

A NUMERICAL STUDY OF ORBIT LIFETIME

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The computation of orbit lifetime is extremely challenging. The abundance of uncertainty makes the results of any one prediction suspect. In this study, we examine how the issues of uncertain atmospheric behavior, selection of an a priori atmospheric density model and the selection of a computational technique affect orbit lifetime predictions. Key to this effort is the development of a stochastic sequence to generate realistic time series of solar weather to drive the atmospheric density models. We hope the results of this study can serve as a guide to analysts making modeling decisions and provide recommendations for qualifying orbit lifetime predictions.

Revision: This version contains revisions to Figure 7, Figure 9, Figure 12, Figures 20-25 and to the conclusions drawn based on Figures 20-25 relative to the version of this paper that was presented at the 2005 AAS/AIAA Astrodynamics Specialists Conference in Lake Tahoe.

INTRODUCTION

The question often arises: what is the most accurate method for predicting orbit lifetime? This question is very difficult to answer due to the large amount of uncertainty related to almost every step in the computational process. Of specific interest to this effort are the uncertainties due to:

- the unknown behavior of the atmosphere in the future
- variations in computed atmospheric density between different atmospheric density models
- the computational technique used to make the predictions

The behavior of the atmosphere is primarily driven by solar flux and geomagnetic activity. Measurements, or predicted values, of these quantities are given as input to an

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atmospheric density model to compute an estimate of the instantaneous atmospheric density at the location of the satellite which is then used in the computation of the atmospheric drag force on the satellite. Since orbit lifetime computations often require flux inputs beyond what is available in short term predicts, a source of longer term predictions is required. One commonly used set of long term predictions are those produced by Schatten^{1,2}. Updated Schatten predicts are produced approximately every 4 months, with each predict covering a time span of approximately 2 solar cycles. The data files contain a predicted mean behavior of the F10.7 radio flux and uncertainty bounds about the mean for nominal, early and late timing of the cycle. Multiple predictions of orbit lifetime are commonly computed based on the nominal prediction of mean solar activity and specified deviations from the mean such as a two-sigma high prediction or a one-sigma low prediction. The intent of forming multiple predictions is to attempt to compute bounds on the actual orbit lifetime. While this methodology is effective for bounding the mean behavior, the real solar flux values will not have such a smooth and predictable trajectory. In the first part of this study, we develop a stochastic sequence to generate realistic future solar flux trajectories. The development of the stochastic sequence involves determining the amplitude and time correlation characteristics of the F10.7 variations from the predicted mean as a function of location within the solar cycle. The simulated F10.7 trajectories will serve as drivers for Monte-Carlo analyses of the orbit lifetime predictions. The results of this analysis will provide a distribution for the predicted orbit lifetime of a satellite based on a particular atmospheric density model.

Another complicating factor in communicating the results of an orbit lifetime study is the use of different atmospheric density models by different organizations. Differences in these *a priori* density models result in differing estimates of the atmospheric density and therefore in differing estimates of orbit lifetime. In order to quantify these differences relative to other uncertainties inherent in the prediction of orbit lifetime, we generate distributions of predicted orbit lifetime based on Monte-Carlo analyses for various atmospheric density models. The results of these analyses provide information on how density model selection may influence lifetime calculations.

Finally, we examine the question: when is Cowell integration a better means of predicting orbit lifetime than using a simplified model designed for longer term computations? To address this issue, we generate Monte-Carlo analyses for trajectories with varying expected mission lifetimes, and compare the results obtained using numerical integration of the complete equations of motion to results obtained using a simplified formulation specifically targeted at the computation of orbit lifetime.

While our results are by no means comprehensive, we hope that this study can serve as a guide to analysts for making appropriate modeling decisions based on the proximity to end of life of the satellite and provide a recommended method for qualifying orbit lifetime predictions.

EXAMPLE ORBIT PARAMETERS

For the purpose of this analysis we selected a set of orbit elements which we expected to produce fairly short orbit lifetimes throughout the solar cycle. We then selected two epochs for the analysis, one near solar maximum and the other near solar minimum. The selected orbit elements and epochs are given in Table 1.

TABLE 1. APPROXIMATE ORBIT PARAMETERS

Epoch (GMT)	Flux	a (km)	Alt(Km)	e	i (deg)	Ω (deg)
4 Oct 2000	max	6753	375	0.0	97	13
1 May 1986	min	6753	375	0.0	97	13
$C_d = 2.0, C_r = 1.0, \text{Area/Mass} = 0.02 \text{ m}^2/\text{kg}$						

Note that while the initial orbit state is the same in both cases, the predicted orbit lifetime will be quite different due to differing solar weather.

CHARACTERIZING SOLAR WEATHER

Modern atmospheric density models require measures of environmental activity, usually the F10.7 cm solar flux measurement (F10.7) and a measure of geomagnetic activity (A_p or K_p) to compute estimates of atmospheric density. This presents a formidable problem in the prediction of orbit lifetime since neither index is easy to predict. Our goal is not to improve the prediction of these drivers of atmospheric density, but to develop a scheme for producing realistic predicted trajectories of F10.7. The production of the realistic trajectories for geomagnetic activity is deferred until a later study. We define a realistic trajectory as one that generally follows an *a priori* prediction model where the variations from the model are statistically consistent with historical solar cycles. While other *a priori* predictions of solar activity exist^{3,4}, we will use the Schatten^{1,2} predicts as the *a priori* prediction model for this analysis. We begin our characterization of solar weather by analyzing variations of the mean solar flux behavior. We then superimpose the effects of daily solar flux variations.

Variation of the Mean

The solar flux predictions from Schatten are to be interpreted as predictions of the smooth, mean behavior of the F10.7 cm solar flux. The designation of "mean behavior" indicates that these predictions do not attempt to model daily variations of the solar flux, but instead model the average trend. The Schatten predictions are given as predicted monthly averages and contain an estimate of the uncertainty associated with the predictions. It is important to note the standard deviation represents uncertainty in the mean behavior of the solar flux, not uncertainty due to daily variation. The predicted mean solar flux for solar cycle 23 is shown in Figure 1³.

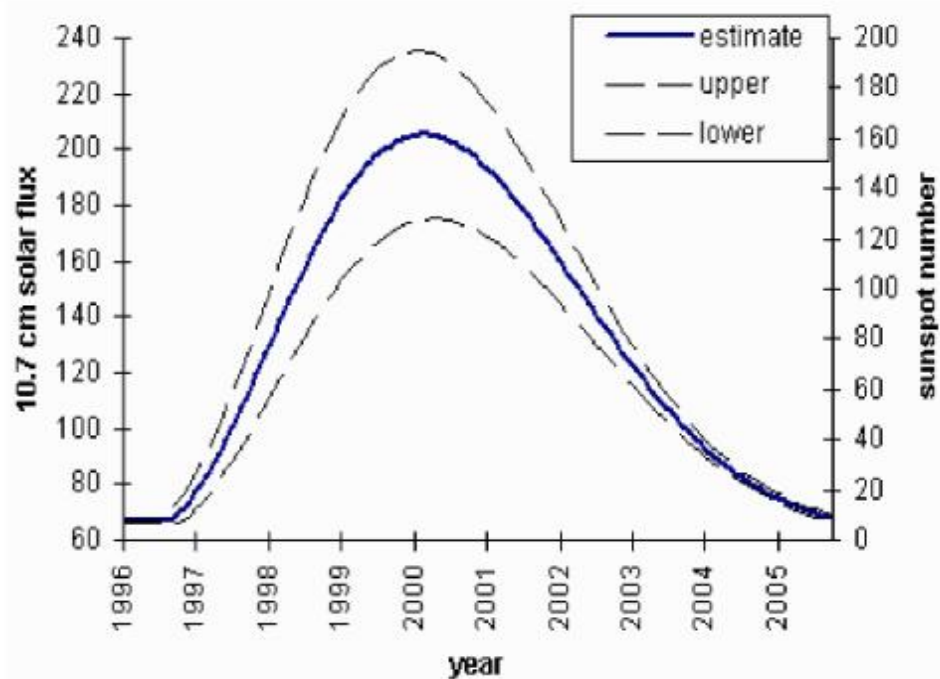


Figure 1. Predicted solar flux and sunspot numbers for solar cycle 23³

To characterize the dependence of orbit lifetime predictions on the mean behavior of the solar flux, we performed Monte-Carlo analyses for time periods in the vicinity of solar max and solar min. The goals of these analyses are:

