An Analysis of State Vector Prediction Accuracy
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Modern precise navigation services are creating increased applications for numerically generated state vectors for satellite operations. Traditional radar and optical techniques can achieve modest accuracy in orbit determination, but on-board GPS satellite receivers are changing the routine accuracy available. Operational requirements usually involve future locations, rather than past locations derived from Orbit Determination (OD) techniques. This paper compares propagation of various satellite initial state vectors to independently produced Precision Orbit Ephemerides (POE's). The initial state of each satellite is varied to reflect expected orbital accuracy achievable through existing OD techniques. Satellite ephemerides are compared to known POE's, and to precise reference ephemerides generated by state-of-the-art orbit determination techniques.

INTRODUCTION

The requirements for precise orbit determination (OD) and propagation are becoming commonplace as numerical operations become standard. A distinction between post-processing and prediction is made in that most historical studies examine the ability to post-fit observational data. With all the input data measured and known, post processing often achieves accuracies of 2-10 cm radial position, even for orbits strongly influenced by non-conservative forces. Operational processing of the entire satellite population (TLE data from the analytical SGP4 theory for instance), achieves only km-level accuracy, but can achieve about 400 m (Phillips 1995) and perhaps 50-100 m today in post processing when using numerical methods. The large discrepancy is primarily due to the lack of sufficient quantity, geographic distribution, and quality of the observations, omitted force models, inadequate calibration, and other computational and technical limitations.

Despite the original source of initial state vectors, the important aspect for an operational planner is the ability to assess the accuracy of propagations from a known initial state. This is essential to make well-informed operational decisions and affects everything from simple mission planning and maneuver operations, to the broader concepts of Space Situational Awareness (SSA) and Single Integrated Space Picture (SISP). Consider the case where a close conjunction is predicted between two satellites. If one satellite is well tracked and has a sophisticated orbit determination resulting in a final estimated position accuracy of 10 m, while the other satellite has reasonable data, but lacks the ability to track a dynamically changing drag perturbation and achieves a 600 m final estimated state, does this tell us anything about the conjunction that is predicted to occur in one week? Actually, it's a very rough indicator, but not very important to the operational planner. What's most important is the ability to understand how those initial differences will translate into the future predicted positions at the time of the encounter.[†]

This paper seeks to clarify the ability to accurately propagate a state vector into the future. Several areas are examined.

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[†] An accurate covariance matrix that can propagate into the future and "predict" how the accuracy will degrade over time will also do this, but that is another issue and the subject of another paper. Most covariance matrices are not regarded as highly accurate and in general, they over-estimate the uncertainty at the future time.

1. How well can the propagator predict the initial state - no errors assumed in an OD process? This section seeks to qualify how well the propagator can emulate the perturbing forces, as used / defined in the Precision Orbit Ephemerides (POEs).

2. How well can the propagator predict using an initial state that includes various errors resulting from the OD process, say 1 m and 1 mm/s? These errors are introduced in the in-track direction as the largest source of uncertainty is generally in this direction, resulting from non-conservative forces.

3. How well can an optimal filter/smoother process the POE as an observation, and then take that state and predict into the future. This section seeks to determine if accurate recent satellite states are more important than an averaged final estimate in achieving accuracy for the prediction period. A side portion of this step examines the ability to process observations and form a reference orbit from which to make comparisons from. While not as accurate as a POE, it provides a glimpse of what is achievable.

4. Establish a framework from which to evaluate additional orbits as observational data and independent post-processed precise orbits become available.

Point #2 could be expanded to "perturb" the final state to be more reflective of what a skin-track orbit might be.

OBJECTIVE

This paper shows numerical results for propagating satellite state vectors against known POE's and against reference ephemerides generated by Analytical Graphics Inc.'s (AGI's) Orbit Determination Tool Kit (ODTK) optimal filter/smoother. All propagations are performed with AGI's Satellite Tool Kit/High Precision Orbit Propagator (STK/HPOP). The time span ranges from several hours to a couple of months, and includes tests for exact initial condition matches and expected real-world difference in initial position. The POE's are considered "truth" for this paper as their uncertainty is much less than the accumulated error in the propagation. When *reference orbits* are generated via OD, they are labeled as such to avoid confusion with truly independent precise reference orbits.

STUDY PROCESS

To effectively examine the behavior of propagation using different initial condition accuracies for various satellites, a multi-step process was used depending on the available data, shown pictorially in Fig. 1.

First, a single state vector from a POE was taken and used without modification in a propagation. The result was then compared to the remaining POE. The external development and post-processed accuracy of these POE's is generally well established, and often is within 2-10 cm radial position for the entire span. Compared to the errors we would expect from propagating an orbit, these can be considered near-zero, and to be truth.

Next, recognizing that orbit determination techniques are virtually never able to determine the satellite position to within 2-10 cm at epoch in real-time, noise was added to the initial states. Even satellites with GPS receivers may have state vectors that are a meter or two from the "true" satellite position. To determine the impact of this uncertainty for real-time operations, I made different displacements to the initial state vectors, and repeated the comparison process of the first step.
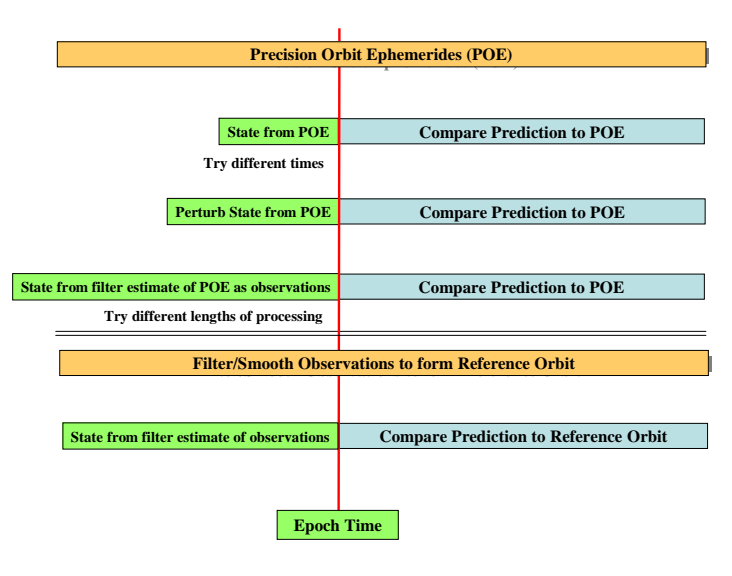
Figure 1: General setup for Prediction Comparisons: Several comparisons are found using the POE's directly. Other orbits require generation of reference orbits. The distinction is important as the independent nature and accuracy are not generally the same.

The last step is more realistic in terms of processing observational data, but less accurate as no POE information was available for the comparison. This step created reference orbits from existing GPS, Satellite Laser Ranging (SLR), or other data that was available. Unfortunately, this processing usually produces reference ephemerides that are accurate to many meters, not a few cm. However, when a small portion of that data is processed by the filter, the satellite parameters can be modeled and allowed to vary dynamically, both in the OD and in the prediction phases. Essentially, the previous steps were constrained to use a given c_D, c_R, etc. (values which may have been in error), but this step allows the OD technique to solve for a time-varying value for each parameter.

Additionally, because the underlying mathematical technique for ODTK is a real-time Kalman Filter (Vallado, 2007, Sec 10.6), there is no fit-span. Thus, almost any length of data will suffice and the final state estimate will be the best solution at that point. Of course *some* data is needed to determine each parameter, but because the filter process sequentially, it always continues to improve. Most studies that examine OD results use extensive algorithms to test and ensure the fit span is adequate, but one never really knows if the choice was correct and that all the dynamics were modeled when looking at the results in real-time. The filter does not suffer these limitations.

The propagation span for each ephemeris was generally kept at about 7-14 days. Although the results at the end of this time showed some large differences, 4-7 days is generally about the event-horizon at which final operational decisions are made. Differences are computed at many times during the ephemeris span to provide the reader with a look at the time-varying trends.

SATELLITES CONSIDERED

The initial task was to select a number of satellites with existing POE information. The satellites selected for this study form a spectrum from LEO to GEO satellites (Table 1). The epochs varied quite a bit due to the presence of available information. A month of data was

selected for as many satellites as possible, and periods of higher solar activity were preferred to show the enhanced effects of atmospheric drag on the lower satellite orbits. The additional satellites in the LEO category were designed to better determine the results in the drag regime, and at the lower end of the solar radiation pressure regime.

Table 1: **Satellite Orbital Parameters:** This table lists the general orbital parameters for each of the study satellites. Note that additional POE information is available, but the orbits are generally very close to those given here.

Category	SSC #	Name	a (km)	e	i (deg)	Period (min)
LEO	27642	ICESat	6973	0.00024	94.00	97
LEO	21574	Ers-1	7151	0.00333	98.24	100
LEO	23560	Ers-2	7162	0.00012	98.54	100
LEO	26997	Jason	7715	0.00075	66.04	112
MEO	25030 PRN-08	GPS Block IIa	26560	0.00375	56.01	718
MEO	28129 PRN-22	GPS Block IIr	26560	0.00375	54.51	718
MEO	29486-PRN-31	GPS Block IIr-m	26560	0.00375	55.11	718
GEO			35780	0.00007	0.03	1436

The study satellites permitted a quick look at various orbital classes, but the specific satellite parameters (c_D, c_{SR}, m, A, etc) were sometimes difficult to obtain. Table 2 lists some approximate values used in the analysis. Note that these values are somewhat arbitrary in most analyses and selection of one parameter to absorb the total error is often done. One could imply that ρ, m, A are completely known and that all the remaining error is from a changing c_D value (Bowman, 2007). However, without extensive processing and knowledge of each individual satellite, its materials, atmospheric composition, etc, this is not realistic and ignores many previous studies on the physical nature of c_D (Gaposchkin 1994 for instance).

Table 2: **Satellite Parameters:** Many of these parameters were assumed, or taken from information derived from the Internet. The mass should generally be assumed as the initial mass. These are not intended to be definitive values!

