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An Analysis of State Vector Propagation Using Differing Flight Dynamics Programs[*]

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Since the demonstration of the first numerically generated space catalog by the United States Navy in 1997, the issue of how to transition from the two-line element sets (TLEs), to routine use of numerical vectors in satellite flight dynamics operations is generating some unique challenges. Specifically, how will organizations efficiently interact with and use orbital data from programs outside their control? The historical TLE operations used analytically generated datasets for a majority of their calculations which required strict adherence to a specific mathematical technique. Use of numerical techniques presents different challenges even though the underlying mathematical technique is the same. This paper provides results of an experiment in which various initial state vectors, representing a cross-section of the existing satellite population, were propagated from several days to a month. The ephemerides, created by several legacy flight dynamics programs, are compared to ephemerides from Analytical Graphics Inc.'s Satellite Tool Kit (STK). There is no assertion of right or wrong answers within the comparisons; rather, the relative differences are shown to gauge the effectiveness of the setup for each case. Most of the comparisons show that mm to cm-level comparisons are possible with careful attention to parameters. Differences are discussed including potential error sources. One goal is to present a format that simplifies transmission and use of state vector information between programs, seeking a standard for better integration of interoperability. This will avoid significant expenses in using entirely new, or unavailable software. Tables are presented to demonstrate the effect of various force models and their contribution to the satellite orbit. Finally, sample ephemeris information, potential new formats to exchange data, and STK scenario setups are included to initiate a community forum on numerical ephemeris propagations.

INTRODUCTION

The use of numerically generated state vectors for satellite operations is not new. However, with the first numerically generated space catalog by the Navy in 1997 (Coffey and Neal, 1998), the potential to replace the existing TLEs with numerical results now poses some unique challenges for the astrodynamics community. To effectively make this transition, several things must occur. Vallado (1999) proposed a fundamental question for all space surveillance functions.

What observations and processing are needed to achieve a certain level of accuracy on a particular satellite, now, and at a future time?

The answer involves tracking and surveillance functions, orbit determination, propagation, and standards. Also implied are the formats to effectively transmit the information to various organizations that will make operational decisions. Vallado and Carter (1997) showed that significantly more observational data is required than is currently being taken on some satellites, and Vallado and Alfano (1999) outlined many of the issues with obtaining and distributing data from a tracking and surveillance system. This paper answers some of the issues surrounding the propagation, interoperability, standards, and transfer of information. All these functions will be needed to transition from TLE data to numerical processing.

For several decades, many organizations have relied on TLEs to perform various flight dynamics operations. This implied the use of certain mathematical theories, and resulted in limited accuracy in analyses[‡]. Numerical state vectors are clearly the current choice for many of these operations, but they are only now beginning to gain mainstream acceptance in some routine space surveillance operations. To accurately propagate numerical satellite state vectors between programs, four primary types of information are required:

- the initial state vector and detailed satellite parameters
- a standard mathematical approach from which various applications can be implemented
- specific details of any tailoring or assumptions made to the processing

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[‡] Cefola and Fonte (1996) showed that even the AFSPC analytical theories could achieve order-of-magnitude accuracy increases by adopting features of semi-analytical satellite theories.

- an understanding of the effects of simplifying assumptions and sensitivities of individual orbital regimes.

The use of numerical techniques for propagation makes the underlying standard quite simple – the math is the same, but there are still numerous flight dynamics programs in use today. The challenge is to understand how to get them to agree so organizations can be interoperable. Schutz et al. (1980) investigated the tasks required to obtain a certain level of agreement between GEODYN and UTOPIA. Although this study examined the orbit determination aspect of the programs, it pioneered much of the discussion in this paper at a time when many space surveillance operations were using analytical theories. About the same time, AFSPC used analytically generated datasets from SGP4 and the Navy Positions Partials and Time (PPT) programs for a majority of their calculations. This required strict adherence to a specific mathematical technique. However, a lack of standardization prompted Hoots (1980) to detail the mathematical technique because organizations were incorrectly mixing datasets between these two diverse mathematical theories.

One approach to the differences introduced by various flight dynamics programs is a notion to "standardize" one particular computer code for numerical techniques (Kaya et al., 2004) via the AFSPC Instructions 60-102 and 33-105. However, such restrictions are unnecessary to achieve interoperability results within the uncertainties of the models and observations, as discussed in the forum at the AAS 2005 conference (Jan 2005, Session 17).

The reality of multiple flight dynamics programs suggests the need for standardization. There are numerous formal standardization efforts today. The CCSDS already has existing standards that prescribe the essential elements needed to convey information between organizations. In fact, the Orbit Parameter Message (OPM) is a direct outgrowth of this work. The AIAA Committee on Standards is nearing completion of an updated and totally revised recommended practice from the original Part I (AIAA 1995) that will encourage commonality of propagation approaches throughout the astrodynamics community. The upcoming AIAA Recommended Practice will recommend many of the assumptions that are routinely made with respect to numerical operations. Vallado (2001) concluded that a standard for astrodynamics is as follows:

> The astrodynamics theory, models, algorithms, and information exchange that are well established by authority, widely available, of overwhelming quality, whose purpose is to promote improved accuracy and interoperability between all organizations that use space.

Using the above definition, it is clear that a single computer code implementation cannot represent a viable standard. If this code were publicly available, fully documented, refereed, and independently validated and verified, then it could potentially be considered as a common practice. The real question is whether to standardize computer code, or just the mathematical theory and equations. This particular question has generated a lot of interest. I have already proposed that computer software does ***not*** represent a standard (possibly a common practice, but clearly not a standard) (Vallado, 2001). In some rare cases, a form of computer code may be presented (as with the IERS and IAU theories), but official standards do ***not*** reference the code, rather the underlying technical approach.

There is renewed interest in how organizations efficiently interact with and use orbital data from programs outside their control. Precision Orbit Determination activities routinely produce centimeter-level results or better (e.g. UT Austin, GPS), and satellite GPS receivers routinely generate meter-level ephemerides (Rim et al., 2000). These accuracy levels are available through numerical techniques[*], and although everyone is now using the same standard (numerical integration), there are new issues which must be addressed to promote interoperability – hence this paper.