- 1) To reveal the distribution of orbit lifetime predictions generated from a Gaussian distribution of smooth F10.7 trajectories
- 2) To identify the location of the orbit lifetimes predicted based on the nominal, +2 sigma and -2 sigma F10.7 trajectories in the lifetime distribution

The Monte-Carlo analyses were generated using the orbit lifetime prediction tool (Lifetime) in Satellite Tool Kit (STK) after augmentation to allow for the selection of the atmospheric density model and to read in daily solar flux values. The lifetime prediction capability in STK is a modified version of the Lifetime program developed at NASA Langley⁴. The Lifetime tool uses a simplified semi-analytic technique for propagating the orbit at large step sizes on the order of one or more orbits per step. Atmospheric drag effects are modeled using one or more Gaussian quadratures per orbit. A similar tool with a similar name was also developed by the Aerospace Corporation⁵. For each trial in a Monte-Carlo run, a solar flux trajectory was generated based on the random selection of the mean solar flux level in terms of a number of standard deviations from the nominal predict. Each Monte-Carlo run contains 1000 trials. Figures 2 and 3 depict distributions of orbit lifetime predictions at solar max and solar min using the Jacchia 70 atmospheric density model. The lifetime predictions based on the nominal and ± 2 sigma mean solar flux trajectories are annotated on each figure.

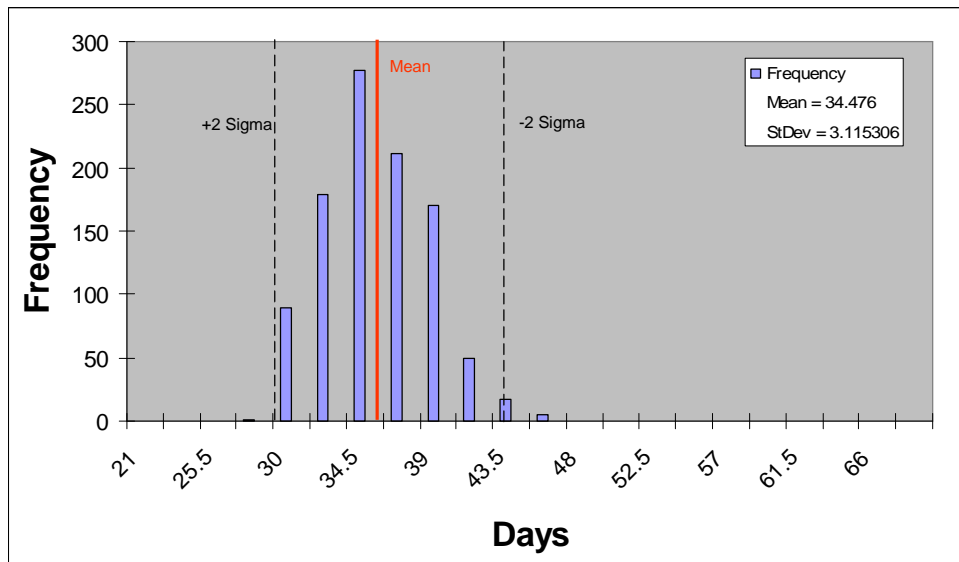


Figure 2. Distribution of orbit lifetime near solar max based variations in the mean solar flux using the Jacchia70 atmospheric density model. Lifetime results based on using nominal and ± 2 sigma input flux predictions are annotated.

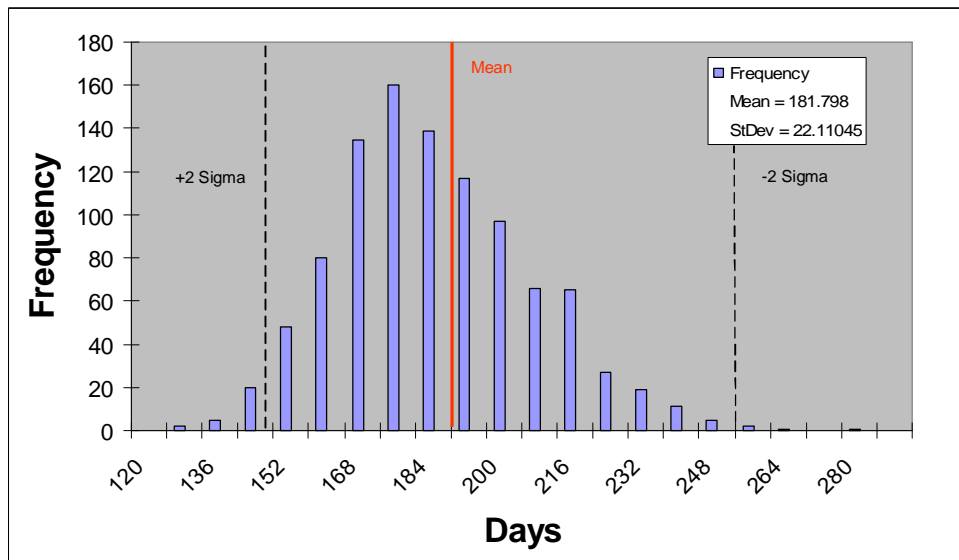


Figure 3. Distribution of orbit lifetime near solar min based variations in the mean solar flux using the Jacchia70 atmospheric density model. Lifetime results based on using nominal and ± 2 sigma input flux predictions are annotated.

Figures 2 and 3 clearly demonstrate the expected longer mean orbit lifetime and the larger standard deviation from the mean during solar min. Also evident is the non-Gaussian shape of both distributions. It is interesting to note the extension of the tail on both distributions.

Daily Variation

Actual solar weather histories have significant daily variations from the mean trend. These variations differ in amplitude and time correlation through the solar cycle. To better characterize these variations, smooth curves were fit to observed solar flux data from the last three solar cycles. Note that solar cycle 23 is not complete since as of the date of this study, we are still in solar cycle 23. These smooth curve fits were used in place of the Schatten predictions due to significant timing errors in the Schatten predictions for these solar cycles. The curve fits and actual observed data are shown in Figures 4-6.