Name	Apogee Alt (km)	Perigee Alt (km)	c_D	Mass (kg)	Area (m^2)	c_{SR}
ICESat	596	594	2.2	970	2.0	1.0
Ers-1	797	750	2.5	2377.13	11	1.0
Ers-2	785	783	2.5	2377.13	11	1.0
Jason	1343	1332	2.2	489.1	9.536	1.2
GPS Block IIa	20438	19927	2.2	970	18.02	1.0
GPS Block IIr	20311	20052	2.2	1100	19.38	1.0
GPS Block IIr-m	20358	20005	2.2	1100	19.38	1.0
GEO	35781	35780	2.2	2000	70	1.0

DATA SOURCES

Most of the POE's came from the UT/Austin/CSR website. These sources are extremely useful for studies as raw observational data are more difficult to come by, and processing the

data adds an extra source of potential error contribution. However, this is changing with some satellites having SLR data, GPS measurements, and on-board accelerometer data.

For this paper, I commonly used the WGS-84/EGM-96 and EGM-96 gravity models shown below. The parameters are important to ensure compatibility with external organizations. STK/HPOP is designed to use an ASCII file for the gravity model, including the defining coefficients.

For EGM-96
1. Gravitational Parameter $\mu = 398600.4415$ km^3/s^2
2. Radius of the Earth $r = 6378.1363$ km
3. Flattening $f = 1/298.257$
4. Rotation rate of the Earth $\omega = 7.292158553e-5$ rad/s

For WGS-84/EGM-96
1. Gravitational Parameter $\mu = 398600.4418$ km^3/s^2
2. Radius of the Earth $r = 6378.137$ km
3. Flattening $f = 1/298.257223563$
4. Rotation rate of the Earth $\omega = 7.292158553e-5$ rad/s

The sources of data for Earth Orientation Parameters (EOP) and space weather are somewhat standard and I used the data from CelesTrak which is a consolidated accumulation of the past, present, and future data from the defining data locations on the web (Vallado and Kelso, 2005).

http://celestrak.com/SpaceData/

Coordinate systems varied (ITRF, TOD, IAU-76/FK5, etc.) but STK/HPOP accepts any of these systems and applies standard reduction techniques to accomplish the propagations (Vallado, 2007:228).

Finally, the study intervals were quite varied, depending on the available data. Some satellites contained maneuvers in the data. When the maneuvers were known, the analysis simply proceeded through each maneuver. When the maneuvers were unknown, different time periods were selected. Table 3 shows these intervals.

Table 3: **Study Intervals:** The time intervals for each satellite varied depending on availability and time to show certain perturbing effects.

Name	Study Interval	Comments
ICESat	Feb 2003	Modest solar flux, a few known maneuvers
Ers-1	Aug 1991	Solar flux increased
Ers-2	Sep 1995	Modest solar flux
Jason	Apr 2002	Modest solar flux
GPS Block IIa	Dec 06 - Jan 07	Maneuver during the first week
GPS Block IIr	Dec 06 - Jan 07	No maneuvers
GPS Block IIm	Dec 06 - Jan 07	No maneuvers
GEO	Jul - Sep 2007	Many known maneuvers

PROPAGATING THE INITIAL STATE VECTOR

The first step was to propagate a single initial state from the POE's, and compare the results to the POE. POE's are often regarded as "truth" because they are developed after all the data is collected, and extensive analysis, processing, smoothing, etc. can be applied to minimize errors throughout the ephemeris time span. This phase was intended to show the ability of the

propagator to match the "truth". While the POE's are derived from independent processing, their accuracy is usually sub-meter, in which case we can consider them as truth for our purposes.

Note that POE's are generally not available in near-real time, so the approach I've taken here is to simply take the first state vector from the POE, and propagate it through time. This should conservatively bound the "expected" results one could see, if the dynamics were known perfectly in the estimation process and you were able to get an OD estimate that was good to about 5 cm in real time. The plots contain the results for an "exact" fit, and subsequent propagation.

As I showed in 2005, any number of propagation programs may be used to move the satellite state forward in time … as long as detailed information about the force models and parameters is known. Unfortunately, most POE sources provide little to no additional information about the specific parameters.[*] The behavior of various satellites to individual perturbations (see Vallado 2007 for individual graphs) shows that atmospheric drag and solar radiation pressure contribute the largest uncertainty by at least an order of magnitude. Thus, difficulties in knowing what c_D, c_{SR} were used created the largest uncertainty. Later, I introduce one solution to this through OD runs of the POE data.

ICESat
Study interval: Feb 19, 2003 21:00:00.000 UTC – Mar 24, 2003 00:00:00.000 UTC

Information on the POD aspects of ICESat was obtained from the GLAS POD document (Rim and Schutz, 2002). For a starting estimate, I used a mass of 950 kg assuming the vehicle had used some maneuvering fuel. I'll show 4 graphs to illustrate the component (normal, tangential and cross-track, NTW, Vallado, 2007:165) behavior of the predictions for varying time periods – 2 hours, 1 day, 1 week, and 1 month. The NTW components remain valid for highly eccentric orbits should they be considered.
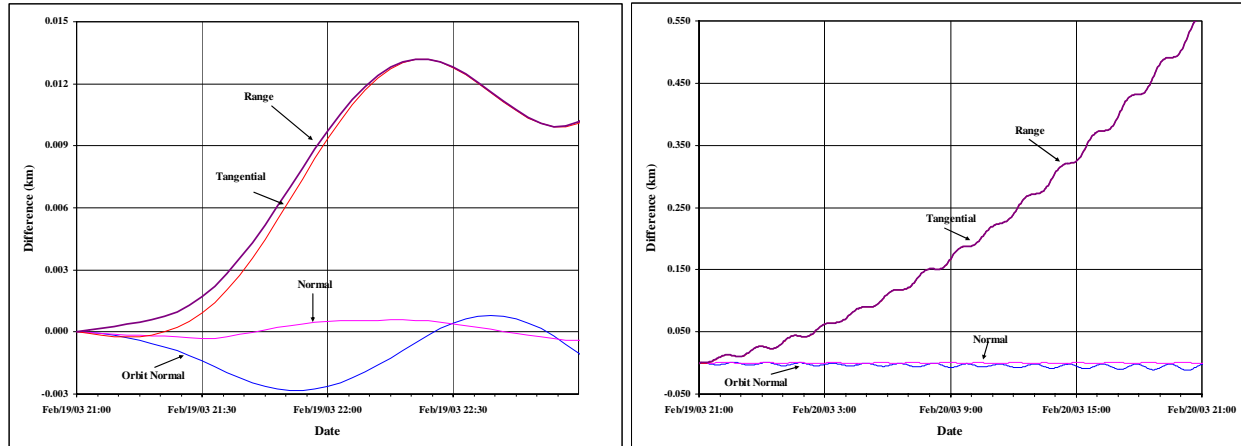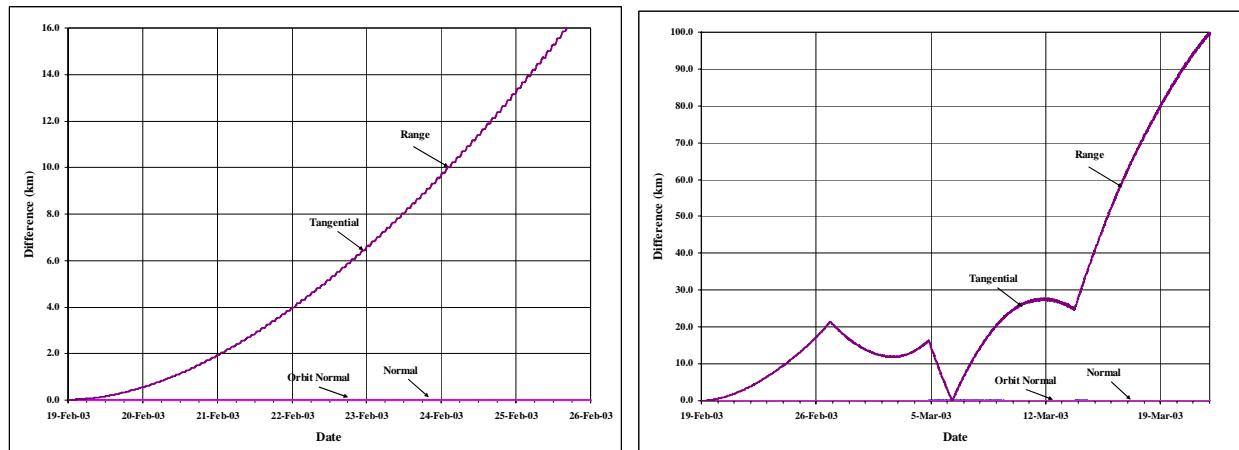

Figure 2: ICESat POE Ephemeris Comparisons: Results are given for ICESat at 2 hours (left) and at 1 day (right). Notice that the tangential component becomes dominant after only about 45 minutes.

When the scope is expanded to one month, maneuvers become evident. The presence of unknown maneuvers in the data limited the length of immediate analysis that could be performed. The maneuvers are apparent for February 27, March 5, and March 14. Time did not

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[*] This is not unusual as single values for c_D, c_{SR}, etc are generally not used in the development of POE's. Rather, each OD program is typically designed and tweaked to best match the particular satellite under consideration. The propagation is a combination of data processing that usually includes unmodeled accelerations, once per cycle variations, etc. These are very hard to match with a single value, and hence, separate OD is recommended for applications requiring analysis outside the interval of the original POE.

permit an adequate evaluation of the maneuvers, but the ability of the filter to solve for the maneuver appears to be the best choice to propagate through the event and extend the analysis.

Figure 3: ICESat POE Ephemeris Comparisons: Results are given for ICESat at 1 week (left) and at 1 month (right). Notice that the pronounced appearance of maneuvers in the long range comparison.