OBJECTIVE

This paper demonstrates interoperability among different computer codes, achieving agreement within the bounds of the dynamics and the model uncertainties. These results enable free, open, and productive exchange among stake holders inevitably fostering advances inaccessible if narrow standardization is imposed. It gives results of sensitivity studies designed to show the envelope of performance for numerical ephemeris generation. Ephemerides generated by various legacy flight dynamics programs are compared to Analytical Graphics Inc.'s Satellite Tool Kit/High Precision Orbit Propagator (STK/HPOP). The time span ranges from several days to a month. There is no assertion of right or wrong answers within the comparisons, rather, the relative differences are shown to gauge the ability to align the two programs. They also provide a guide to the differences one should

[*] Highly accurate processing is also possible with *some* high-fidelity analytical and semi-analytical techniques.

expect for certain orbits and force model configurations. The sensitivity studies provide a general frame of reference for these comparisons. The conservative force model (gravity and third-body) comparisons show that mm and cm-level comparisons are possible with careful attention to input parameters. Comparisons showing lesser agreement were not subjected to meticulous investigation due to time restrictions, however, the sensitivity studies indicate the likely sources of the differences. An important outcome is the development of a format to permit improved transmission and use of state vector information between organizations. The goal is to promote a realistic standard for better integration and interoperability, thus avoiding significant expenses in using entirely new, untested, or unavailable software. Finally, availability of sample ephemeris information and STK scenarios to initiate a community forum on numerical ephemeris generation is discussed.

POTENTIAL ERROR SOURCES

Schutz (1980) suggests several potential error sources that can be encountered when comparing programs:

1. Inaccurate or mis-applied mathematical models
2. Measurement errors (not addressed in this paper because we did not examine orbit determination)
3. Truncation error when approximating a mathematical function
4. Round-off error resulting from computational precision
5. Mathematical model simplifications and approximations
6. Human error

Over the course of work for this paper, each of these were noted, but easily fixed. This paper adds additional nuances that I'll also discuss later. These were not so easily dismissed:

7. Identifying the precise parameters used in the setup of an individual program
8. Treatment of input data for various force models

INITIAL STATE VECTORS

The initial task was to devise a number of tests to sample the resident space objects that are in orbit today. There is a combination of altitude (semimajor axis) and eccentricity that shows where the concentrations of existing satellites are, as well as a set of force models to use for analysis. A simple analysis was conducted to determine if there was a way to differentiate the catalog into a form that would permit an evaluation of where the highest concentrations of satellites existed. Several approaches were considered – simple binning of eccentricity, inclination, and semimajor axis, equal binning of percentiles for each of the 3 elements listed, and orbital classes. The latter can be used, but combining the traditional 2-dimensional plot with inclination provides an important element. Figure 1 shows the 2-D and 3-D results.

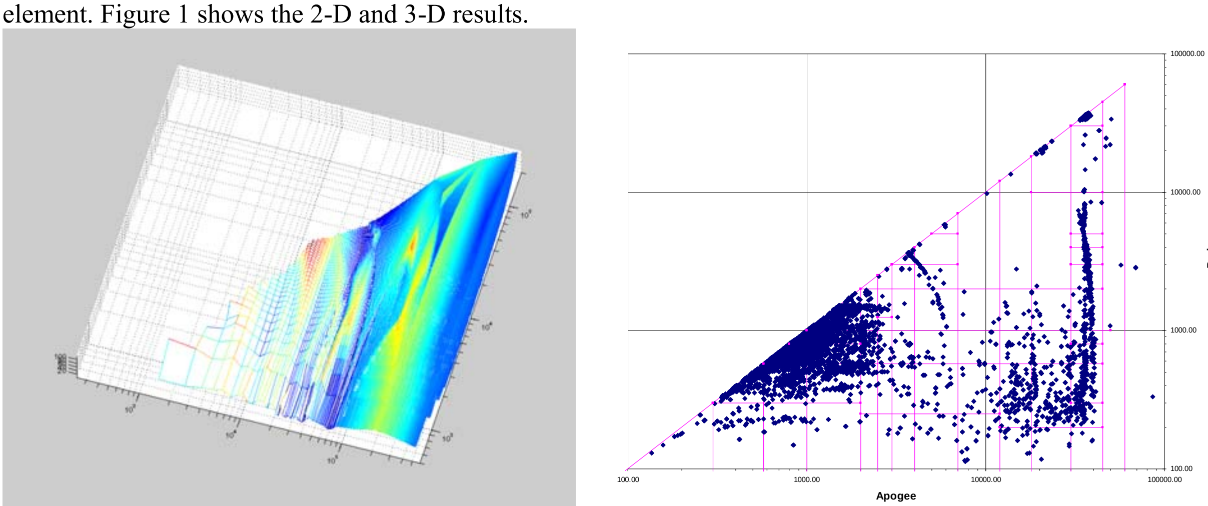


Figure 1 : Visualizing the Space Catalog: This figure shows two perspectives for the space satellite catalog. The left plot shows apogee vs perigee values, with inclination being the vertical axis. Note that the red color indicates higher inclination values. The dark blue orbits are generally below about 30 degrees inclination. The right plot shows the traditional two-dimensional apogee – perigee relations. Between the two, you can estimate the number of satellites in a particular region.

The satellites selected for this study form a spectrum from LEO to GEO satellites (Table 1). The epochs were generally arbitrary, especially in cases where drag or solar radiation pressure effects were examined, but some were selected to match previous data comparisons (Vallado, 2000). An important consideration was to have all the initial state vectors in the past where complete Earth Orientation Parameter (EOP) and solar weather data were available. The vagaries of predicted data (sources, values, interpolation methods, etc.) would have easily confused the results. The additional satellites in the LEO category were designed to better determine the results in the drag regime, and at the lower end of the solar radiation pressure regime.