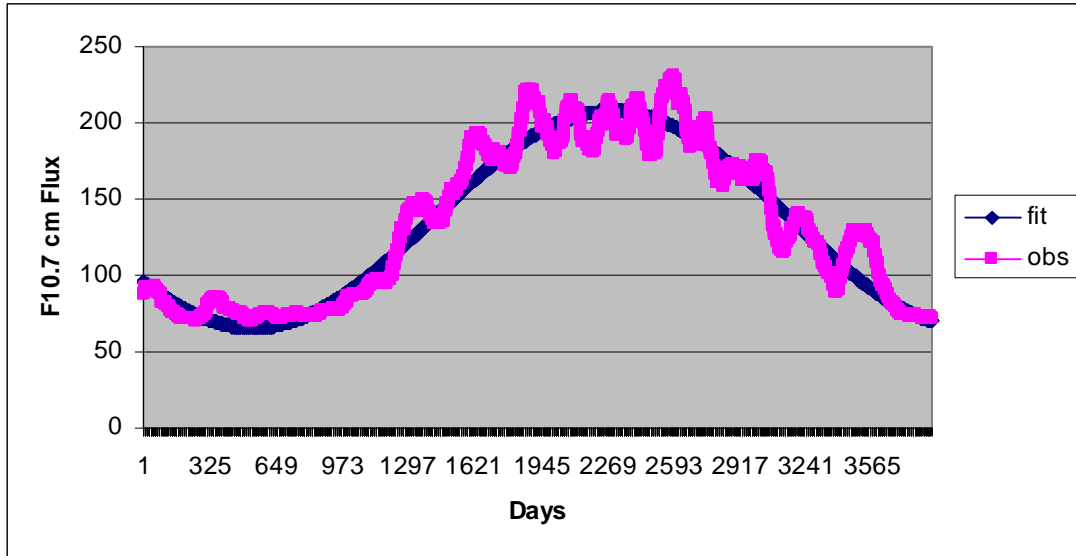


Figure 4. Observed and fit values of F10.7 cm solar flux for solar cycle 21

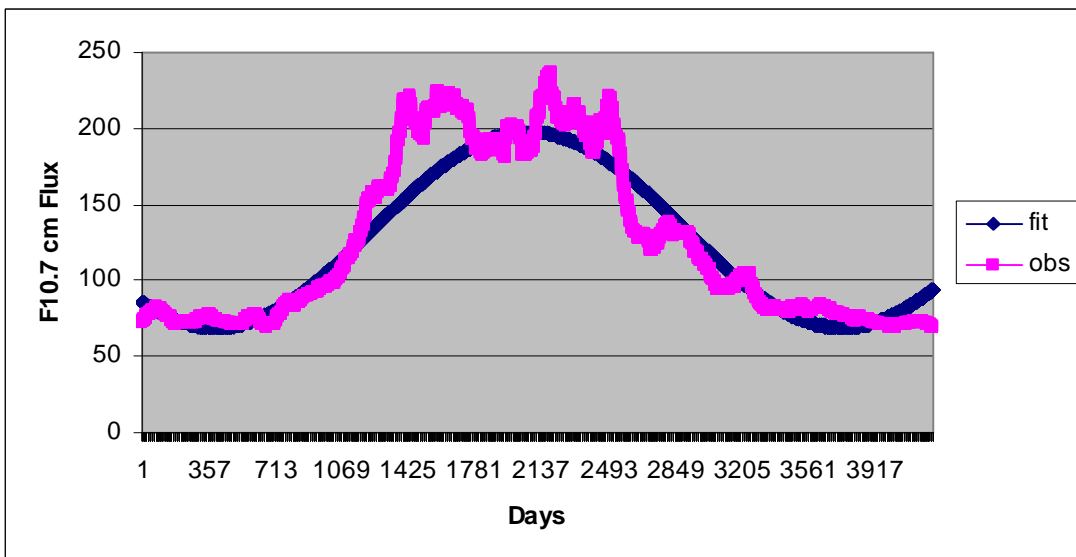


Figure 5. Observed and fit values of F10.7 cm solar flux for solar cycle 22

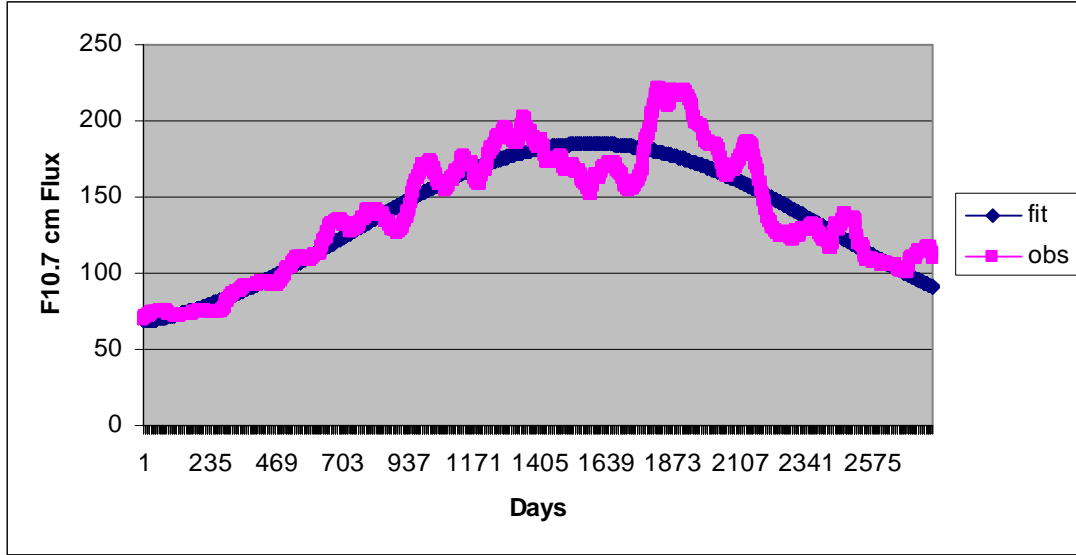


Figure 6. Observed and fit values of F10.7 cm solar flux for solar cycle 23

We chose a scalar exponential Gauss-Markov sequence as the stochastic model for the amplitude of the daily variation. The transition equation for this sequence is given by,

$$x(t_{k+1}) = \Phi(t_{k+1}, t_k)x(t_k) + \sqrt{1 - \Phi^2(t_{k+1}, t_k)}w(t_{k+1}), \quad (1)$$

where $w(t)$ is a Gaussian white noise variable with zero mean and variance s_w^2 ,

$$\Phi(t_{k+1}, t_k) = e^{a|t_{k+1} - t_k|}, \quad (2)$$

And

$$a < 0. \quad (3)$$

The random sequence is initialized via

$$x(t_0) = w(t_0). \quad (4)$$

The parameter a is related to the exponential half-life by

$$e^{at} = 0.5, \quad (5)$$

$$a = \frac{\ln(0.5)}{t}. \quad (6)$$

The connection between the stochastic model described by Eqs. 1-6 and the daily variations of F10.7 cm flux is made by determining the sample standard deviation, s_w and sample time correlation half-life values, t , throughout the solar cycle.

We divided each solar cycle into 8 segments to characterize the changing behavior of the daily variations. Data for each segment from the three solar cycles, Figures 4-6, were combined to produce composite sample standard deviations and time correlation half life values for each segment. Each segment was specified by a name and a center point measured in terms of a normalized solar cycle. The normalization is performed such that the length of the normalized solar cycle is one, the segment

centered on solar minimum has a normalized center of zero (and one) and the segment centered on solar maximum has a normalized center of 0.5. The results are given for each segment in Table 2.

TABLE 2. DAILY F10.7 VARIATION

Segment	Normalized Center	Std Dev	Half-Life (Days)
Min	0.0	7	112
Min Up	0.125	4	66
Up Slope	0.25	10	49
Up Max	0.375	16	63
Max	0.5	24	83
Max Down	0.625	17	51
Down Slope	0.75	12	44
Down Min	0.875	10	41
Min	1.0	7	112

Our original goal was to use the data from Table 2 to derive functional forms for the standard deviation and half life based on the location in the solar cycle. These simple functions could then be used to drive stochastic sequences which would allow for the generation of realistic solar weather trajectories. Unfortunately, we were unable to achieve an adequate representation of the general behavior of either data set using simple functional. To overcome the difficult nature of the data, we used a cubic spline to approximate a continuous variation through the solar cycle. The resulting functional forms relative to the location in the solar cycle are shown in Figures 7 and 8. In both cases, our goal is to produce functional forms that are representative of the historical data. Precise fits to the imprecise data in Table 2 are not required.

