region the uncertainty progresses the more unreasonable it becomes to associate low probability with low risk. If the positional uncertainty is large enough, the resulting low probability may mislead the user into thinking the encounter poses little or no threat. Therefore, a low probability in the dilution region may be the result of poor quality data and should be treated accordingly.

The dilution region boundary should be used to determine the minimum accuracy requirement for a meaningful probability assessment. When calculating true probability from equation (1), the reader is advised to always consider this region. If the positional data is not of sufficient quality to avoid this region, then get better (more accurate) data and reassess the true probability. If better data is not available or still insufficient, consider using the maximum probability as opposed to the true one. This will ensure that a decision maker is not lulled into a false sense of security by a low probability calculation that is specious.

MANEUVER PLANNING METHODOLOGY

Collision avoidance maneuver planning begins by first determining if a collision is likely to occur. This may be determined by an FAA-like advisory service such as SOCRATES[15] (http://celestrak.com/SOCRATES/) or by individual operators running their own collision probability assessments. The user should determine the adequacy of the data to support the calculation by comparing the standard deviation associated with the projected, combined-covariance, minor axis to the value obtained in determining the maximum probability. If operating in the dilution region the user should get better data and reassess the likelihood of the conjunction.

If the risk of collision is considered valid, it must then be weighed against the risk of performing a maneuver. A user should consider how the maneuver may shorten the satellite's operational lifetime by consuming fuel. Mission degradation should be considered if the satellite departs from its designed orbit. Return to nominal orbit and the fuel it takes to get there are additional factors. Thruster failure is a possibility. To assist in collision probability reduction, a MATLAB analysis tool was created that can perform parametric studies of single-axis and dual-axes maneuvers. The tool reads the object pair's positions, velocities, covariances and physical sizes from a Satellite Tool Kit (STK) Advanced Close Approach Tool (AdvCAT) scenario and allows the user to modify the covariances and physical object sizes. MATLAB then maps regions of collision probability as a function of maneuver time and velocity change.

The example that follows is taken from an April 20, 2005, conjunction assessment involving IRIDIUM 80 and a piece of COSMOS 886 debris. Having read in all pertinent data from STK/AdvCAT, the user is given the option of changing each object's radius and/or changing the components of the covariance through the graphical user interface (GUI) as shown in Fig 6.

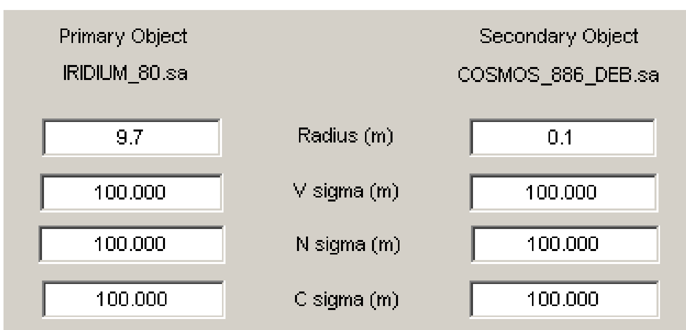

Fig. 6 MATLAB GUI Input Window (Upper Left)

A radius change can be used to examine the reorientation of non-spherical objects when attempting to reduce the collision-tube radius and its subsequent footprint in the encounter plane. Holding all other parameters constant, this action alone might reduce collision probability to an acceptable level. Changing the covariance components would indicate the probability calculation's sensitivity to different positional uncertainties.

Before attempting a one-dimensional parametric analysis, the user must set the upper and lower bounds of maneuver time as indicated in Fig. 7.

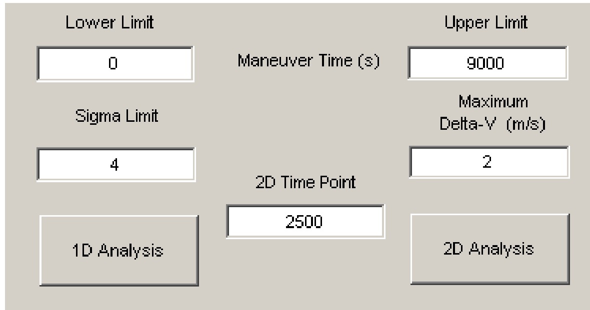

Fig. 7 MATLAB GUI Input Window (Middle Left)

The Sigma Limit (n) assigns the size of the n-σ ellipsoid shell and fixes the computational bounds of the probability calculation. The Maximum Delta-V sets the maneuver ΔV search limit. The 2D Time Point establishes a specific maneuver time for two-dimensional analysis and is not initially needed.