Table 1: Study Satellites

Category	SSC #	Name	Perigee Alt (km)	Apogee Alt (km)	*a* (km)	*e*	*i* (deg)	Period (min)
LEO	25544	ISS	377	389	6762	0.00085	51.60	92
LEO	21867	JERS	475	490	6860	0.00106	97.60	94
LEO	07646	Starlette	800	1100	7331	0.02107	49.80	104
LEO	00011	Vanguard 2	550	3023	8164	0.15147	32.86	122
MEO	22076	TOPEX	1340	1347	7723	0.00051	66.06	113
MEO	26690	NAVSTAR 50	20082	20282	26560	0.00375	55.24	718
HEO	25054	SL-12 RB	186	21371	17157	0.61736	46.70	373
HEO	20052	Molnyia 3-35	285	38026	25533	0.73904	62.05	677
GEO	26038	Galaxy 11	35785	35790	42165	0.00007	0.03	1436

PROGRAMS

Next, we needed to obtain data from several legacy numerical integration flight dynamics system programs. The following flight dynamics programs were used in the study:

GEODYN	(Goddard, NASA, NRL, other)
GTDS	(Goddard Trajectory Determination System, GSFC, MIT)
HPOP	(Analytical Graphics Inc., Satellite Tool Kit)
SPECIAL-K	(Navy)
TRACE	(Raytheon/Geodynamics)

GEODYN has long been a standard reference program for high-precision analyses. It was developed decades ago for, among other things, gravity field coefficient determination. It has numerous high-fidelity options and is used extensively by NASA and the Satellite Laser Ranging (SLR) community. The Goddard Trajectory Determination System (GTDS) exists in several forms resulting from the original development started in the early 1970s at the Computer Sciences Corporation (CSC). An operational version is used by NASA, MIT/Lincoln Laboratory uses a UNIX version, as does the Charles Stark Draper Laboratory (CSDL). Dr. Cefola led much of the development of the CSC, CSDL, and MIT/LL versions of GTDS, and continues to refine its operation and capabilities. HPOP was originally written by Microcosm and subsequently enhanced by Analytical Graphics with the goal of taking the legacy experience and programming it within "modern" theories and architecture. Special-K was undertaken as a project by Naval Research Laboratory and Naval Space Command several years ago to upgrade the Naval operational system software, and to exploit the new processing capabilities of the modern computer. Special-K was successfully used in the first full space object catalog to be processed by numerical techniques (Coffey and Neal, 1998). TRACE has a long lineage dating back to the 1960s when it was developed as a general-purpose flight dynamics program. There have been several spin-offs, including the Geodynamics version used by Raytheon, and the original version still maintained by The Aerospace Corporation. Vallado (2007:1013) identifies a few other programs that are in the category of legacy numerical flight dynamics programs, but they were unavailable for study at this time. Some, like UTOPIA from UT Austin, OCEAN from Naval Research Laboratory, and GIPSY/OASIS have already seen extensive analysis with programs like GEODYN. The remaining ones have not.

We had intended to conduct an expanded analysis with Aerospace TRACE, but the STK Verification and Validation report (Chao et al., 2000) covered all aspects that we would have examined in this paper. The results showed that the two numerical integration programs compared at the sub-meter level (even atmospheric drag). Because the goal was not to force any development or change to a particular program, we set up a framework in which each organization could provide ephemerides resulting from the state vectors in their "usual" format, coordinate system, etc. This showed important interoperability considerations in that the comparison program

(STK/HPOP) needed to be able to accept a variety of input formats, coordinate systems, etc. An important inference is that since all these programs compared closely with STK, they would also compare closely to each other.

INPUT DATA SOURCES

As mentioned earlier, an important new result from this paper, not examined in Schutz (1980), is the need to completely define the parameters required to generate the ephemerides in different programs. This includes detailed information on what data is used (source of data, frequency, etc.), as well as how the data is used within a program (lag times, offsets, etc.). Both of these are contained in a sample state vector format proposed at the end of this paper.

The use of a gravitational model assumes that the coefficients are identical with each program. During testing for this paper, small differences were noted within the "same" models, however, the differences in coefficients did not introduce more than sub-millimeter variations in the answers. The gravitational model also implies a set of physical constants – differences in which cause substantial variations in the ephemerides. The gravitational field (coefficients) and constants are generally related, but not rigorously as some organizations mix quantities. The best solution is for any software program is to be able to manually insert the mixed values, however, there are often embedded constants that make this process extremely difficult, and ultimately unreliable. For this paper, we commonly used the WGS-84/EGM-96 and EGM-96 gravity models shown below, although a few simulations used the older WGS-84 model. STK/HPOP is designed to use an ASCII file for the gravity model, including the defining coefficients. This simplified matching the other programs.

For EGM-96

1. Gravitational Parameter μ =398600.4415 km^3/s^2
2. Radius of the Earth r = 6378.1363 km
3. Flattening f = 1/298.257
4. Rotation rate of the Earth ω = 7.292158553e-5 rad/s

For WGS-84/EGM-96

1. Gravitational Parameter μ = 398600.4418 km^3/s^2
2. Radius of the Earth r = 6378.137 km
3. Flattening f = 1/298.257223563
4. Rotation rate of the Earth ω =7.292158553e-5 rad/s

The sources of data for Earth Orientation Parameters (EOP) and solar weather are somewhat standard. However, there are typically small differences between sources of EOP data (IERS, NGA, and USNO, for instance), but the impact on overall accuracy is usually small (a few meters or so). Vallado and Kelso (2005) study the various sources of data and show how to produce current consolidated data files which are located on the Internet. With the exception of the atmospheric drag tests (where additional flexibility was needed), the EOP and space weather files are from

http://celestrak.com/SpaceData/

With the publication of the IAU 2000 Resolutions (McCarthy and Petit, 2003 and Kaplan 2005), the choice of coordinate systems would seem to be easy. However, the practical reality is that large programs often lag behind the current standards. In addition, many programs still use an architecture that was based on a previous coordinate system or time system that was in place when the original code was written. In some cases, this can be nearly 40 years ago. Nevertheless, it's possible to compare the results given that the updates were performed properly. The level of accuracy in the resulting comparisons suggest that this is the case with the programs examined.

USING THE INPUT DATA

The use of EOP data and solar weather parameters also present challenges and it's potentially the largest source of differences in the results. The predicted values often do not exist at the same frequency interval as the actual data (monthly vs. daily, etc.). Programs often use some form of interpolation (Lagrangian interpolation scheme is recommend by IERS), and this can result in measurable differences in the final results. To minimize this effect, I chose examples that occurred in the past so actual data would be available. Tanygin and Wright (2004) documented the discontinuities in solar weather data and the effect of various smoothing operations. While it is obvious that programs should attempt to match interpolation techniques, the larger differences in atmospheric density model results would appear to overwhelm any smaller contribution from the indices. We'll discuss this later.