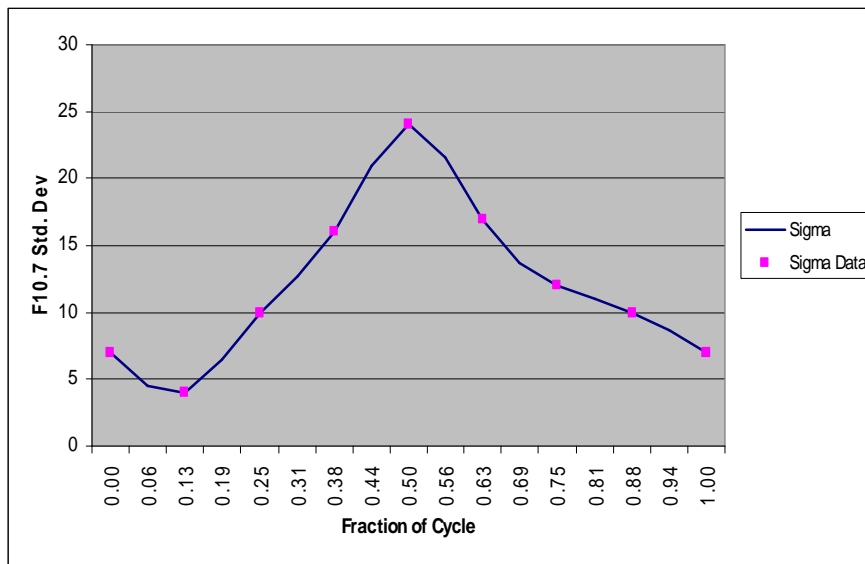


Figure 7. Functional representation of F10.7 cm flux standard deviation

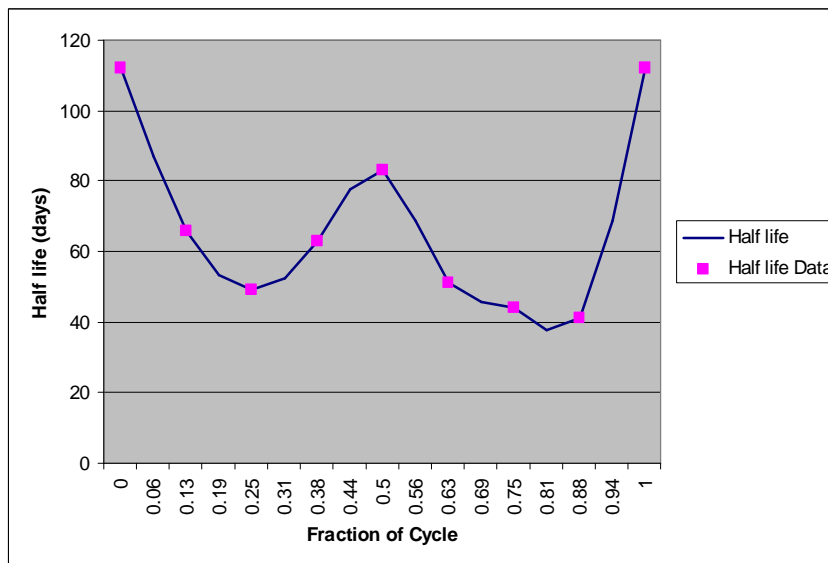


Figure 8. Cubic spline fit of F10.7 cm solar flux half life

To analyze the effects of the daily solar flux variations on the orbit lifetime, we used the nominal Schatten predict as the *a priori* model and added random sequences of daily variations. One such trajectory is shown in Figure 9. Monte-Carlo analyses were used to characterize the amount of variation in the orbit lifetime which can be expected based purely on the daily variations. This type of analysis is representative of an orbit lifetime prediction for an existing satellite where the current mean behavior of the solar weather is fairly well defined. In these cases, the initial daily variation has been constrained to a reference history to simulate the case where we constrain the random future variations to start with current solar weather predictions. Results for the solar max and solar min test cases are shown in Figures 10 and 11 respectively. As expected, the means of the distributions based on daily variations are very similar to the means of the distributions based on mean variations. A somewhat more interesting

result is that the standard deviations are also significant relative to those resulting from variations in the mean behavior.

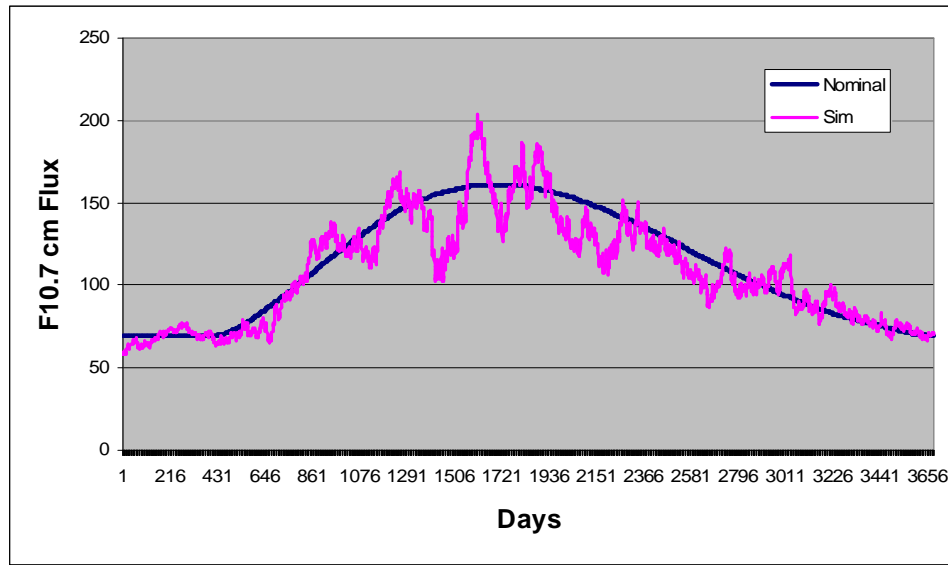


Figure 9. Simulated daily F10.7 cm flux

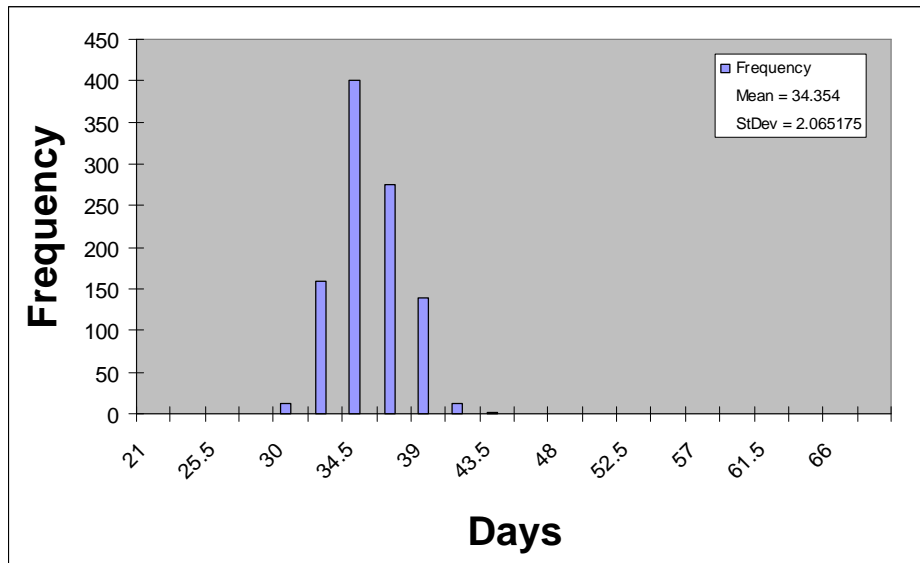


Figure 10. Distribution of orbit lifetime near solar max based on daily variations of F10.7 cm flux using the Jacchia70 atmospheric density model

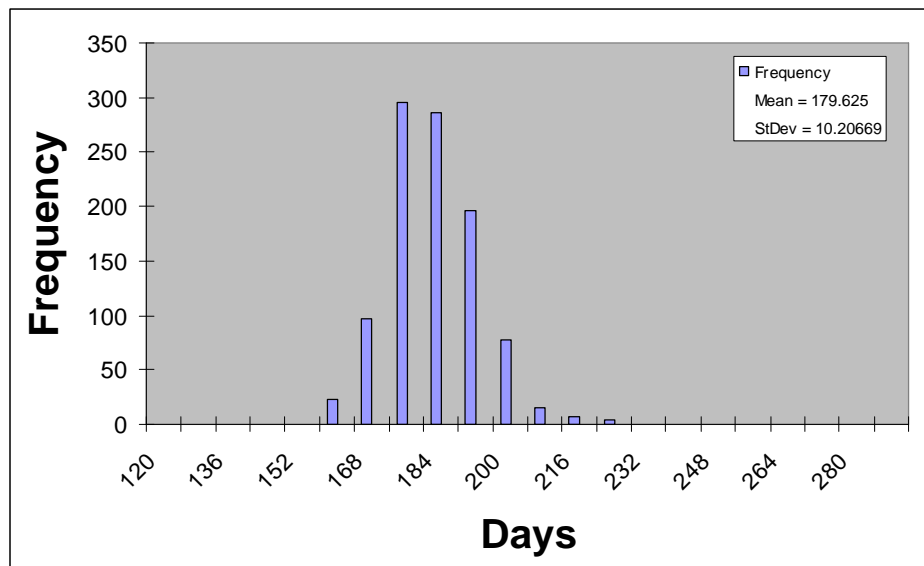


Figure 11. Distribution of orbit lifetime near solar min based on daily variations of F10.7 cm flux using the Jacchia70 atmospheric density model

Finally, we constructed solar flux trajectories combining the mean and daily variations. A sample trajectory is shown in Figure 12. Monte-Carlo results for the solar max and solar min test cases are shown in Figures 13 and 14. Additional histograms are given for other density models in the next section. This type of analysis is appropriate for future mission where the solar weather conditions are more uncertain.