The satellite parameters were investigated and the area was changed to 5.21 m^2 because the satellite was in a primarily "sailboat" orientation (solar panel edges into the velocity direction). The c_D was also changed to 2.52 to better align with initial POE generation values. The results show a dramatic improvement (note that the maneuvers are still present).
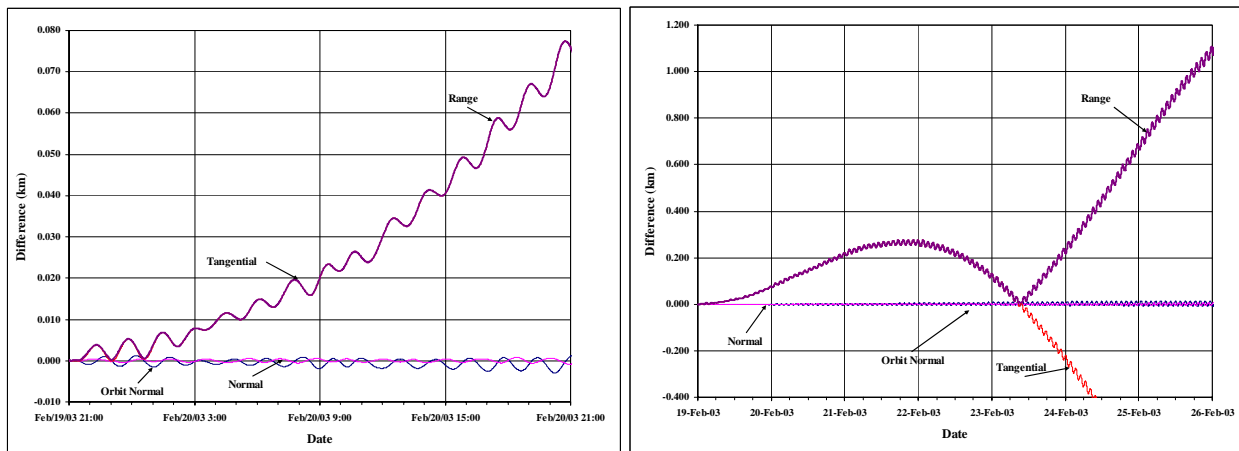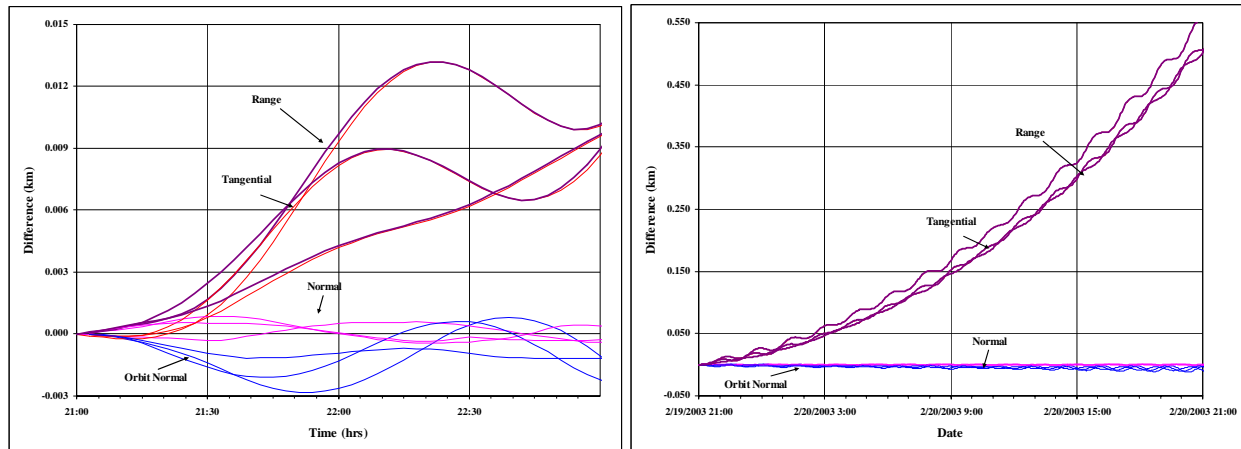

Figure 4: ICESat POE Ephemeris Comparisons Using Improved Satellite Parameters: When the satellite parameters are updated, the performance is much better. Shown are the 1 day performance (left), and the 1 week performance (right). Comparing to Fig 2, the 2 hour result is 4 m versus 15 m, and the 1 week performance is about 1 km versus 16 km.

The important point here is that the results changed dramatically by simply changing the initial input parameters. This reinforces the importance of modeling each satellite with as much precision as possible – and again points to OD of the POE data to determine the individual satellite parameters. One could take the initial estimate and allow a single parameter to change in the OD, with the resulting simulation absorbing all the differences in that single variable and claim that the problem is uniquely determined. However, by removing as much uncertainty from

each parameter as possible, the orbit determination is better able to find the "true" orbital state, and the results are reproducible over various time periods and orbital regimes.

Several different start times were examined to determine if the initial state from the POE affected the results. The results were different for the short term, but quickly matched the original results. The following figure shows start times of 21:00, 21:20, and 21:40 UTC. Because the data starts at different times, the data is actually displaced, but the figure shows all three times beginning at the initial time for easier comparison. The general trend was the same, but especially in the 2 hour interval, the shapes were quite different. Past 1 day though, the results were essentially the same.
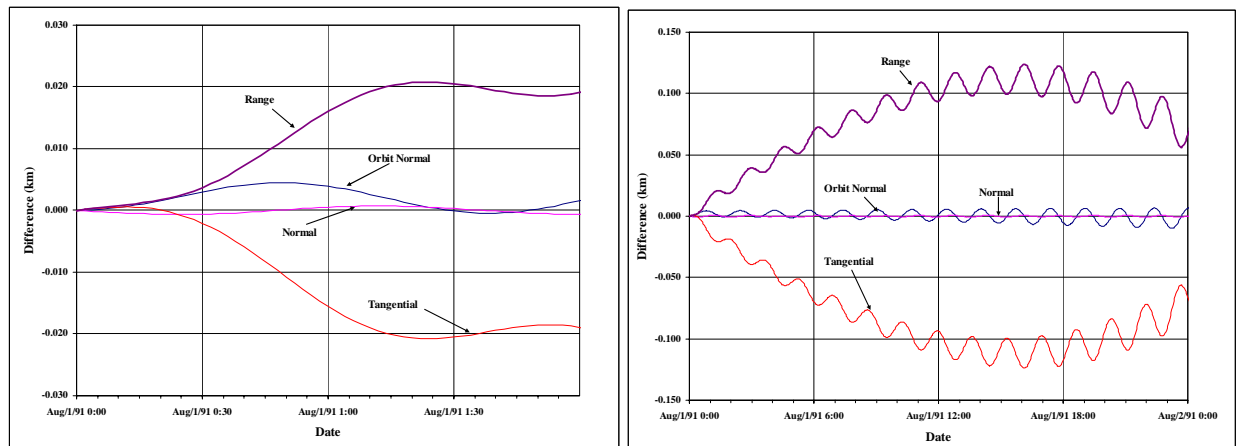

Figure 5: ICESat POE Ephemeris Comparisons: Results are given for ICESat at 2 hours (left) and at 1 day (right) using three different starting locations (each differs by 20 minutes). Notice the difference in the short term results.

This may be an artifact of the smoothing process used to create the original POE's or some aliasing resulting from poorly known satellite parameters, but time did not permit an adequate investigation.
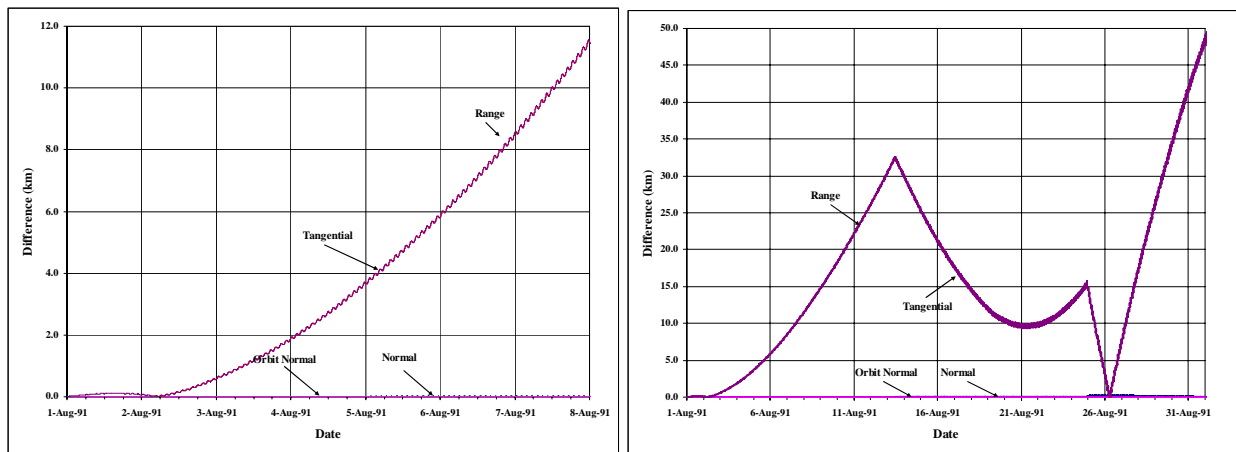
ERS-1