Integration techniques contribute generally small errors to the propagation process. However, the step-size choice for fixed-step techniques became important for highly eccentric orbits. Several runs were made with Runge-Kutta 7/8 and Gauss-Jackson propagators with fixed, variable, and regularized time step sizes. Time steps were generally chosen that produced similar results at the mm level. Notice that I've included the initial step size and other information in the data format as it is important information, especially with eccentric orbits. The exact implementation of a particular numerical integrator appeared to be pretty constant throughout the programs tested and, therefore, no additional time was spent in this area.

Finally, most of today's message formats are incomplete as they provide insufficient information with which to reconstruct an ephemeris propagation. To meet the demands of interoperability, a new message format is clearly indicated. The format should include sufficient information to replicate the force models, coordinate systems, input data sources, and treatment of this information. While some formats exist today (SP3, IIRV, VCM, etc.), they omit key points needed to align the resulting ephemeris generations, with the exception of the OPM message from CCSDS. Given that one can match force models and efficiently and accurately propagate other organizations' vectors, a revised format will provide a much needed method to exchange information. An important note is the use of XML for the actual message, allowing you to convert to a particular organizational standard without affecting the interoperability of the basic message. The proposed format is given at the end of the paper was an early prototype for the OPM message.

STUDY PROCESS

To effectively discuss legacy program comparisons (and any other numerical propagation comparisons), many runs were made to lay a foundation from which any differences could be discussed. It was apparent from the start that none of the programs would agree exactly, thus the proper basis was needed to accurately address any observed differences.

First, the effect of each force model on a particular satellite orbit was examined. This showed the "macro-level" importance that could be attached to each force model for a particular satellite – e.g. examining solid Earth tides for a sun-synchronous orbit may be more important than on a lower altitude, less inclined orbit.

Next, a series of sensitivity studies examining gravitational field truncations, atmospheric drag, and solar radiation pressure differences was performed. Although these studies did not include an OD for each test, they provided a sense of the differences that would be expected by varying certain input and model parameters.

With the preceding tests accomplished, the ephemeris comparisons themselves were examined. The ephemeris comparisons were easiest to pursue when a series of tests were conducted to "buildup" the force models. In this way, we could determine if a particular force model was contributing the majority of the difference to the overall solution. In general, each of the following tests check a different portion of the programs under consideration. Note that for some of the comparisons, the gravity field was included with other forces once it was examined separately. This also permitted an evaluation of any coupling between force models (such as drag and gravitational acceleration).

- Two-body
  - Checks numerical integrators, coordinate and time systems
- Gravity field
  - Checks μ, Earth radius, gravitational coefficients, etc.
- Two-body plus atmospheric drag
  - Checks atmospheric density model, solar weather parameter treatment
- Two-body plus third-body
  - Checks incorporation of JPL DE/LE files, other files and constants
- Two-body plus solar radiation pressure
  - Checks Earth shadow assumptions, solar constants, etc.

The propagation span for each ephemeris was generally kept at about 4 days. Although the results at the end of this time showed some large differences, 4 days is generally about the event-horizon in which operational decisions are made. Differences are computed at each ephemeris time step to provide the user with a look at the time-varying trends.

PRELIMINARIES

Although not formally researched in this paper, it is important to recognize that certain propagators can perform well in certain environments, and for certain orbital regimes.

I. INDIVIDUAL FORCE MODEL CONTRIBUTIONS

This section looks at the magnitude of the various forces on different satellite orbits. An important assumption was that no orbit determination (OD) was performed with each different force model. Usually, there is a match between the OD and propagation processes. The OD adjusts the initial state used for each propagation based on the available force models during the OD run. By keeping the same initial state vector, we added a certain amount of uncertainty that would have been minimized with individual OD runs. However, because the purpose was to illustrate only the approximate trends, the additional computations were not warranted.

The graphic in Fig. 2 shows a general applicability of each force model for various orbital regimes. Although this is not intended to be exhaustive, it is intended to summarize some of the more detailed results that follow in the sensitivity studies.

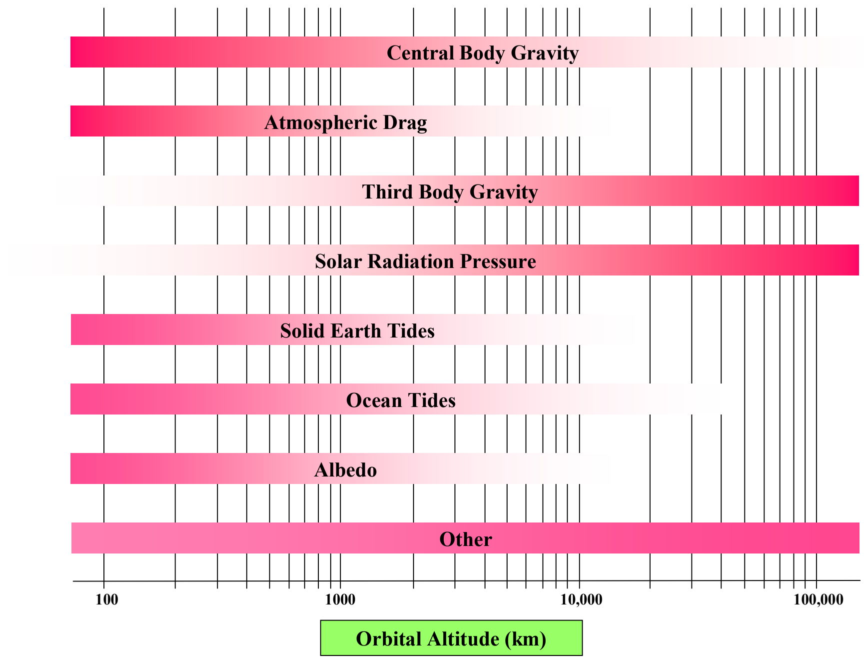


Figure 2 : Generic Force Model Setup: This figure shows approximate force model setups for various orbital altitudes. Note that specific accuracy requirements may extend the areas of applicability, and hence the faded color bars. Additionally, the eccentricity will also require additional force models not depicted here.

Time limited a comprehensive study, but several tests were chosen to highlight the overall trends and to show the approximate trends for various satellites. These runs formed the basis of ephemeris baseline runs which are located on the web for comparison studies. Users of STK/HPOP can also download scenario files to simplify the process of replicating and generating additional runs.