Pushing the 1D Analysis button produces a probability contour plot (log 10) for each axis (Velocity-**V**, Normal-**N**, and Co-Normal-**C**) as seen in Fig. 8.

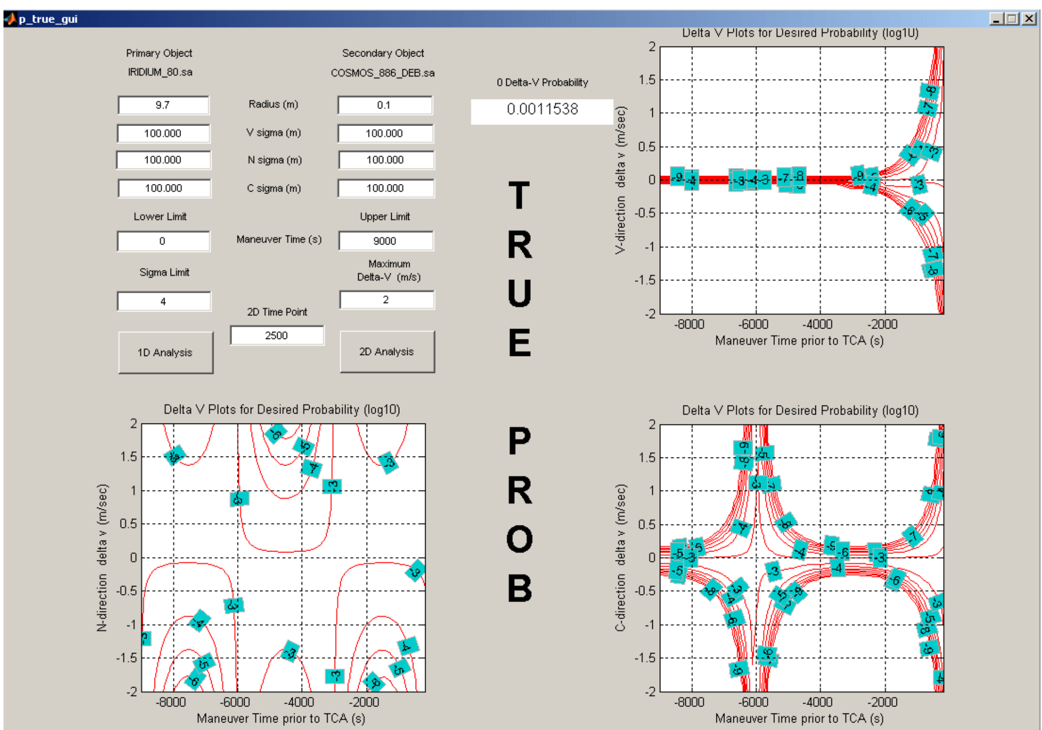

Fig. 8 MATLAB GUI 1D Analysis output

Figure 9 zooms in on the Co-Normal plot where equal probability contours (log 10) are mapped for the user-defined ranges of ΔV and maneuver time.