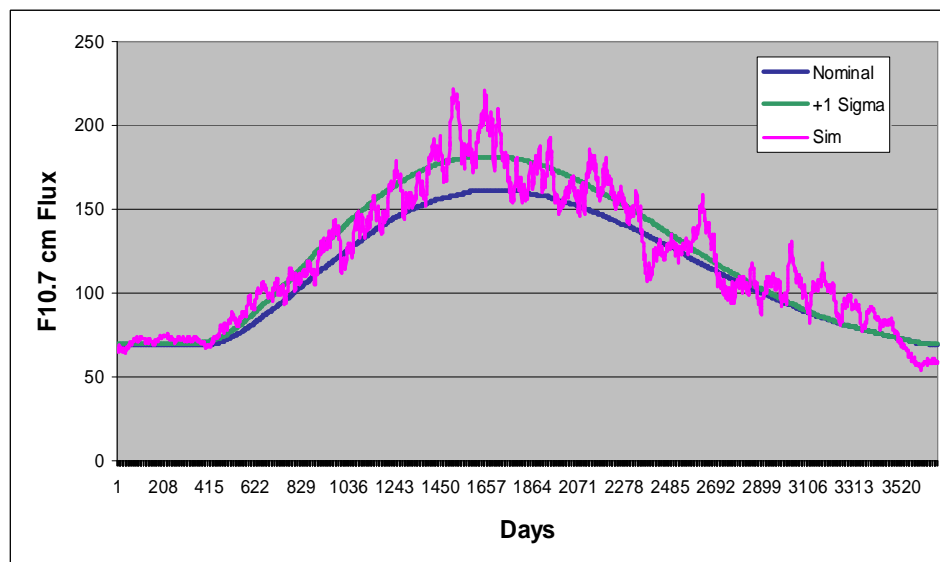


Figure 12. Simulated daily F10.7 cm flux with +1 Sigma deviation of the mean

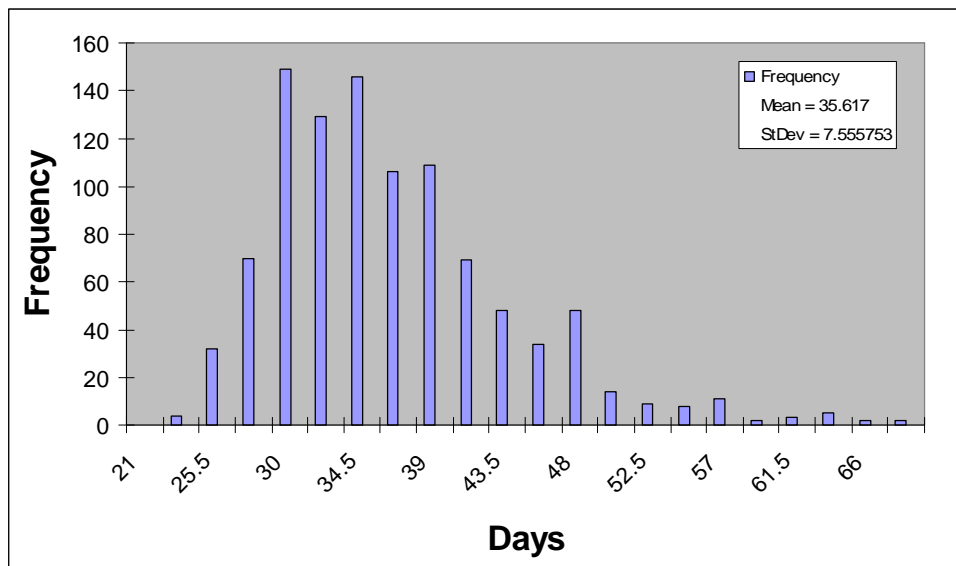


Figure 13. Distribution of orbit lifetime near solar max based on mean and daily variations of F10.7 cm flux using the Jacchia70 atmospheric density model

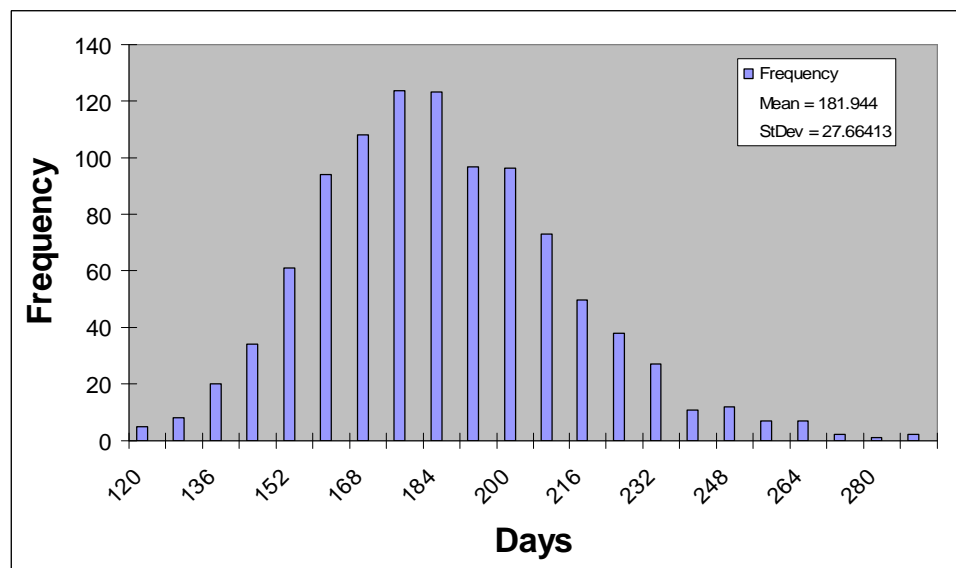


Figure 14. Distribution of orbit lifetime near solar min based on mean and daily variations of F10.7 cm flux using the Jacchia 1970 atmospheric density model

A comparison of Figures 13-14, which contain both mean and daily variations with Figures 1-2, which contain only variations of the mean solar flux trajectory, shows that the addition of the daily variations did not make a significant change in the mean orbit lifetime, but did substantially increase the uncertainty about the mean.

ATMOSPHERIC DENSITY MODEL SELECTION

Since different atmospheric density models produce different estimates of atmospheric density, they also produce different predictions of orbit lifetime. We include several of the more commonly used atmospheric density models in this analysis

for the purpose of characterizing the effect of the density model selection relative to the effect of the uncertainty in the predicted solar weather. We seek to provide an answer to the fundamental question: is the selection of a specific atmospheric density model important relative to the uncertainty in future solar weather conditions for the purpose of predicting orbit lifetime. We are also interested in whether the difference in orbit lifetime predictions resulting from the use of different atmospheric density models is consistent with the commonly accepted value of approximately 15% uncertainty in modern density models⁷. Table 3 lists the density models considered and associated dependencies on solar weather and geomagnetic activity.

TABLE 3. ATMOSPHERIC DENSITY MODELS

Model	Daily F10.7	Avg. F10.7	3 Hourly Ap/Kp	Daily Ap/Kp
Jacchia 1970	X	X	X	
Jacchia 1971	X	X	X	
MSIS 1986	X	X	X	
MSISE 1990	X	X	X	
NRL MSISE 2000	X	X	X	X
Harris Priester		X		

The NRL MSISE 2000 model has a switch which allows for the use of 3 hourly Ap data or daily Ap data. For the purposes of this test, the daily Ap option was used.

The Monte-Carlo results for all density models including random variations in the mean solar flux trajectory and random daily variations about the mean trajectory are shown in Figures 13-19. The random daily variations in these tests were initialized with random draws based on the daily F10.7 uncertainties listed in Table 2. The same set of random solar flux trajectories was used for all atmospheric density models. These plots should therefore be considered to be representative of predictions for a future mission where current solar weather conditions are not relevant to the predicted lifetime.