Study interval: 1 Aug 1991 00:00:00.000 to 2 Sep 1991 00:00:00.000 UTC

There was not as much information available on the POE formation for the ERS-satellites, thus the estimates in Table 2 were used as a starting point. The mass was 2200 kg, and the area about 11 m^2. The results are very similar (in general trends) to the ICESat results including some apparent maneuvers in the long time period plot.
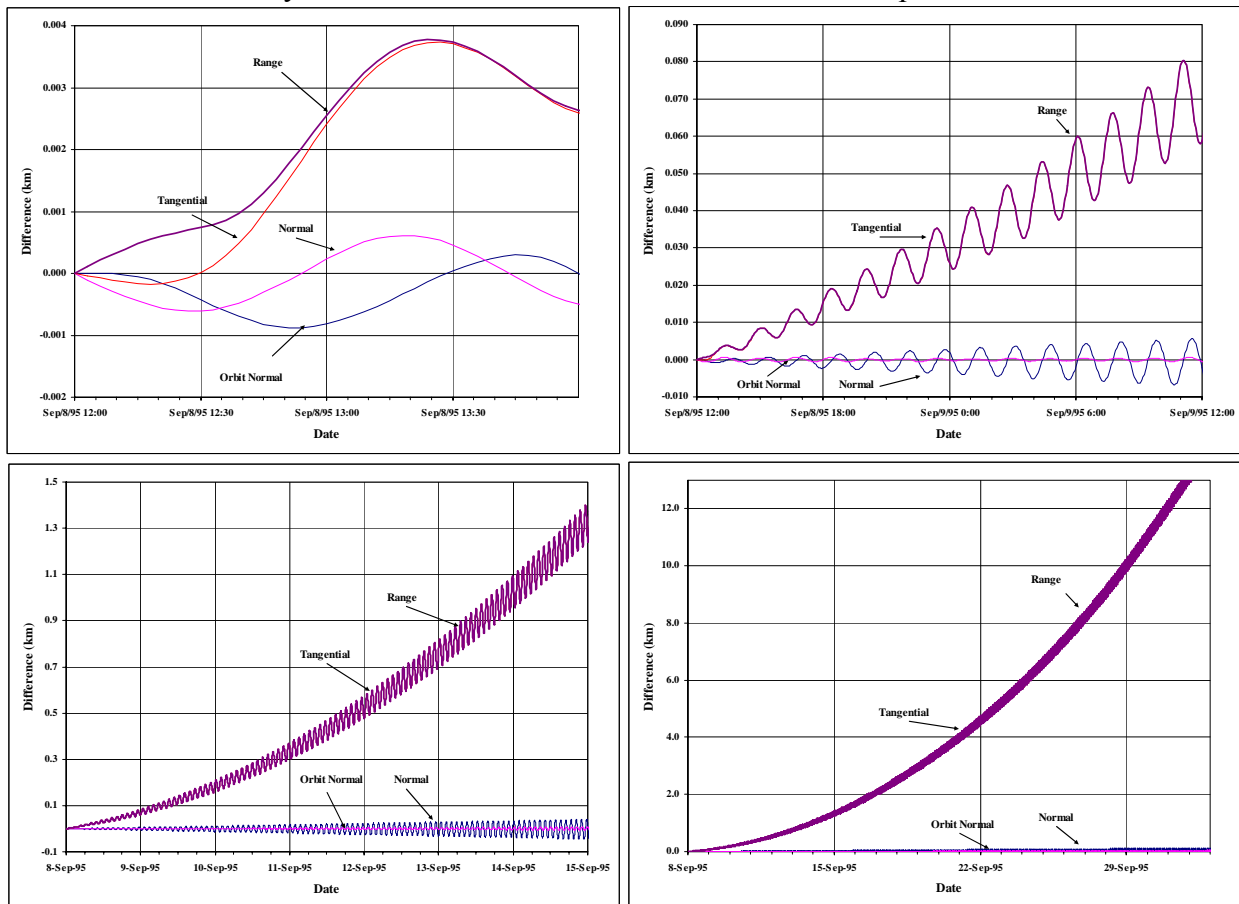
Figure 6: ERS-1 POE Ephemeris Comparisons: Results are given for ERS-1 at the four time intervals. Notice that the pronounced appearance of maneuvers in the long range comparison.

ERS-2

Study Interval: 8 Sep 1995 12:00:00.000 to 10 Oct 1995 00:00:00.000 UTC

The satellite parameters were the same as ERS-1. The mass was 2200 kg, and the area about 11 m^2. The study interval was chosen to avoid maneuvers on September 5th and 6th.
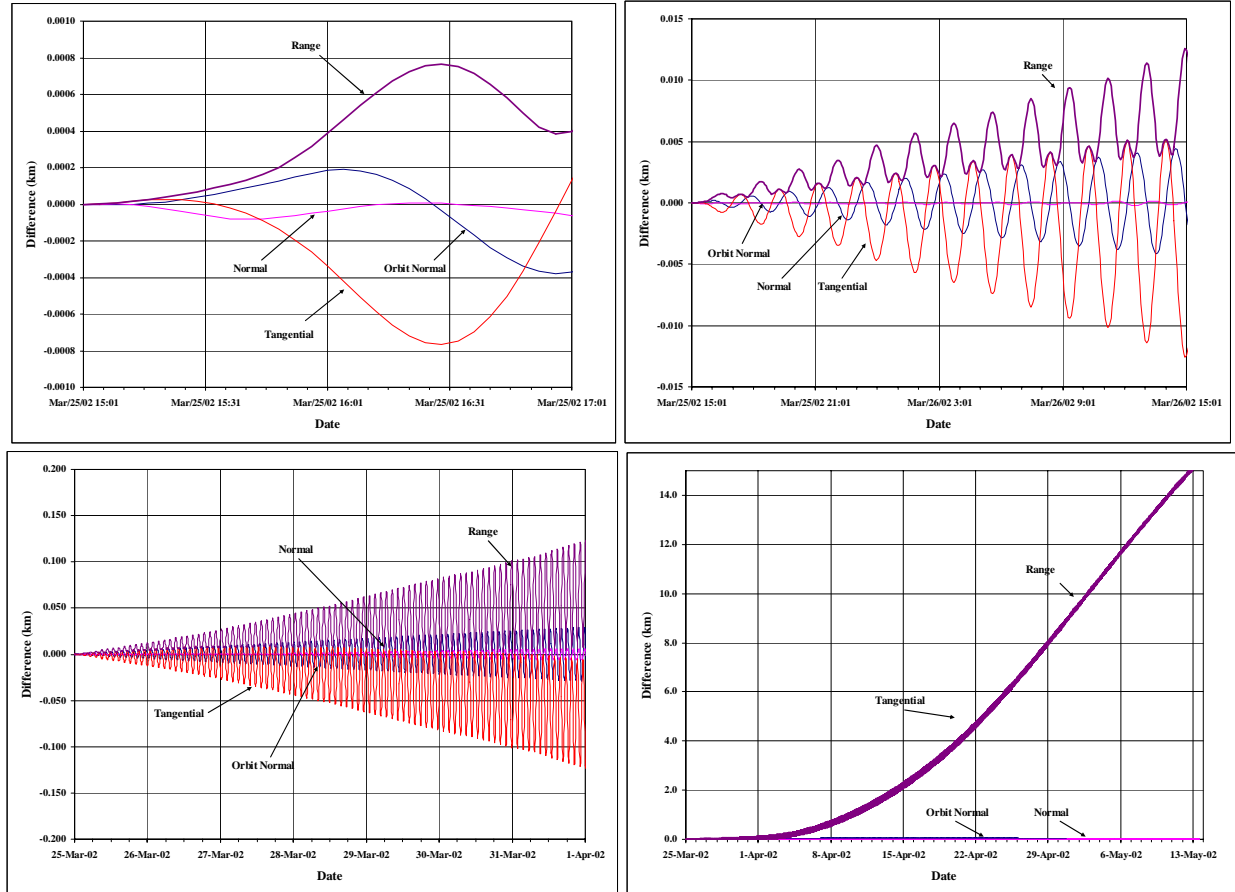

Figure 7: ERS-2 POE Ephemeris Comparisons: Results are given for ERS-2 at the four time intervals. There did not appear to be maneuvers in the long range comparison.

JASON

Study interval: 25 Mar 2002 15:01:00.000 – 14 May 2002 04:42:00.000 UTC

We would expect that the results for Jason would be better than the lower altitude satellites because atmospheric drag is significantly reduced at this altitude. There was not detailed information on the POE formation, so I used the general values from Table 2. The mass was 475 kg and the area about 9.536 m^2. After about 24 hours, the following figure shows the prediction to be accurate to about 15 m. Notice the variation in the 1 day and 1 week plots. The scale is small, but the effect is simply a result of solving the second-order, nonlinear, equations of motion. The degree with which the oscillations occur is a function of the orbit, the relevant perturbations, the modeling of the satellite parameters, and mostly the scale.

Figure 8: Jason POE Ephemeris Comparisons: Results are given for Jason at the four time intervals. The results are very similar to the ERS results.

GPS

To adequately examine the GPS satellites, specific information is required on the solar radiation pressure modeling. There are different models for each block of satellites, and these make a large difference when trying to compare results. All the GPS models use 2 solar radiation pressure scaling coefficients (k_1, or Scale, and k_2, or Y-Bias). Scale multiplies the model in body-specific equations in the X and Z directions, while Y-Bias estimates the acceleration in the body Y-direction. Both values are usually near unity, but will differ over time. Table 4 lists some of the models available in STK/HPOP, and used in the GPS ephemeris generation.

Table 4: **GPS Solar Radiation Pressure Models:** The various GPS satellite blocks use different solar radiation pressure models, although they do share many similarities. Note that NGA uses the Bar-Sever models in the formulation of the GPS POE data.

Block	Name	Reference	Update	Update
Block I	ROCK 4	Rockwell, Fliegel, Gallini, and Swift	T10	
Block II, II-A	ROCK 42	Rockwell, Fliegel, Gallini, and Swift	T20	GPSM.IIA.04, Bar Sever at JPL
Block II-R	Table look-up	Lockheed		GPS.IIR.04, Bar Sever at JPL
Block IIR-M	Table look-up	Lockheed		GPS.IIR.04, Bar Sever at JPL