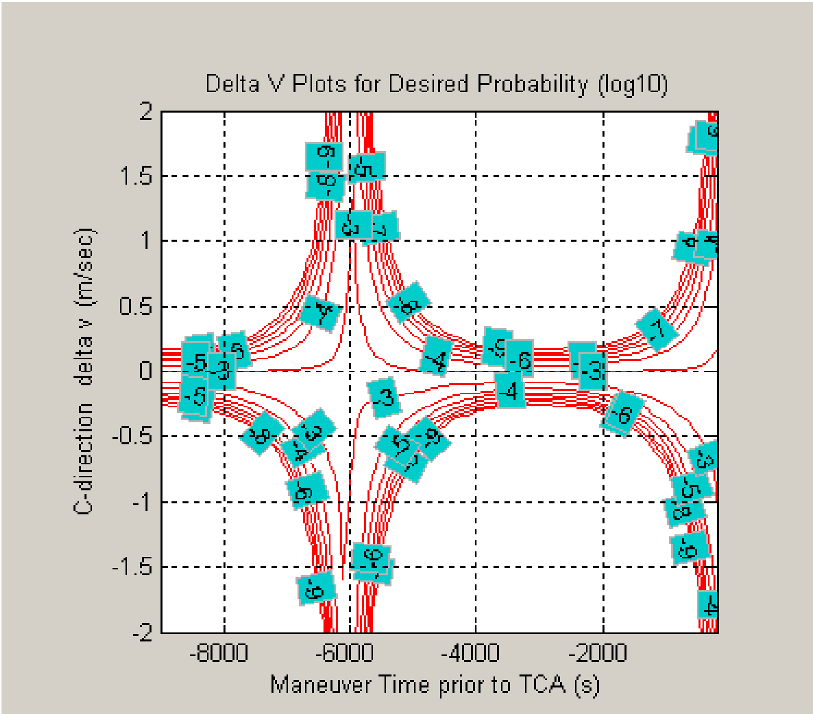

Fig. 9 MATLAB GUI 1D Co-Normal Plot

The plots are created by starting with each object's position and velocity data at the time of closest approach (TCA). The orbits are propagated backwards to a maneuver time using simple two-body dynamics, a velocity change is applied along the appropriate axis and then propagated forward to TCA. The covariance at/near TCA is assumed static and therefore not propagated at all. If the results need refining the user can change any parameter(s) and push the 1D Analysis button to create new plots. This is a simple approximation to render an initial assessment of maneuver options.

The one-dimensional analysis spans a range of maneuver times. The user must input a specific maneuver time (Fig 7, 2D Time Point) to perform the two-dimensional parametric analysis. Pushing the 2D Analysis button produces probability contours for dual-axes maneuvers over the range of allowable velocity changes as seen in Fig 10.

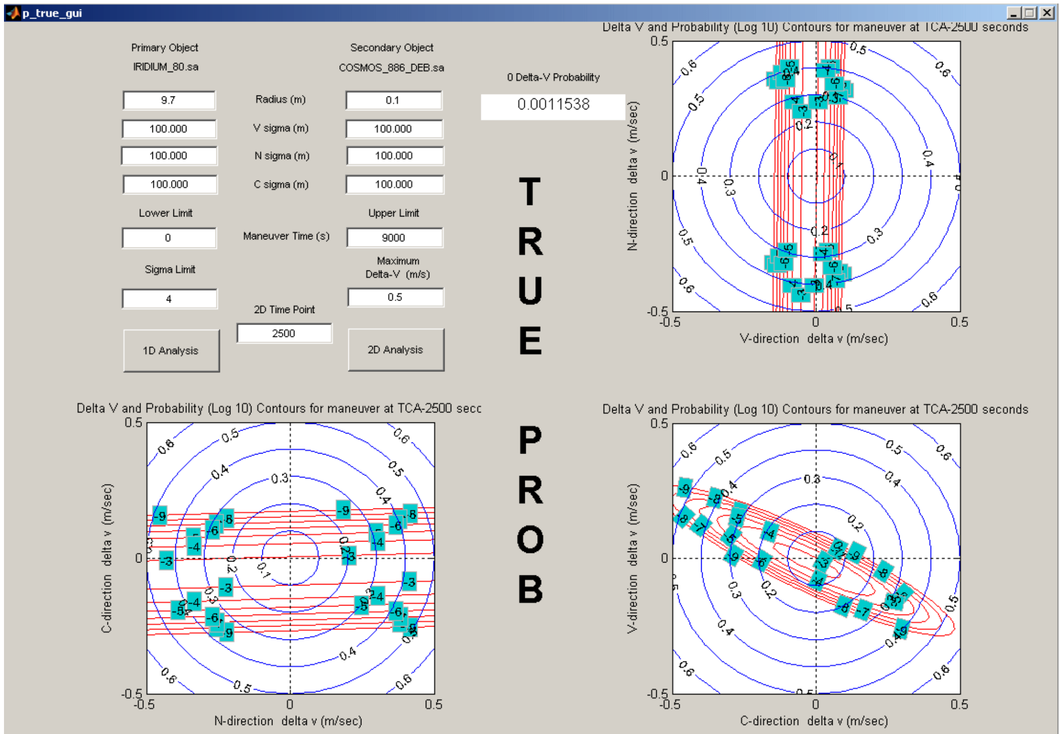

Fig. 10 MATLAB GUI 2D Analysis output

Figure 11 zooms in on the Co-Normal/Velocity plot where equal probability contours (log 10) are superimposed over rings of equal ΔV at the specified maneuver time.