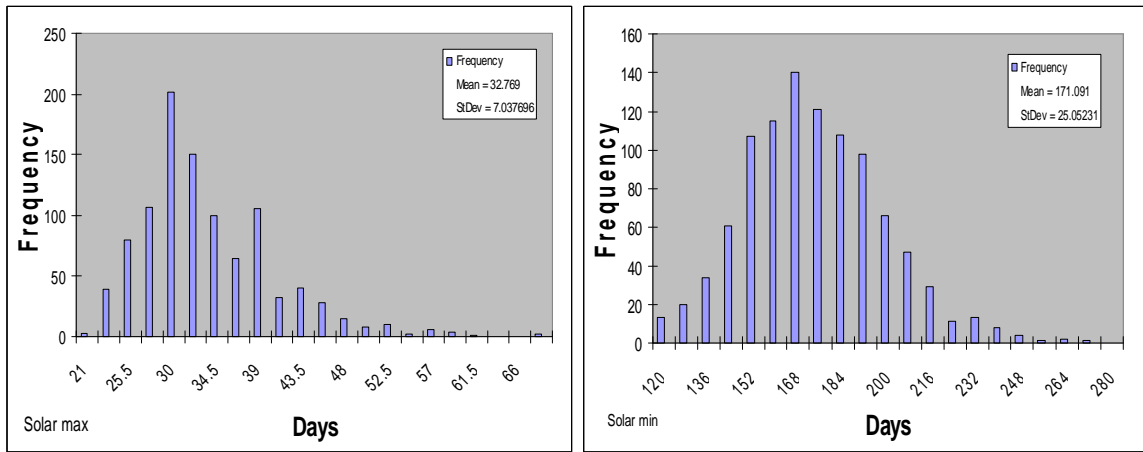


Figure 15. Distributions of orbit lifetime based on mean and daily variations of F10.7 cm flux using the Jacchia71 atmospheric density model

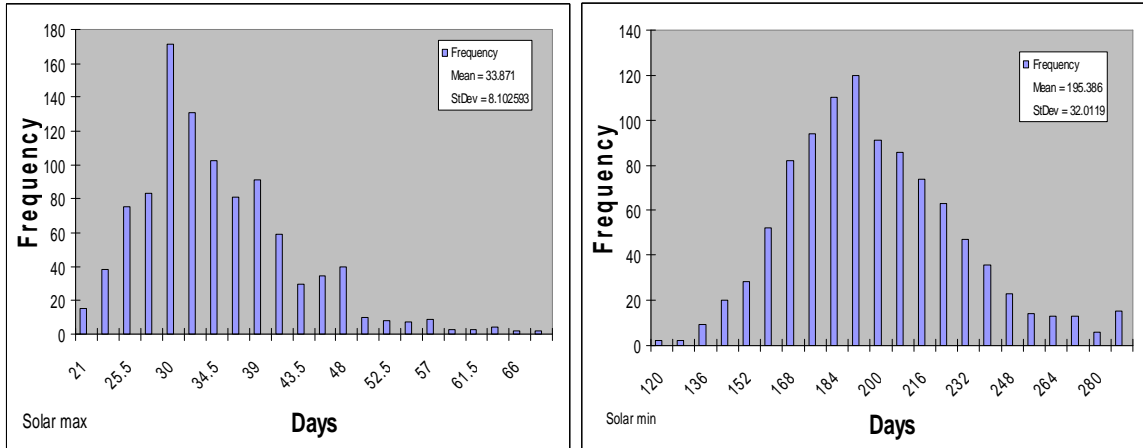


Figure 16. Distributions of orbit lifetime based on mean and daily variations of F10.7 cm flux using the MSIS 1986 atmospheric density model

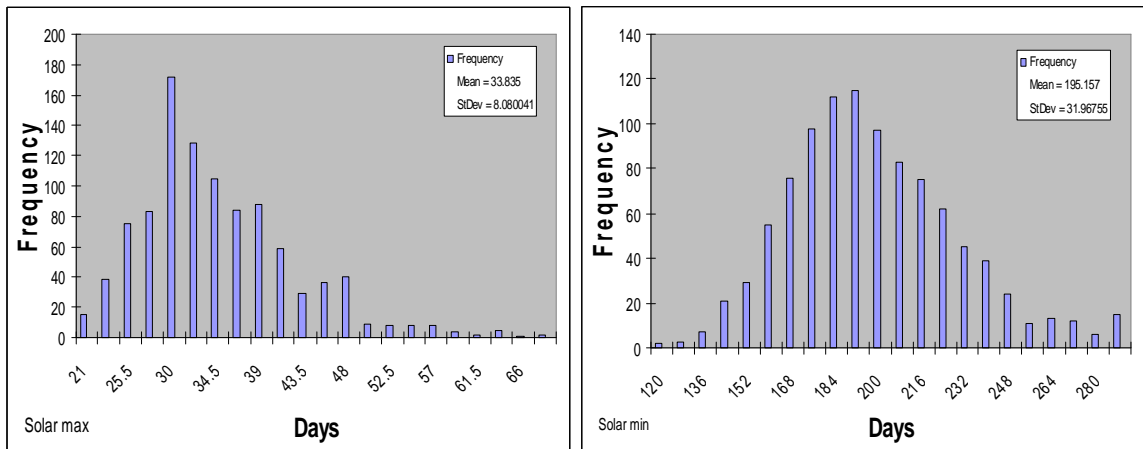


Figure 17. Distribution of orbit lifetime based on mean and daily variations of F10.7 cm flux using the MSISE 1990 atmospheric density model

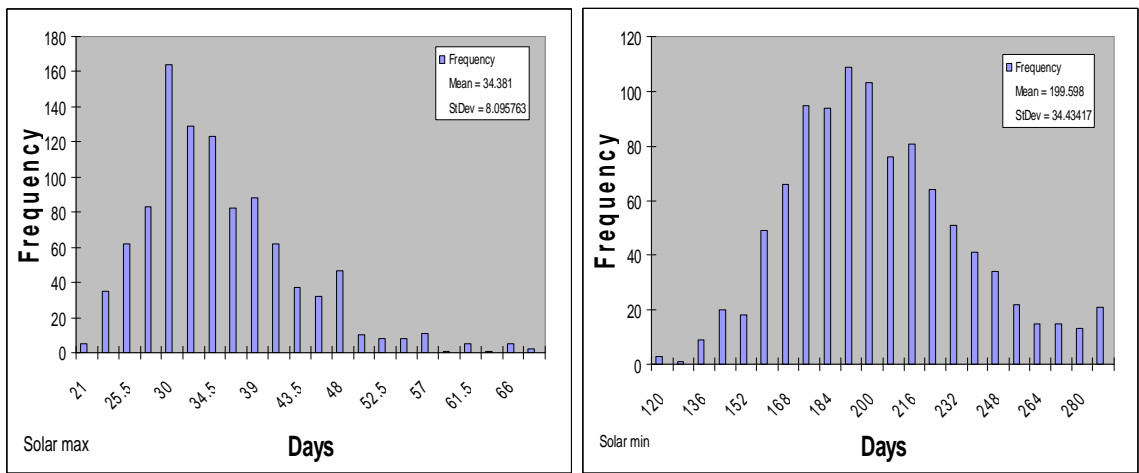


Figure 18. Distribution of orbit lifetime based on mean and daily variations of F10.7 cm flux using the NRL MSISE 2000 atmospheric density model

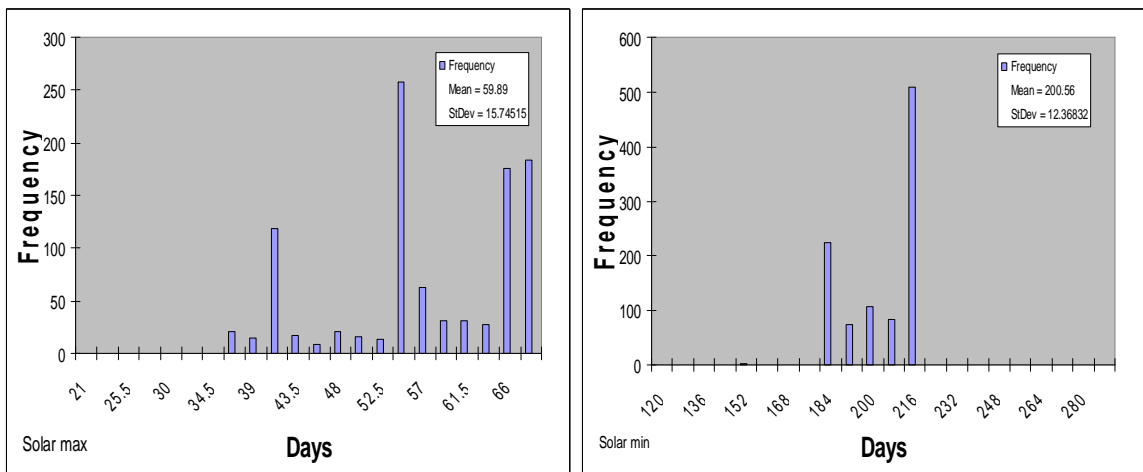


Figure 19. Distribution of orbit lifetime based on mean and daily variations of F10.7 cm flux using the Harris Priester atmospheric density model

Note the remarkable agreement in terms of the mean, standard deviation and general shape of the orbit lifetime distribution for all of the models, with the exception of the Harris Priester model. The distributions resulting from the use of the Harris Priester model are actually quite troublesome and suggest some sort of modal behavior.