GPS SVN-22

Study interval: 26 Nov 2006 00:00:00.000 to 31 Jan 2007 23:45:00.000 UTC

Using a/m = 0.01569, the mass was about 1100 kg. The k_1/k_2 parameters were both left at 1.0. We'll see later that this is not a very good approximation, but it serves to initiate the problem.
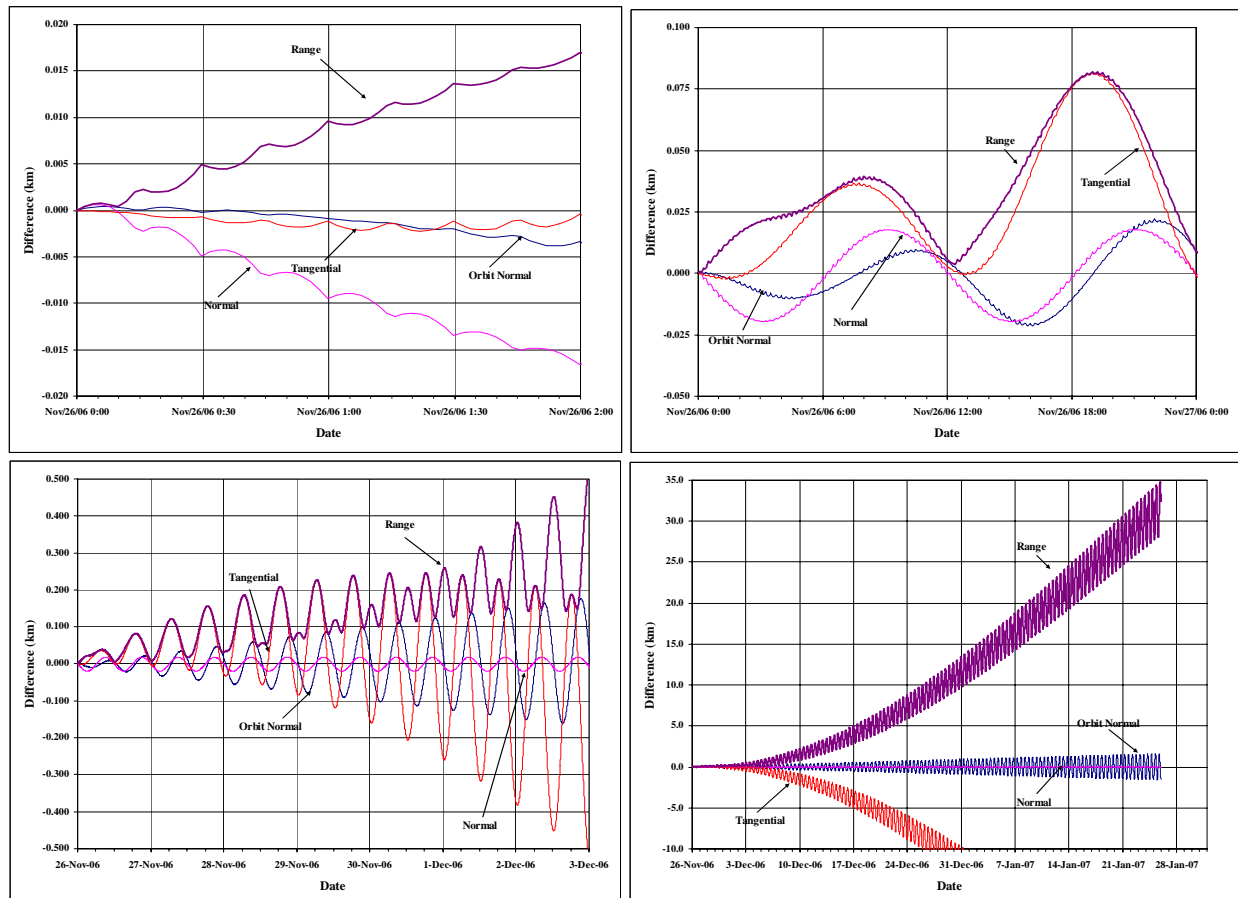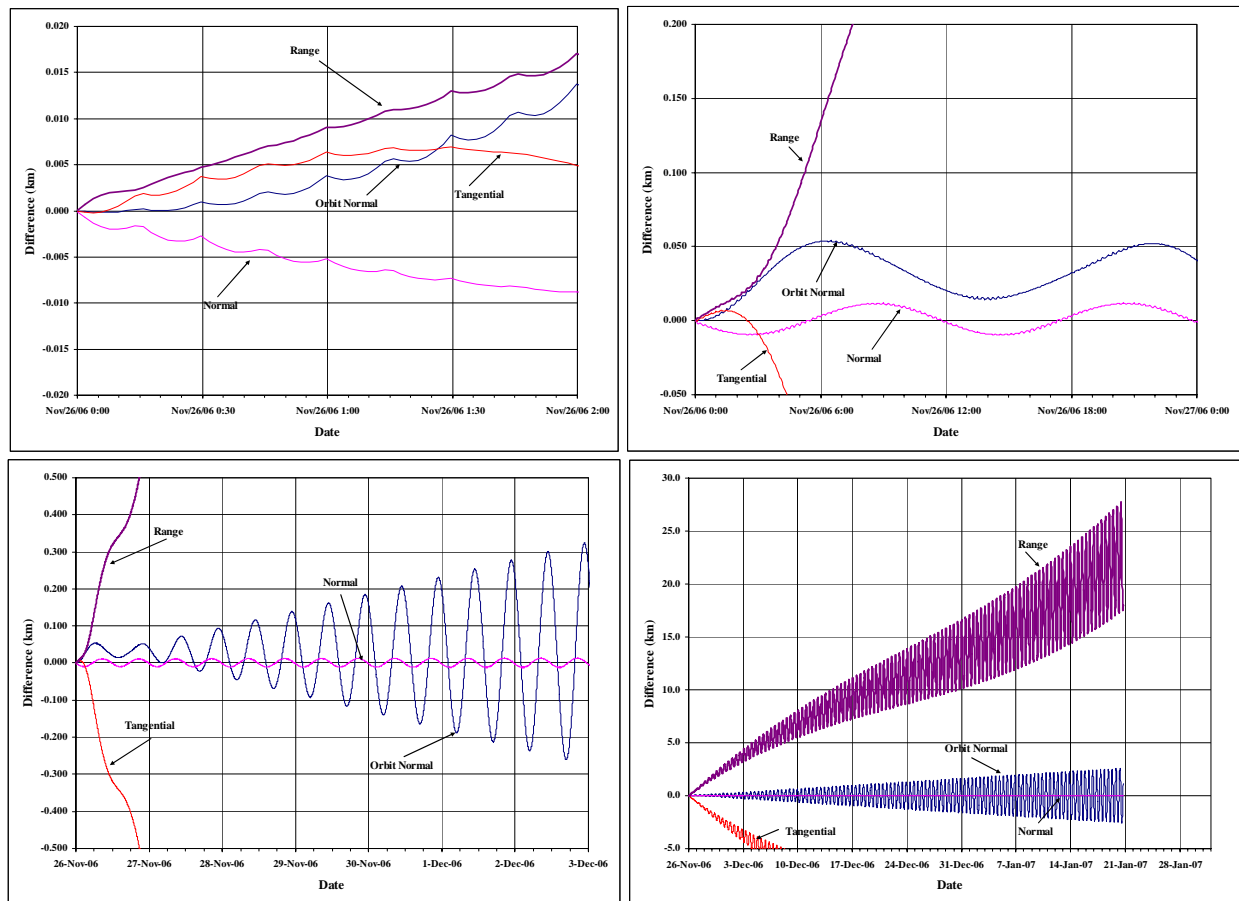

Figure 9: GPS SVN-22 POE Ephemeris Comparisons: Results are given for GPS SVN-22 at the four time intervals. The final plot is for about 2 months. Notice that the slight change in behavior at about the 2-3 week point.

GPS SVN-31
Study interval: 26 Nov 2006 00:00:00.000 to 31 Jan 2007 23:45:00.000 UTC
Using a/m = 0.01569, therefore mass was about 1100 kg. The satellite enters eclipse in January 2007 and both k_1/k_2 parameters were set to 1.0.

Figure 10: GPS SVN-31 POE Ephemeris Comparisons: Results are given for GPS SVN-31 at the four time intervals. The rapid departure suggests some of the modeling is not properly set.

PERTURBING THE INITIAL STATE VECTOR

For real operations, the state is not known with the same level of accuracy as is available in post-processing. Thus, even rapid ephemerides from a GPS receiver will differ by perhaps 1 m, and / or 1 mm/s at any epoch. To simulate this more realistic scenario, the initial state vector from the POE was changed to include this offset (in the tangential direction), and the prediction through time is then performed. This should conservatively bound the "expected" results one could see, if the last update (say from a Kalman filter of on-board GPS measurements) was used for near real-time prediction calculations.

The plots contain the results for an initial state that was perturbed in the tangential position axes by 1 m, and each velocity axis by 1 mm/s. These errors were intended to simulate performance under actual conditions when the initial state would not have cm-level accuracy (as in the POE), but something larger.

Because these analyses are being done against "truth" (the POE), one would expect the errors to grow due to two factors. First, the inadequacy of the force models, space weather data,

etc. causes error growth (as seen in the first tests where the exact initial state was used). Next, the initial displacement from the POE also introduces an error – which turns out to be an even larger effect in some cases. Although the argument can be made that in practice, you would have better knowledge of the satellite characteristics, the departure from "truth" will still result from both sources. For satellites experiencing significant atmospheric drag, this effect can overwhelm any benefit of a good initial state vector, or satellite characteristic, in just a few revolutions, depending on the orbit. From previous analyses conducted by the author, the effect of velocity uncertainty is sometimes more influential than that of position, although I did not investigate that thoroughly here. The primary goal was to examine what impact the additional uncertainty in the initial state vectors had on the transient behavior. Thus, only the first few hours were examined as after that time, the non-conservative forces would mask any initial uncertainty.

The first runs were conducted on ICESat and ERS-2. As expected, the results look very similar, with the ERS-2 performance being slightly better because it's in a slightly higher orbit.
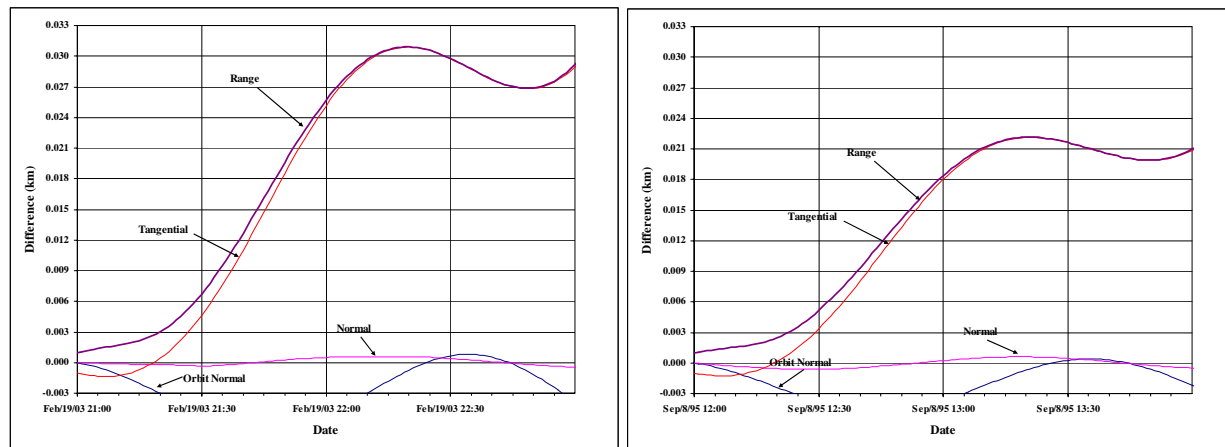
Figure 11: POE Ephemeris Comparisons: Results are given for ICESat (left) and ERS-2 (right). The similarity of the orbital altitudes produces very similar results. Notice the difference in the orbit normal component. In both cases, the dominant feature is the initial tangential displacement. The scales are the same between the two graphs.