The default setup for the propagation runs that are located on the Internet as part of the force model contributions portion are as follows:

- Integrator – Gauss-Jackson
- Full correction
- 10 sec initial step size
- WGS-84/EGM-96 gravity field
- EOP file from EOP20030101.dat from CelesTrak. Beware that past EOP values are sometimes updated!
- Space weather file SPW20030101.txt from CelesTrak
- Use polar motion = true
- Update EOP every step (NutationUpdateInterval = 0 sec)
- Use new equation of the equinoxes (additional 2-terms)

The satellite parameters were chosen to illustrate force model effects. Although specific satellites are listed, only their orbital characteristics were used. Each parameter was held constant (c_D = 2.2, c_R = 1.2, A/m = 0.04 m^2/kg). The simulation time, January 4, 2003, was chosen as the epoch to propagate as this was a moderate period of solar activity ($F_{10.7}$ ~ 140). The baseline for comparison in all cases was a two-body orbit, except for the gravity cases which were compared to the next nearest case (2×0 compared to two-body, 12×12 compared to 2×0, and 70×70 compared to 12×12). This was selected to best show the individual contributions. There is coupling between some forces, particularly gravity and atmospheric drag, but the effects are generally less than the other individual forces. Over time, their growth can become noticeable, but they are still usually much less than the predominant forces. Figure 3 shows representative results for some low-Earth satellites.

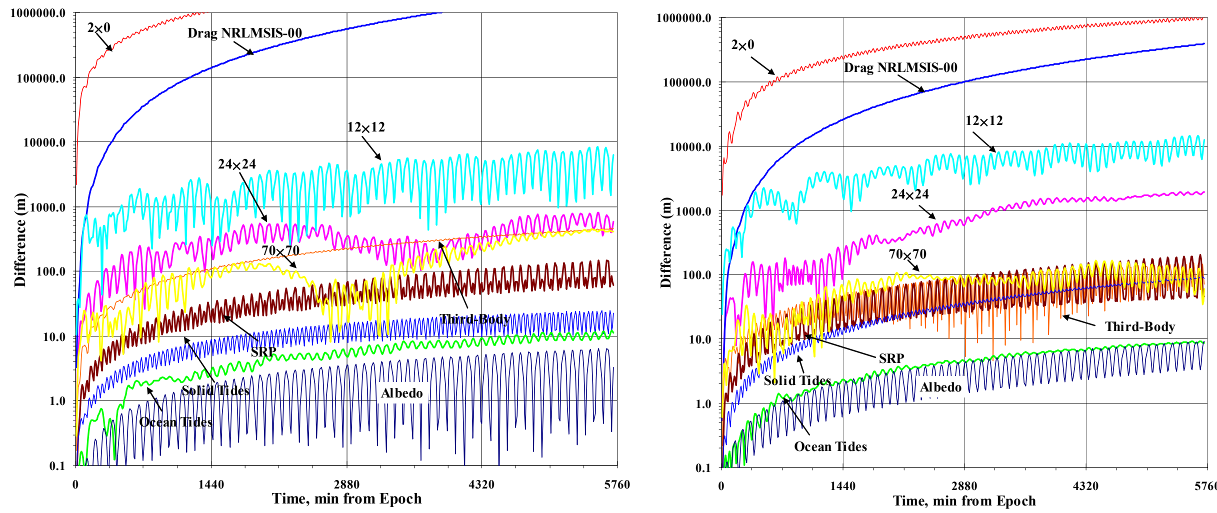


Figure 3 : Force Model Comparisons – LEO 380x390 km, 51.6° (left) and LEO 500x500 km, 51.6°: This figure shows the positional difference over time (four days) from using various force models on the same initial state. Each comparison is made with respect to a two-body ephemeris, except for the gravity runs which compare to the nearest gravity case. Thus, "12×12" is a comparison of a 12×12 WGS84/EGM96 gravity field to a WGS84/EGM96 2×0 gravity field ephemeris, and a "24×24" is a comparison of a 24×24 WGS84/EGM96 gravity field to a WGS84/EGM96 12×12 gravity field ephemeris, etc. The "third-body" is a comparison of a two-body ephemeris to a third-body ephemeris. This is for the ISS, SSC# 25544 and JERS, SSC# 21867 satellites.

For LEO orbits, several characteristics are noted. The high contribution of drag to the results are always prominent. The gravity field also has a large effect. Truncating the gravity field is not recommended as neglecting even the portion from 12×12 to 70×70 can contribute km-level errors. Note that tides represent a small contribution, but one that is needed for precise (cm-level) work. The JERS satellite in Fig. 3 is slightly higher, sun-synchronous, repeat groundtrack orbit. Note the relatively large effect of the solid tides. Because the inclination is approximately sun-synchronous, the satellite experiences additional contributions from solid tides. Essentially, the satellite is orbiting over similar locations on the Earth, at similar times during each day. The repeated exposure to the gravity produces this effect. Ocean tides do not exhibit this behavior.

Next, we examine some LEO orbits that are somewhat elliptical. These orbits are interesting in that they experience gravitational and atmospheric drag effects, but also some third-body and solar radiation pressure forces. Consider Fig. 4.

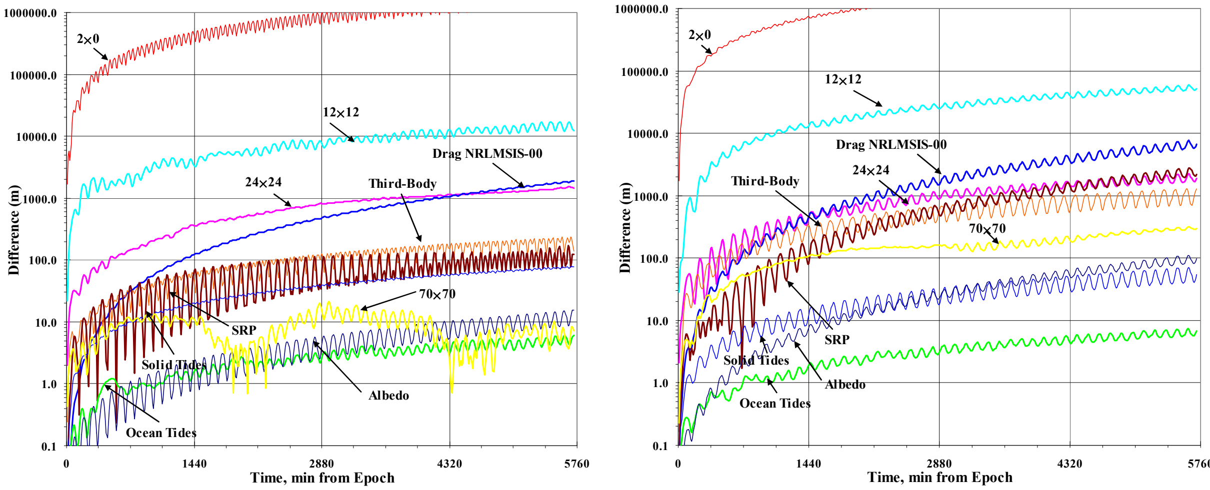


Figure 4 : Force Model Comparisons – LEO 800×1100 km, 49.8° and LEO 550×3020 km, 32.8°: This figure shows RSS position differences for a simplified numerical propagator and force model truncation. This is the Starlette, SSC# 7646 and Vanguard II, SSC# 11.