The important conclusion from this analysis is that the uncertainty in solar weather appears to dominate the difference in the mean orbit lifetime predictions resulting from the selection of different atmospheric density models. This result is consistent with previous finds by Owens et al⁸, who note the same dominance of the solar weather uncertainty over uncertainty in the coefficient of drag. We also note that the differences in predictions using various atmospheric models does not appear to be inconsistent with the 15% uncertainty figure associated with each individual model, with the exception of the Harris Priester model which does not appear to be suitable for this type of analysis.

NUMERICAL INTEGRATION

Another common question when performing lifetime analysis is: when is it appropriate to use a simplified numerical method such as the Lifetime tool that has been used for all examples up to this point versus full numerical integration of the equations of motion. Conventional wisdom on the numerical integration of the Low Earth Orbits (LEO) is that only short term predictions are possible due the large uncertainty in atmospheric density. This is certainly true for cases where the accuracy of the intrack position of the satellite is a concern, but what is the effect on the prediction of orbit lifetime?

To analyze the relative merit of full numerical integration, we designed an experiment starting with our solar min and solar max initial conditions. In the experiment, we generate solar flux *truth* trajectories for both time frames. We then generated a *truth* orbit trajectory for each case using numerical integration based on the solar flux truth trajectory. The lifetime of these truth orbits are then considered to be the true lifetimes. Starting with our original initial conditions and repeating at regular intervals throughout the orbit lifetime, we perform Monte-Carlo analyses computing orbit lifetimes using both the simplified Lifetime method and full numerical integration. Each time we select a new starting epoch, we reset our initial conditions to match the truth trajectory and regenerate a new set of random solar flux trajectories which are constrained to the current location on the solar flux truth model. Lifetime errors were then computed as the difference between the lifetime computed with each trial and the lifetime of the reference *truth* trajectory. This comparison is expected to favor the numerical integration results since numerical integration with the same force models was used in the generation of the reference lifetime. Each Monte-Carlo analysis consists of 500 trials and the results are displayed in Figures 20-25. The starting epoch for the runs was advanced in increments of 1 week for the solar max case and 1 month for the solar min case.

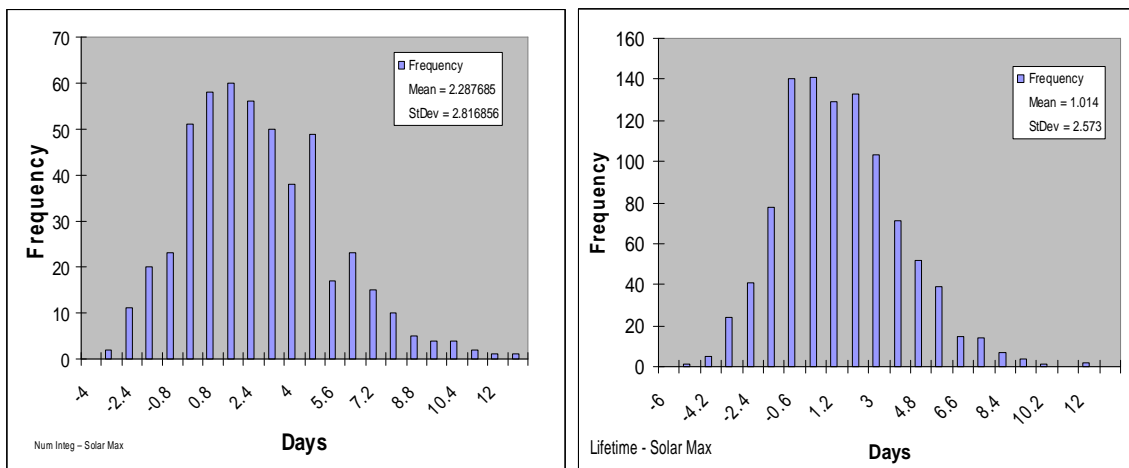


Figure 20. Orbit lifetime errors at solar max, approximately one month out

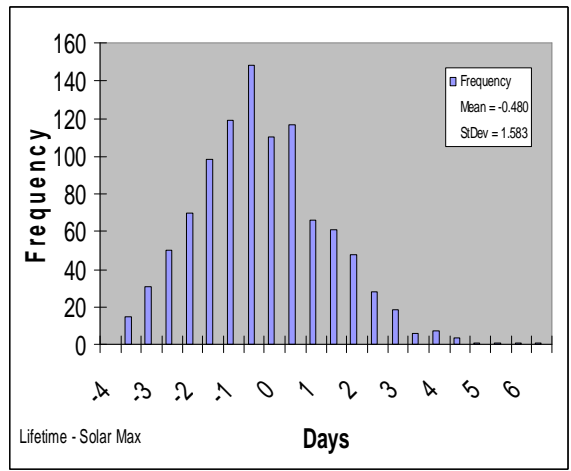
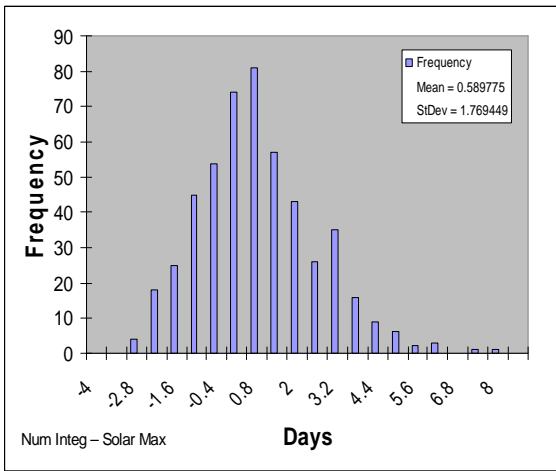


Figure 21. Orbit lifetime errors at solar max approximately three weeks out

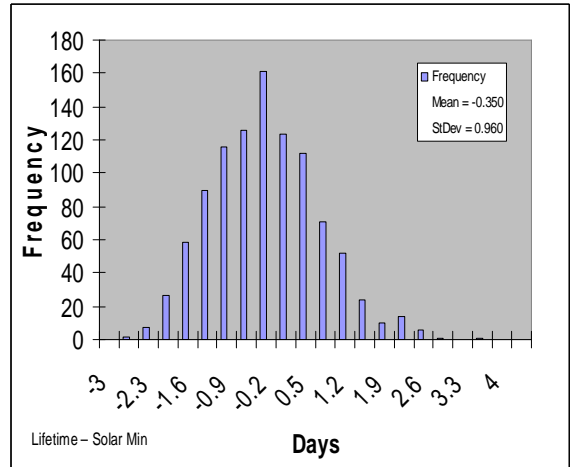
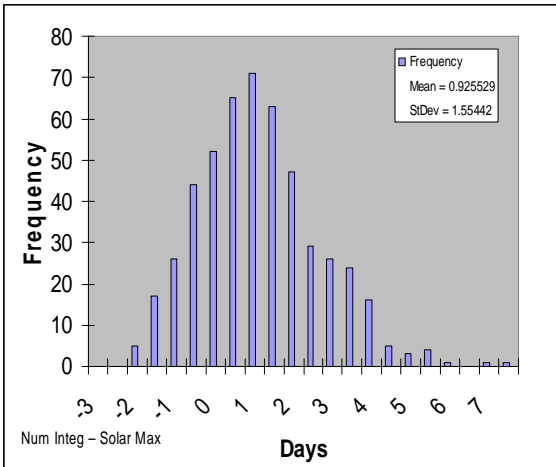