Offsets were applied in different axes to determine if the transient effects would differ greatly. For example consider ERS-2. If we perturb the initial normal component, the tangential difference still quickly masks any transient changes (note the scale change from Fig. 7). ICESat is very similar as in Figs. 2 and 3.
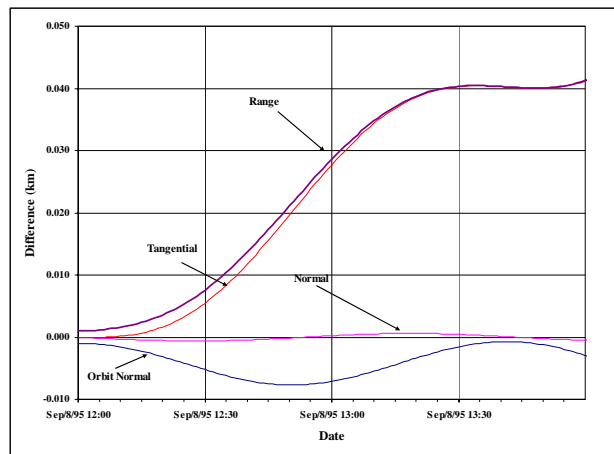
Figure 12: ERS-2 POE Ephemeris Comparisons: Results are given for ERS-2 with an initial normal displacement. The initial normal difference is quickly masked by the tangential component.

OD USING THE POE'S

The most realistic approach to investigate prediction accuracy is to use actual observational data that would be available in near-real time, perform a differential correction, and then propagate the result into the future. Comparing to the POE (at a later time), would then reveal the uncertainty involved with this operation. This process has several important benefits, including comparison with a truly external and independent source for producing the POE. In addition, most POE's are developed by fusing data from several sources, thus providing an additional level of confidence in the final result.

The difficult step is obtaining the data. Satellite Laser Ranging Data (SLR) exists for many satellites, and GPS and accelerometer data are available from some sources. However, obtaining these data at the same time a POE is developed and available is difficult. In the absence of raw observational data, one can also use the POE itself as a data source – the case used here. This requires that the POE be long enough that you could get a "converged" solution and still have enough data to perform a comparison with. The absence of maneuvers is also desirable. Several satellites met these criteria.
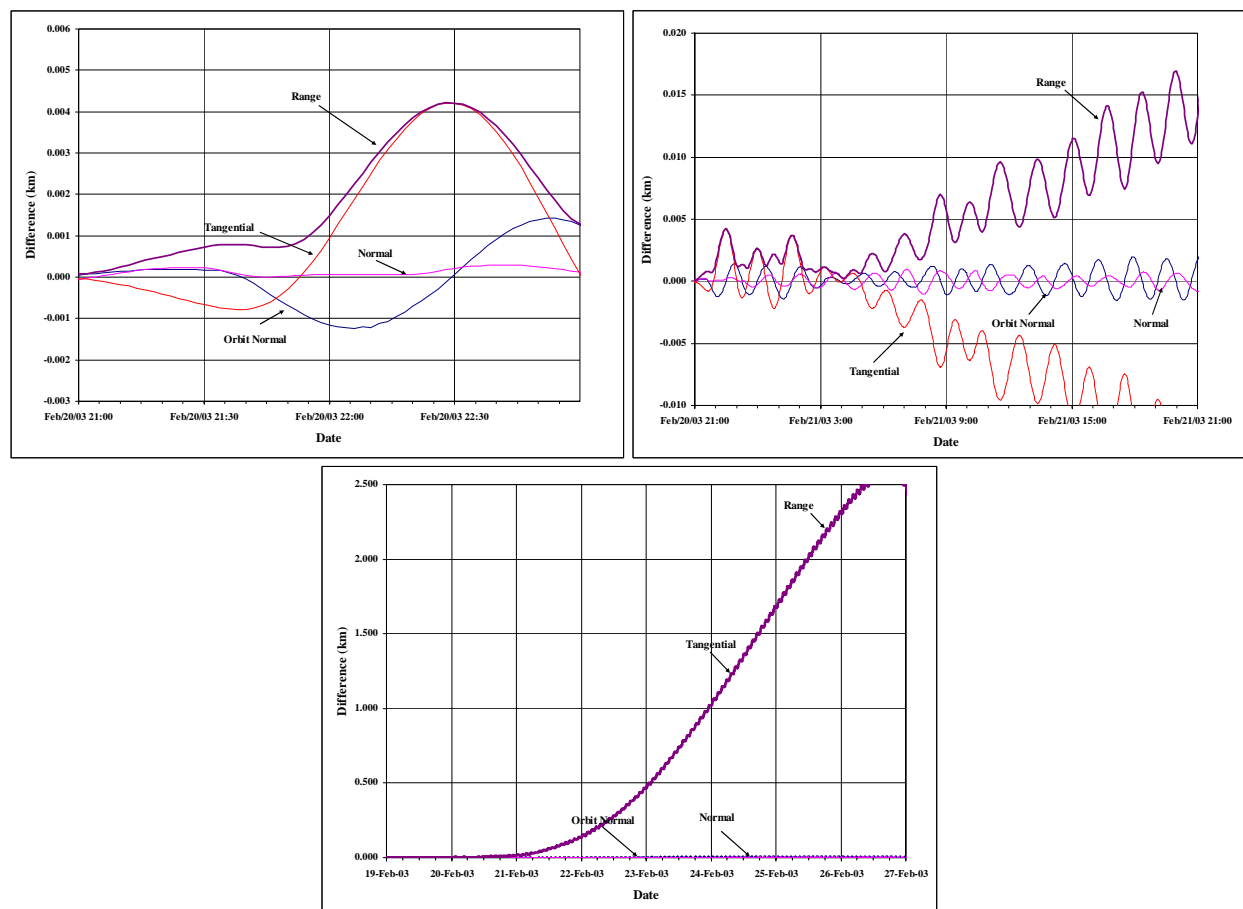
ICESat

Study period: Feb 19, 2003 21:00:00.000 – Mar 1, 2003 21:00:00.000 UTC

The initial satellite parameters were used as identified previously. The GLAS document (Rim and Schutz, 2002) provided additional details about the formation of the POE. From this report, several key points were obtained to assist the setup of the prediction runs. The *"position of the GLAS instrument should be known with an accuracy of 5 and 20 cm in radial and horizontal components, respectively."* To obtain this accuracy, *"the adopted approach for ICESat/GLAS POD is the dynamic approach with gravity tuning and the reduced-dynamic solutions will be used for validation of the dynamic solutions.* In addition, to *"account for the deviations in the computed values of density from the true density, the computed values of density, ρ_c, can be modified by using empirical parameters which are adjusted in the orbit solution. Once-per revolution density correction parameters [Elyasberg et al., 1972; Shum et al., 1986] have been shown to be especially effective for these purposes.* As is often done, to "account for the unmodeled forces, which act on the satellite or for incorrect force models, some empirical parameters are customarily introduced in the orbit solution. These include the*

empirical tangential perturbation and the one-cycle per- orbital-revolution (1 cpr) force in the radial, transverse, and normal directions [Colombo, 1986; Colombo, 1989]. Especially for satellites like ICESat/GLAS which are tracked continuously with high precision data, introduction of these parameters can significantly reduce orbit errors occurring at the 1 cpr frequency and in the along track direction [Rim et al., 1996]."

The number of "additional" parameters (1 cpr, unmodeled accelerations, reduced dynamics, etc) in the POE formation confirms the need for separate OD runs using the POE data as an input. In ODTK, the process is relatively straightforward to estimate these parameters from a POE. The .sp3 vectors are formatted as NAV solutions (.navsol) and the filter then processes the state vectors as observations. The bias values were set to zero, and the bias sigmas were set to 0.5 m. With full force models (70x70 EGM-96 gravity, NRLMSIS-00 atmospheric drag, Sun and Moon third body, solar radiation pressure, solid and ocean tides, albedo) we run the filter using 1 day of the POE information as observations, and then predict for about a week and a half. The results show a marked improvement over the state propagation in Figs. 2 and 3. Note the maneuver at the end of the week prediction, and thus, no month long prediction plot.

Figure 13: ICESAT OD and POE Ephemeris Comparisons: Results are given for ICESAT after a 1 day OD of the POE data. The tangential difference still dominates very quickly, but the overall results are much better than the original prediction because the satellite parameters are better known from the OD portion of the run.

Several things stand out.

1. The expected result of using information from an estimation process to predict into the future is demonstrated. The OD provides the detailed information on the satellite parameters and

coefficients that may not be available from the associated literature of the POE (c_D, c_{SR}, A, m, etc). The importance of obtaining the proper satellite parameters is also acknowledged in Vallado (2005).

2. The prediction can be quite accurate, even in the presence of non-conservative forces if the proper input data is used. A difference of 2.5 km over about a week for a LEO satellite should not be considered as too bad in the presence of atmospheric drag, but is not quite as good as the original prediction (Fig. 4). This is because the filter solves for satellite parameters, but lets them decay via several half-life parameters. Thus, much like the batch least squares techniques that follow the general rule of thumb that they will predict best in the period equaling the fit span, filters have a period for which they are most accurate (although this example seems to indicate a period of about 3-4 times the length of the data processed). Time did not permit a detailed look at this trend, but it will be the subject of a future paper. More importantly, the error is only about 300 m at 3-4 days in the future. This should be quite adequate for many near-term planning operations.

GPS

The GPS satellite orbits provide a very useful set of data as several years of POE information are available. The higher orbital altitude removes the possibility of evaluating atmospheric drag, but introduces the analysis of solar radiation pressure, the other dominant non-conservative force. Because the original results differed for the two satellites, both were examined here.

Once the mass and area were set (as close as possible), the filter was used to solve for the k_1/k_2 parameters. Several parameters were found. The half-life of the sigmas seemed to perform best when set to 43200 minutes (30 days). The sigma values provided a tighter estimate to the subsequent POEs when they were larger than the state estimates.