Note the rapidly diminishing gravitational effects compared to the LEO satellites in Fig. 5. Also, as the apogee gets larger, the third body and solar radiation pressure forces increase.

As the altitudes for the satellites increase, there is a large change as third body and solar radiation pressure become dominant. The pronounced dips in solar radiation pressure are generally due to the satellite entering eclipse. Precise GPS modeling involves complex solar radiation pressure models that account for the attitude of the satellite over time.

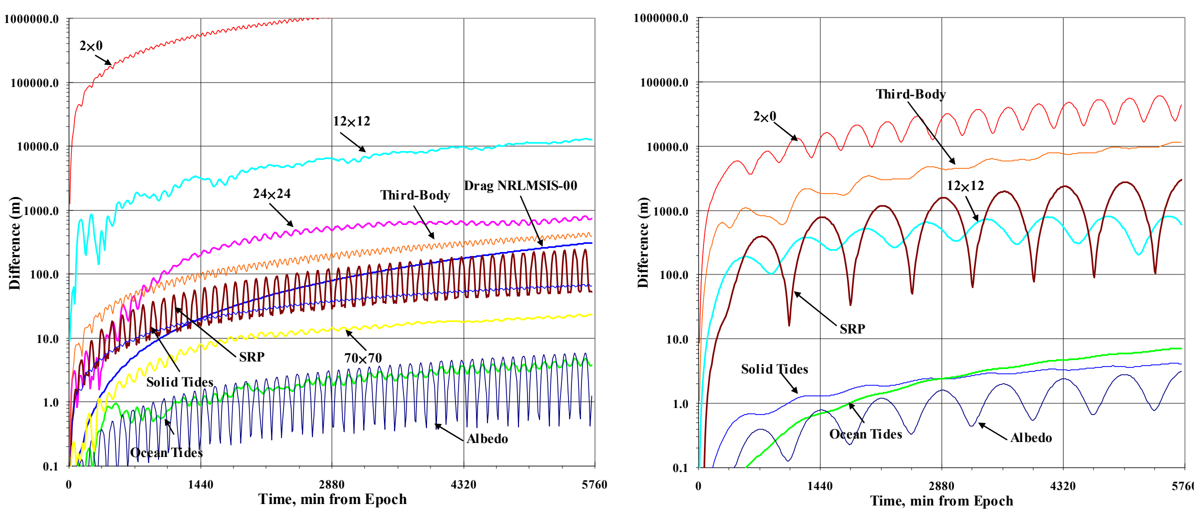


Figure 5 : Force Model Comparisons – LEO 1300×1300 km, 66° and MEO 20000×20000 km, 55.2°: This figure shows RSS position differences for a simplified numerical propagator and force model truncation. This is the TOPEX, SSC# 22076 satellites, and the GPS, SSC# 26690 satellites.

Highly eccentric orbits are usually the Molnyia or geosynchronous transfer orbit satellites. These are among the most difficult orbits to model because they have very high velocities in the atmosphere, enhancing the effect of atmospheric drag, and they spend long periods of time at apogee where the third body and solar radiation pressure forces can influence the orbit. The peaks/valleys are associated with atmospheric entry and satellite eclipse. Notice that even the gravitational effects become important in these satellites.

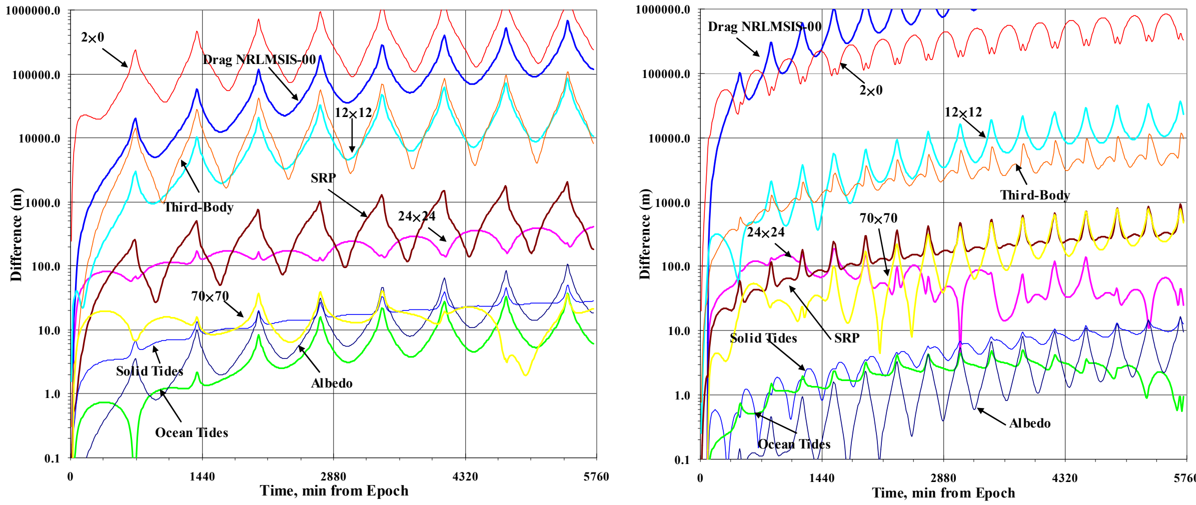


Figure 6 : Force Model Comparisons – HEO 285×38000 km, 62.1° and HEO 200×20000 km, 46°: This figure shows RSS position differences for a simplified numerical propagator and force model truncation. These are a Molnyia, SSC# 20052, and a SL-12 RB, SSC# 25054.