Figure 22. Orbit lifetime errors for solar max approximately two weeks out

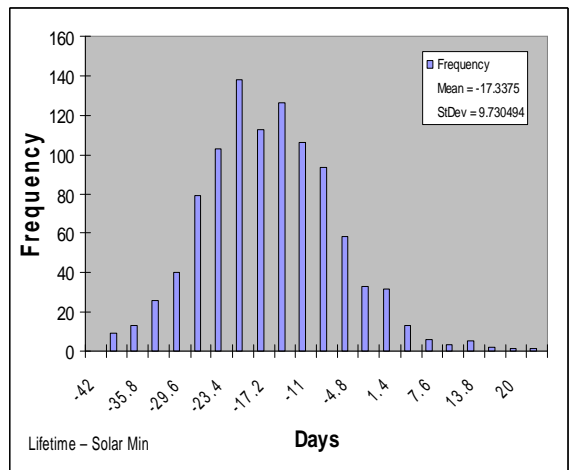
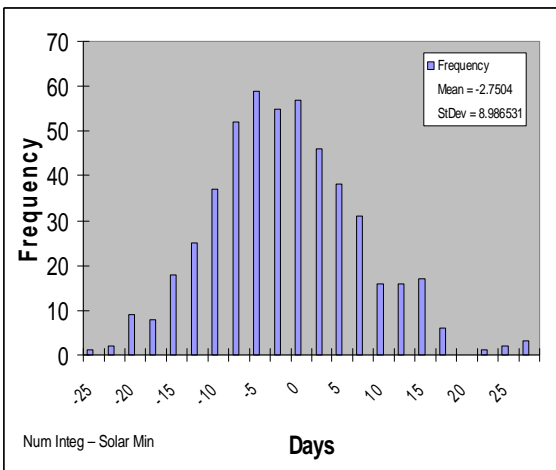


Figure 23. Orbit lifetime errors for solar min approximately six months out

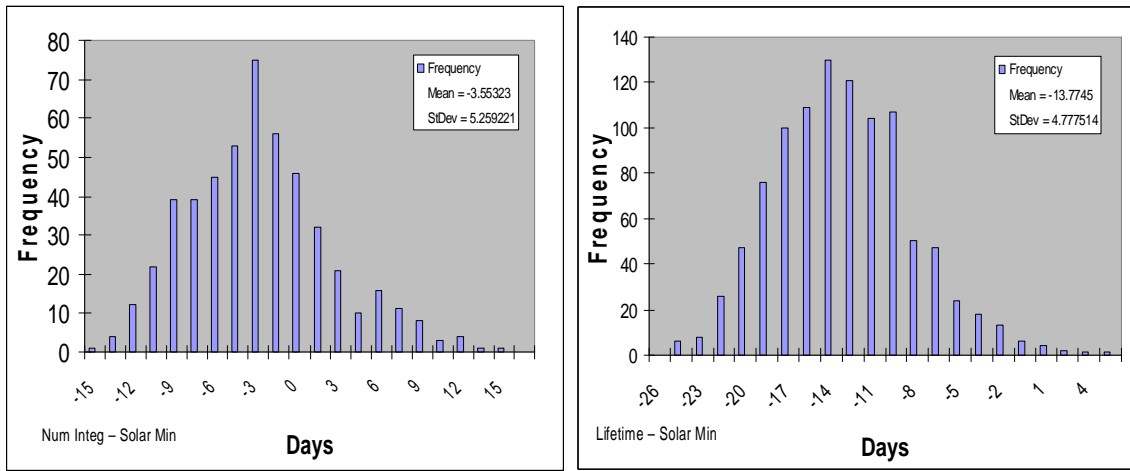


Figure 24. Orbit lifetime errors for solar min approximately four months out

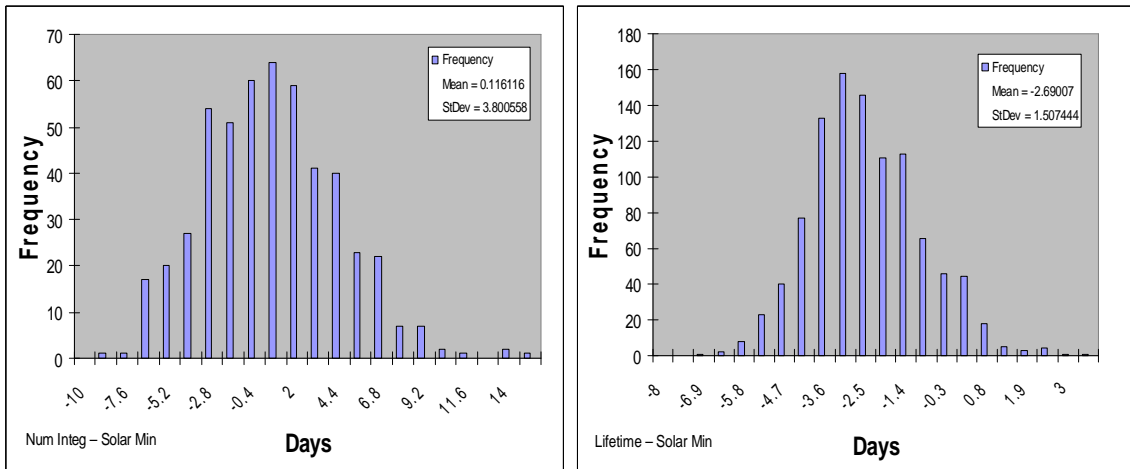


Figure 25. Orbit lifetime errors for solar min approximately two months out

In the solar max test case, both the numerical integration and the Lifetime result are biased high, meaning that predicted lifetimes are longer than the lifetime of the truth trajectory, when the remaining orbit lifetime equals approximately 1 month, see Figure 20. The numerical integration remains biased high, but with decreasing bias and decreasing standard deviation as the time remaining until orbit end of life decreases, Figures 21 and 22. The Lifetime result shows a change in the sign of the bias as the time remaining until orbit end of life decreases. In all cases the bias is within one standard deviation. A different bias at each progression is expected since we are starting from different daily offsets from the mean behavior at each time. It is also expected, though not observed for the solar max test case, for the numerical integration result to perform better in comparison to a numerically integrated reference that used identical force modeling.

The numerical integration result shows similar behavior in terms of the decreasing bias magnitude in the solar min test case, see Figures 23-25. In this case, the numerical integration result starts out biased low, switches to being biased high and the bias almost disappears in the run with approximately 2 months of lifetime remaining. In all three runs, the bias is well within one standard deviation. The lifetime

result starts out and remains biased low. The progression of the standard deviation is similar to that of numerical integration, but the mean behavior is much less satisfactory.

It is difficult to form any strong conclusions from this experiment regarding the accuracy of Lifetime on cases of short orbit lifetime other than that additional investigation is required to determine the cause of the observed behavior. One possibility that cannot be ignored, unfortunately for the authors, is that of operator error in configuring the inputs. An input error, the relative effect of which decreases as the duration of the computation increases, would be consistent with the observed results. On the other hand, the numerical integration results appear to behave nicely over the examined time frames.

ADDITIONAL SOURCES OF VARIABILITY

There are many other uncertain inputs to the orbit lifetime computation than what have been considered here. These areas of uncertainty must also be explored to determine the relative effect of each on the overall orbit lifetime uncertainty. Additional investigation is required to determine the effects of:

- Uncertainty in the timing of future solar cycles
- Uncertainty in the level of predicted geomagnetic activity
- Uncertainty in the a priori atmospheric density models
- Uncertainty in the area to mass ratio for uncontrolled attitude motion
- Uncertainty in initial conditions (position, velocity and drag states)

CONCLUSIONS

Monte-Carlo analyses have proven to be an effective tool for qualifying the results of orbit lifetime predictions. The results of Monte-Carlo analyses indicate that consideration of the effect of unknown daily variations on the distribution of predicted orbit lifetime is important for both current time and future analyses. In the case of future analyses, the effect of the daily variations is seen to be slightly smaller than the effects of uncertainty in the mean behavior. We have also seen that atmospheric density model selection is of lesser importance as compared to proper consideration of the uncertainty in future solar weather. The one exception to this rule, among the density models examined in this study, is that the Harris Priester model is seen to not be appropriate for orbit lifetime predictions. In the case of existing satellites, the recommendation is to use the same density model as is used in the orbit determination process since any global bias in the density model will be absorbed into the estimate of ballistic coefficient during the estimation process. Our results in the comparison of methods for the computation of orbit lifetime were partially inconclusive. Longer orbit lifetimes need to be analyzed to determine the practical range of application for numerical integration. The Lifetime capability, which executes in a fraction of the time required for full numerical integration, has not produced results with the same quality of results as full numerical integration. Improper use of the tool cannot be ruled out as an explanation of this behavior.

ACKNOWLEDGEMENTS

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