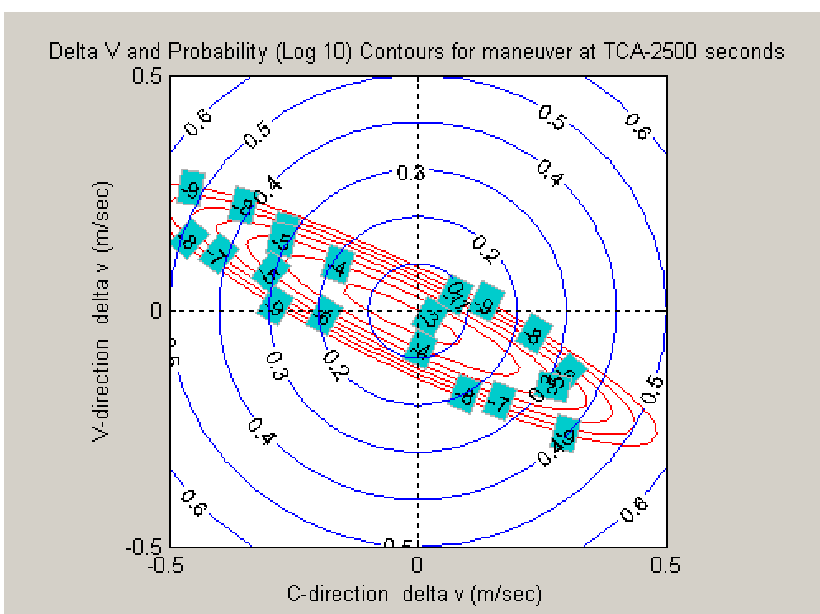

Fig. 11 MATLAB GUI 2D Velocity/Co-Normal Plot

For this example, Fig 10 shows that there is little to be gained by combining the Velocity/Normal axes or the Normal/Co-Normal axes. Figure 11 shows that a combined Co-Normal/Velocity maneuver will have the greatest effect on reducing probability for the smallest ΔV. To alter the search space the user can change any parameter(s) and push the 1D or 2D Analysis button to create new plots.

The user now chooses the maneuver time as well as the ΔV magnitude and direction to reduce the collision probability to an acceptable level. These MATLAB results are for a one-on-one conjunction prediction using simple dynamics. The candidate maneuver must now be fed back into STK for further analysis using better force models and covariance propagation (if desired) to re-assess the one-on-all risk. It is possible that the chosen maneuver is adequate to address the greatest single threat but actually moves the primary satellite into harms way of a formerly lesser threat. The STK scenario can also be used to assess any mission capability reduction that might result from the proposed maneuver.

CONCLUSION

A MATLAB analysis tool has been developed that can perform parametric studies of single-axis and dual-axes maneuvers. This is accomplished by assessing changes in the combined object's cross-sectional footprint in the encounter plane as well as examining candidate times and ΔV for evasive maneuvering. For the time of closest approach, the tool reads the object pair's positions, velocities, covariances and physical sizes from STK. The user is given the ability to modify the object sizes and covariances to determine if an attitude change and/or more accurate positional data is/are sufficient to reduce collision probability to an acceptable level. Given an upper bound on ΔV and a range of permissible maneuver times, MATLAB maps out collision probability contours for single-axis maneuvering using a simple dynamic model. By selecting a specific time, similar contours are produced for dual-axes maneuvering. The chosen maneuver parameters can then be fed back into STK to examine the effects of higher fidelity models or explore other concerns such as mission degradation or inadvertently increasing collision probability with respect to another satellite.

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