Geosynchronous satellites are very different from the previous satellite orbits. Gravity "wells" result from the gravitational effect, thus the placement of the satellite in the geosynchronous belt becomes important. Notice the longer period for effects from the perturbations from the longer orbital period.

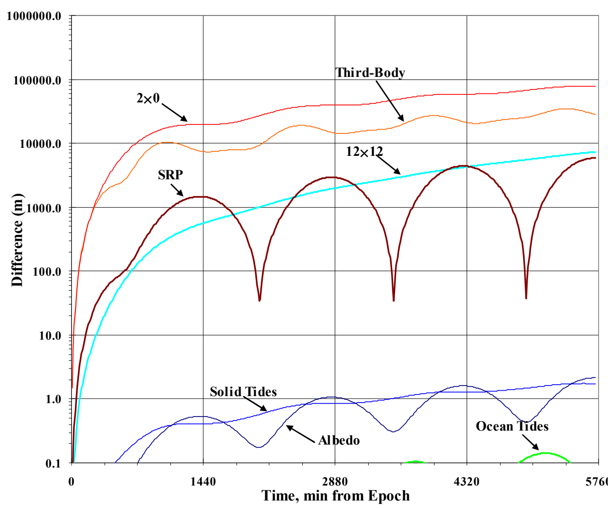


Figure 7 : Force Model Comparisons – GEO 35786×35790 km, 0.0°: This figure shows RSS position differences for a simplified numerical propagator and force model truncation. This is the Galaxy 11 satellite, SSC# 26038.

The logarithmic scale was chosen to permit viewing all the forces on a single graph. Table 2 summarizes the individual results. Included are the final value and an average of the differences during the last period of the satellite's orbit. This average is intended to give an estimate of the variability in the results.

In general, gravity was the largest single perturbation source (shown in km in Table 2 for the 0×0 case), so additional tests were conducted to determine the sensitivity of this perturbation force. Atmospheric drag was generally second for lower orbits, but third-body effects were much higher for higher altitude satellites. Because the study results indicated the conservative forces could be matched to cm-level, no additional studies were performed on third-body forces. Drag was considered separately. It is important to note that these are prediction differences are based on the propagation of identical state vectors with differing acceleration models. A study of orbit determination accuracy using differing acceleration models would produce a very different set of results.

Table 2: Summary Force Model Comparisons: This table lists the overall results from the force model comparisons of several satellites (all values are in meters). Two sections are provided – secular and periodic. The secular is the average over the last revolution at the end of the time span, in this case 4 days. The periodic values are the standard deviations of the differences over the last revolution before the 4 day time. The baseline for comparison is usually against a two-body orbit. Because the effect is so large, the gravity cases refer to the previous case, thus "vs 24×24" is the difference of a 24×24 gravity field, and a 12×12 gravity field propagation.

Secular (Average over the last period)													
Name	Apogee alt (km)	Perigee alt (km)	Incl (deg)	vs 2x0	vs 12x12	vs 24x24	vs 70x70	vs MSIS00	vs ThirdBody	vs SRP	vs SolidTides	vs OceanTides	vs Albedo
ISS	389	377	51.60	4179448.02	4475.43	558.03	418.60	2200700.77	455.60	95.74	18.47	10.60	3.62
JERS	490	475	97.60	559985.25	25983.24	325.83	16.85	222855.71	29925.24	725.71	29.23	12.11	34.99
Starlette	1100	800	49.80	1649332.00	14790.04	1505.60	5.62	1868.78	200.95	121.35	76.73	5.10	11.97
Vanguard 2	3023	550	32.86	2799810.40	54047.23	1956.43	305.60	6727.46	1008.79	2443.25	55.92	6.52	97.51
TOPEX	1347	1340	66.06	2140804.23	12809.29	768.07	22.66	309.47	414.42	142.41	67.08	4.31	3.54
NAVSTAR 50	20282	20282	55.24	43563.66	521.52	0.00	0.00	0.05	10137.84	1632.52	3.93	6.58	1.95
SL-12 R/B	21371	186	46.70	600259.59	17167.00	47.24	372.21	4981564.90	5996.97	407.38	11.28	1.50	6.20
Molnyia 3-35	38026	285	62.05	1649332.00	14790.04	1505.60	5.62	1868.78	200.95	121.35	76.73	5.10	11.97
Galaxy 11	35790	35785	0.03	69198.46	5779.77	0.00	0.00	0.00	27247.77	2849.94	1.52	0.08	1.21
Periodic (std dev over the last period)													
Name	Apogee alt (km)	Perigee alt (km)	Incl (deg)	vs 2x0	vs 12x12	vs 24x24	vs 70x70	vs MSIS00	vs ThirdBody	vs SRP	vs SolidTides	vs OceanTides	vs Albedo
ISS	389	377	51.60	160767.20	2061.85	113.52	7.85	22565.24	3.64	33.29	5.04	1.03	2.23
JERS	490	475	97.60	429353.68	18980.24	67.95	7.39	142934.82	26313.26	487.30	5.42	7.85	22.69
Starlette	1100	800	49.80	261369.53	1704.03	22.70	1.73	43.25	32.78	33.94	1.97	0.70	2.62
Vanguard 2	3023	550	32.86	92256.76	4209.46	218.57	14.44	806.20	222.20	311.03	11.72	0.71	12.05
TOPEX	1347	1340	66.06	93836.85	100.98	42.77	0.53	4.12	24.41	79.76	1.81	0.44	2.03
NAVSTAR 50	20282	20282	55.24	12064.61	213.37	0.00	0.00	0.00	832.04	972.10	0.18	0.42	0.80
SL-12 R/B	21371	186	46.70	184280.90	7621.10	16.55	152.63	2514036.34	2242.38	168.06	1.66	0.63	4.10
Molnyia 3-35	38026	285	62.05	261369.53	1704.03	22.70	1.73	43.25	32.78	33.94	1.97	0.70	2.62
Galaxy 11	35790	35785	0.03	7567.86	931.76	0.00	0.00	0.00	4819.47	1840.33	0.17	0.05	0.56

II. FORCE MODEL SENSITIVITY

It is also important to understand the variability, or sensitivity of each force model. This is especially important for the non-conservative forces which exhibited larger differences between programs. It's imperative to stress that these are differences, and not one being right and the other being wrong.