For SVN-22, the International GPS Service (IGS) rapid orbits were used as observational data. They have stated accuracies of about 5-10 cm radial position, so they are essentially truth for our purposes. After 3 iterations, the k_1/k_2 /sigma values were "steady" and the 2 month prediction was performed, using full force models (70x70 WGS-84/EGM-96 gravity, Sun and Moon third body, solar radiation pressure with Bar Sever model). The final k_1/k_2 values were 1.00433 / -0.55936, with sigmas of 20 and half-lifes of 43200 minutes. The results are shown in the following figure. Notice that the initial uncertainty is about 20 cm which is reasonable considering only 4 days of observations were used.
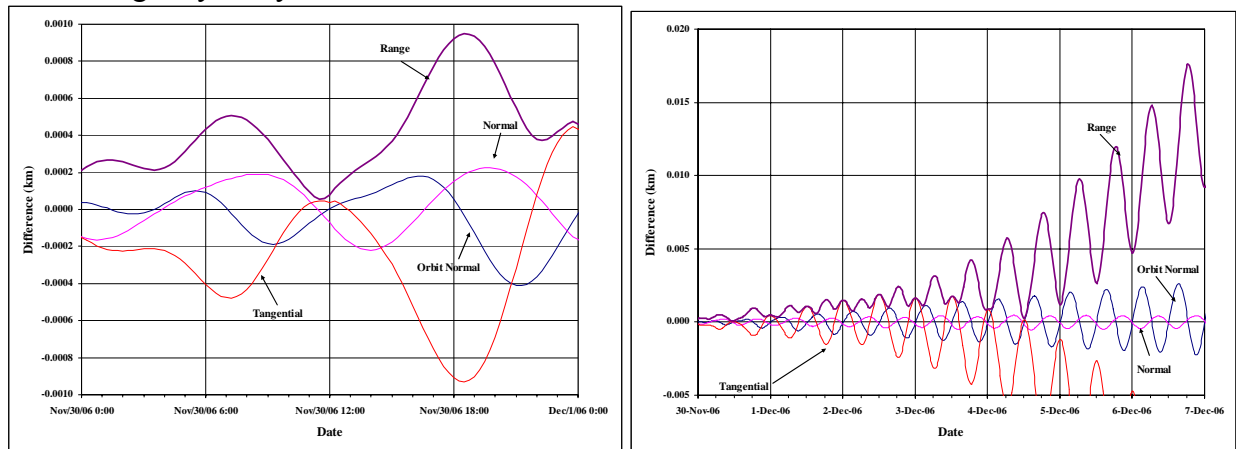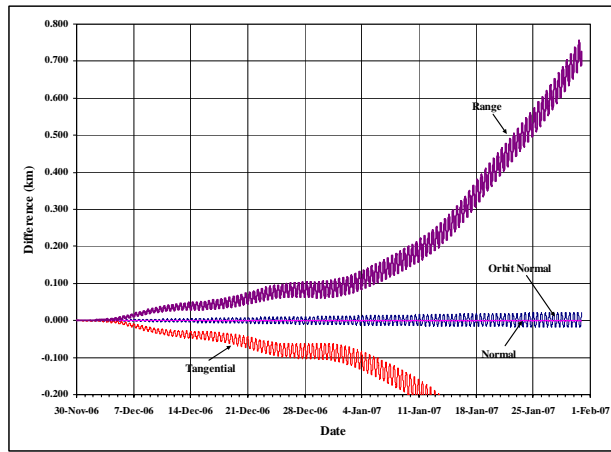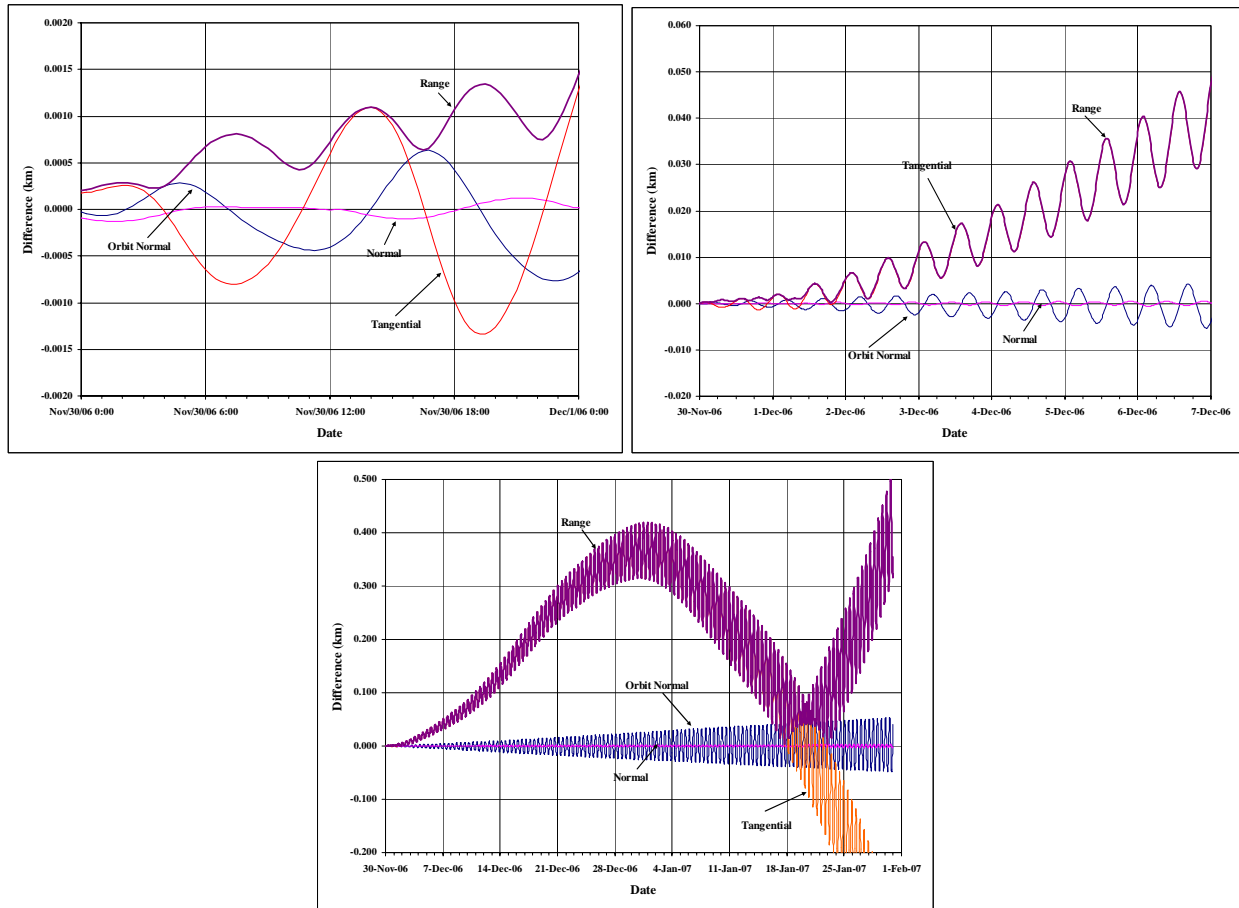
Figure 14: GPS SVN-22 OD and POE Ephemeris Comparisons: Results are given for GPS SVN-22 after a 4 day OD of the POE data. Notice the smoother overall performance of the differences.

For SVN-31, also using a 4-days worth of POE data as the observations, the results are as follows. The final k_1/k_2 values were 0.986206 and -0.334303, with sigmas of 20, and half lifes of 43200 minutes.

Figure 15: GPS SVN-31 OD and POE Ephemeris Comparisons: Results are given for GPS SVN-31 after a 4 day OD of the POE data. Notice the smoother overall performance of the differences and the significant improvement from Fig. 10.

The results in Fig. 15 show the dramatic improvement when more accurate satellite parameters are known.

SELF GENERATED REFERENCE ORBITS

The previous section examined using a portion of the POE's as observations, and then comparing to the entire POE. Unfortunately, POE's aren't available for all satellite classes, and there are cases where the satellite observational data is available without any POE. We examine that case here.

LEO

LEO orbits are probably best analyzed using accelerometer and GPS data that exist for some satellites. Time did not allow for these evaluations for this paper.

HEO

Highly Elliptical Orbits (HEO) are probably the most difficult satellites to model as they experience all forms of perturbations, including solar radiation pressure and atmospheric drag, In addition, data is extremely limited for these satellites. These facts, combined with a lack of time did not allow for these evaluations for this paper. I am seeking observational data to complete this portion of the analysis.

GEO

POE's are not readily available for Geosynchronous satellites, but I obtained some data which permitted me to form a reference orbit (using the filter-smoother combination in ODTK). The approximate accuracy was about 200 m throughout the time interval. I then took a subset of that data, performed an orbit determination, and predicted the remainder of the time. While it would not be valid to perform an OD over the entire interval of data and then take a point and predict through times that were already processed, the filter can rapidly establish the orbit, enabling a slightly longer prediction time analysis.

For the case in which the OD used about 2 days of data, the results are as follows.
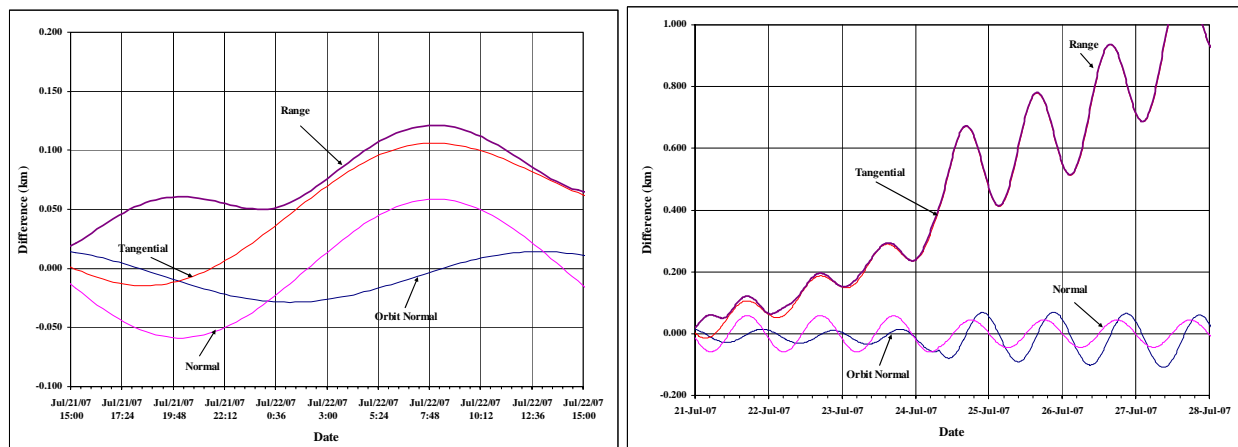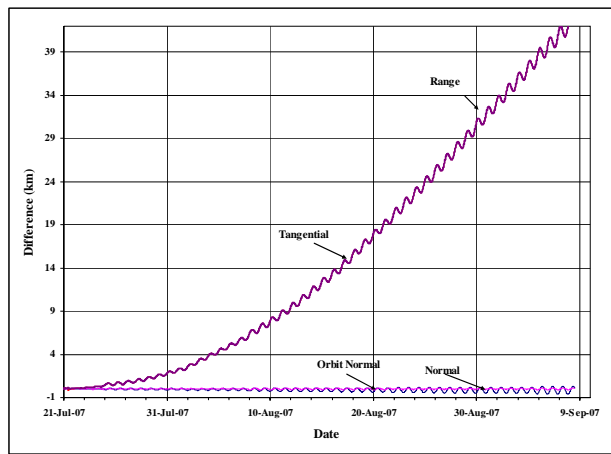
Figure 16: GEO OD Ephemeris Comparisons: Results are given for a GEO satellite after processing 2 days of data. The time intervals are 1 day, 1 week, and about 1 ½ months. The difference after 1 week is about 1 km.

Notice the immediate departure in the data, although reasonable performance considering the orbit is geosynchronous. The behavior is also remarkably similar (though not in scale) to the LEO cases. If we had used 4 days to establish the orbit, we get the results shown in Fig. 17.
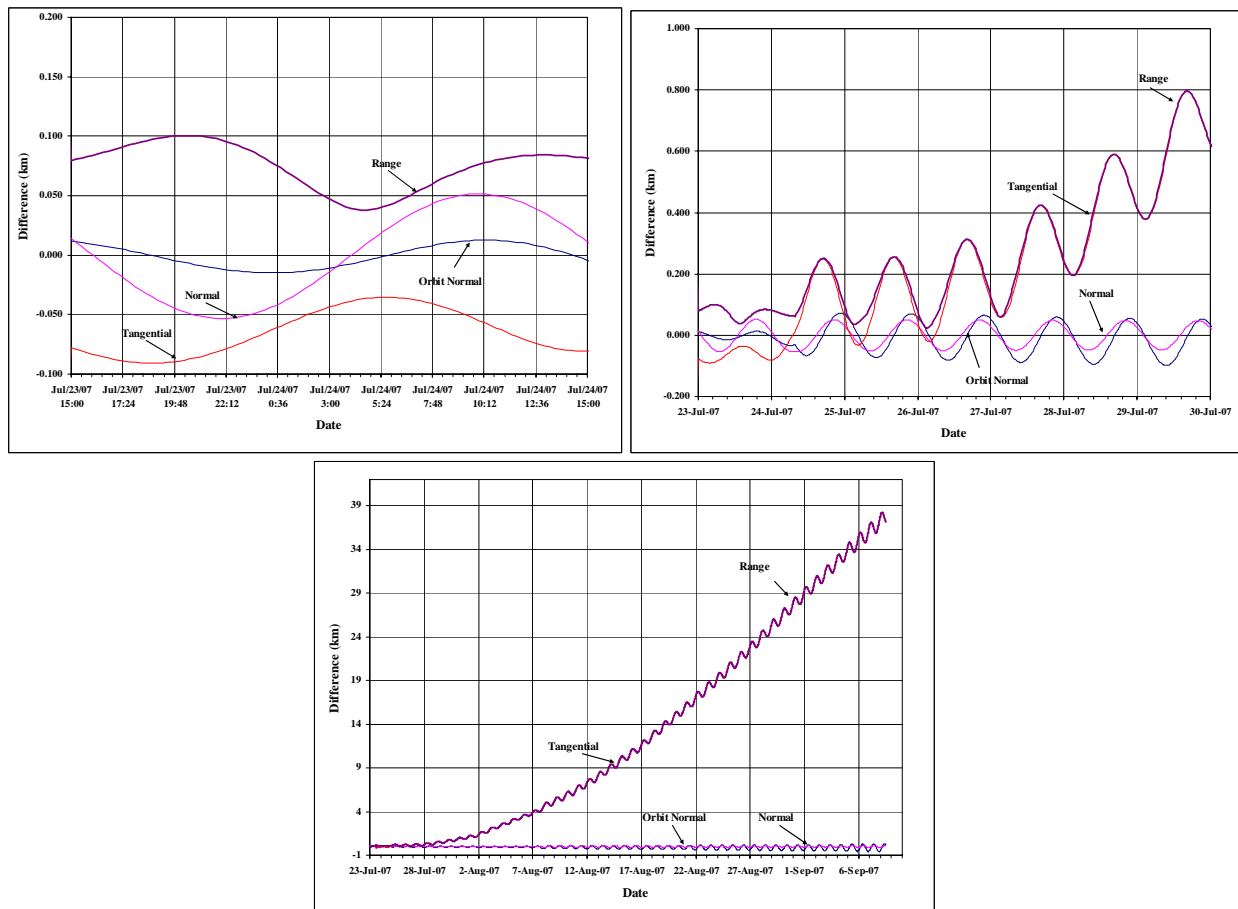
Figure 17: GEO OD Ephemeris Comparisons: Results are given for a GEO satellite after a 4 day OD of observational data. The difference after 1 day is about 200 m.

The graphs (Fig. 16 and 17) do not start on the same date due to the difference in the length of the initial OD. However, notice that at 4 days into the future, the 4-day initial OD performs nearly 2-times better than the 2-day OD case. Obviously with more time (data) in the original OD, the filter estimate will get better – up until a point. For this case, the filter continually improves with the new data, although after about 3 days, the improvements become much smaller. The key is that with just a few days of observational data, fairly accurate predictions can be made for a "long" period into the future. The dominance of the tangential error growth is shown again.

SUMMARY AND CONCLUSIONS

The basic nature of prediction accuracy has been introduced. Four types of prediction examples were examined. The first took the exact state from the POE and processed the data. While not realistic in an operational sense, it showed what is reasonably possible with some modern propagation techniques. Unfortunately, little information about the satellite parameters limited the performance of each case. The second section examined a few cases where the initial POE state vector was perturbed by an amount approximately equal to what one could expect from an on-board GPS receiver (about 1 m and about 1 mm/s). The results departed very quickly from the POE as expected, and no further tests were run. The third phase examined a filter OD on a portion of the POE, and a subsequent prediction with comparison to the POE. Again, while not entirely independent, it showed that more precise modeling of the satellite parameters could dramatically improve the performance of the prediction accuracy. Part of this is due to the fact that the force models are aligned exactly between the OD and prediction runs. The other part is the improved orbit determination, and time-varying of the satellite parameters. The final case examined creating a reference orbit from observations using a filter-smoother result over the entire span of data. The filter was then run on a portion of the observations, and compared to the longer reference ephemeris. While this is perhaps the most realistic scenario from an operational perspective, the precision of the reference orbit is clearly not as good as the independently developed POE's used in the previous tests. This phase did permit a look at what is possible for these orbital classes. Table 5 summarizes the results from the various runs. The satellites are grouped with the various tests so you can quickly see the effect of changing conditions. A similar comparison using Two-Line Element set (TLE) data was conducted (Kelso, 2007) with GPS orbits. While the magnitudes of the differences were much larger, some of the same behavior was noted.

Some general observations:

- Even using cm-level accurate initial state vectors, the best predictions are immediately in the tens to hundreds of meters or more.
- After about half the orbital period, the tangential error begins to dominate the error characteristics, no matter what orbital regime.
- Performing an OD significantly improves the results due to the consistent force modeling, and ability to solve for potentially unknown satellite parameters.
- Almost all results were less than 1 km at a week, which should suffice for many operations.
- A properly formulated filter/smoother can easily process through short segments of data and provide valuable information about a variety of operations.

Table 5: **Summary Prediction Values:** This table summarizes the maximum error, per time interval, for each of the study satellites. All values are in km. The Condition describes the test undertaken, for instance, "Params" are the improved satellite parameters and "Pert" are the runs in which the tangential or normal components were perturbed.

Satellite	Condition	2 hours	1 day	1 week	1 month	2 months	Period (min)
ICESat		0.014	0.550	16.000	manv	X	91
ICESat	Params	0.004	0.078	1.100	manv	X	91
ICESat	Pert Tang	0.032	X	X	X	X	91
ICESat	OD 1 day	0.004	0.017	2.500	manv	X	91
ERS-1		0.020	0.125	12.000	manv	X	97
ERS-2		0.004	0.080	1.400	13.000	X	100
ERS-2	Pert Tang	0.022	X	X	X	X	100
ERS-2	Pert Norm	0.040	X	X	X	X	91
Jason		0.001	0.012	0.120	15.000	X	100
GPS SVN-22		0.017	0.080	0.400	X	34.000	112
GPS SVN-22	OD 4 day	X	0.001	0.017	0.100	0.750	112
GPS SVN-31		0.016	1.000	5.000	X	26.000	718
GPS SVN-31	OD 4 day	X	0.002	0.048	0.420	0.500	718
GEO	OD 2 day	X	0.120	1.000	19.000	42.000	1436
GEO	OD 4 day	X	0.100	0.800	16.000	39.000	1436

It appears there are three time "phases" with respect to the orbital prediction accuracy. During the first phase of a few hours, the current models do rather well in accounting for each perturbation. In the second phase (days to weeks), larger systemic problems emerge – primarily in the non-conservative forces. This manifests itself in the tangential direction, along the velocity vector, and quickly masks any transient effects in the other axes. The third phase appears in the long term behavior (weeks to months) of the satellite motion. This last area is interesting because it encompasses the area in which almost all modern POE formulations use un-modeled accelerations, track weighting, segmented ballistic coefficient, white noise sigmas, bias half-lifes, etc. These methods, while useful to match data in a post processing situation, appear to be of diminished use for long term prediction. While the imprecision of future indices (space weather and the associated atmospheric drag indices, EOP parameters, etc.) is well known and likely accounts for a good deal of the imprecision, it is the authors opinion that additional fundamental research is needed to describe the exact physical effects that produce these long term discontinuities. Solutions may need to re-examine the fundamental formulation of the non-conservative forces.

ACKNOWLEDGMENTS

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