The rationale for the sensitivity analyses was that if a gravity field truncation produced more error than another force model, it would be better to increase the size of the gravity field before trying to "fix" any other force model difference. STK/HPOP performed all the analyses. The resulting ephemerides have been placed on the web to serve as a community resource.

Some sensitivity studies exist in the literature. Barker et al. (1996) showed results of gravitational truncation, however, the results were averaged (RMS) over a long period of time. While this may be useful for long-term trends, or viewgraphs, it is inadequate for the mission planner who is concerned about an upcoming maneuver and the selection of which force models to include in any analyses. In addition, statistics of overall satellite catalog performance are nice for presentations, but again, are not sufficient for an operational planner. A more recent study effectively examined the accelerations required for various orbital classes (Register, 2003) and in part, inspired this effort.

GRAVITATIONAL SENSITIVITY ANALYSIS

This series of tests examined gravitational truncation. While a rigorous approach to astrodynamics requires the complete field, many applications use reduced gravity field orders to speed computational processing and because of program limitations. Some operational systems (AFSPC) often use a blanket 24×24 (for example) field for LEO orbits, rapidly truncating the gravity field as the orbits get higher in altitude. This may not be the best approach to accurately determine the orbit. Barker et al. (1996) suggested a link between accuracy and the zonal truncation. Other studies have almost all examined the average behavior of a square gravity field on the satellite orbit ephemeris. However, it may not tell the proper story for precise operations. Vallado (2005) investigated the behavior of truncations for several satellites. One example is shown here for a satellite in about a 500 km altitude circular orbit.

As in the force model section, there was no orbit determination (OD) performed with each different force model. Usually, there is a match between the OD and propagation processes. The OD adjusts the initial state based on the available force models during the OD run. Each time the gravity field is changed, the potential energy of the system changes, and an OD process produces a different state vector to reflect this change. Although the most precise way to evaluate each force model would be to perform an OD on each individual case, the process would be unnecessarily long because we are only trying to establish the relative trends for each perturbation, not specific values for an individual case. By keeping the same initial state vector, we added a certain amount of uncertainty that would have been minimized by individual OD runs.

There are two plots in Fig. 8 that demonstrate this effect – a square gravity field is first and the non-square follows. The scales are the same for each to allow easy scanning of the results. Each gravity field examined a 4-day period. The plots show the effects of gravity field truncations at each time step.

Several conclusions may be drawn. First, as the gravity field size increases, the magnitude of the difference generally decreases (for non-square fields). An analysis of the analytical variations (Vallado, 2007, 643-667) could reveal a more precise cause of this difference, but as computers have become faster, the easiest approach is to simply use a complete gravity field. Another important conclusion is that if one incorporates tidal effects, albedo, etc., the gravity field should be large enough to ensure errors from the truncated gravitational force modeling do not mask the effect of the additional included force models. Also note that certain gravity field sizes (6×6 for instance with satellite 07646) exhibit more error than just a 4x4 solution. This appeared in several test cases, hence the runs to test a complete zonal field (to 70th order) with a truncation of the tesseral and sectoral terms. These results followed the more "normal" pattern where larger fields, for instance 70×6, performed better than smaller fields (70×4). Finally, the inadequacy of an averaging scheme or reporting scenario is readily apparent. Some organizations use a single value to represent the "error" of a satellite. For the most accuracy, the covariance should be used, and the resulting propagation can show the expected performance through time. In the absence of covariance data, one should examine the performance over time (as in the plots in this paper). Consider an analysis for two highly elliptical satellites for about 1 day in the future, vs. the planning at 3 days ahead for a close conjunction. Because the variation in a single period of the satellite can be several km, the difference at 2 days will be considerably more – definitely important if a maneuver decision is to be made from the data.

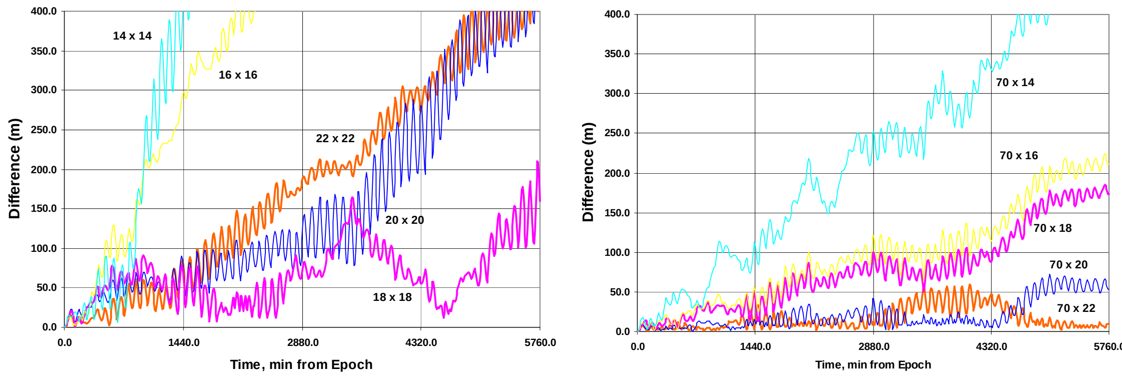


Figure 8: **Gravity Field Comparisons:** Truncated gravity fields are compared to ephemeris runs for a complete EGM-96 70×70 field for a satellite at about 500 km altitude. The left plot is for a square gravity field. The right plot includes all the zonals (70) in the truncations. The results do not always improve with a larger field (due to neglecting the OD contribution in forming the initial state), but the accuracy generally improves as the non-square truncation is reduced (the differences from 70×70 for 22×22 are greater than 18×18 on the left, but the 70×22 is smaller than the 70×18 on the right).

ATMOSPHERIC DRAG SENSITIVITY ANALYSIS

Atmospheric drag is probably the most elusive of the force models examined. There are several reasons for this. Before discussing the potential sources for the differences, it's useful to review the basic acceleration equation.

$$\vec{a}_{drag} = -\frac{1}{2}\rho \frac{c_D A}{m} v_{rel}^2 \frac{\vec{v}_{rel}}{|\vec{v}_{rel}|}$$

ρ The density usually depends on the atmospheric model, EUV, $F_{10.7}$, k_p, a_p, prediction capability, atmospheric composition, etc. There is wide variability here, and many parameters that can cause significant changes. The popular parameters to examine today are the density and the exospheric temperatures. This single parameter represents the largest contribution to error in any orbit determination application.

c_D The coefficient of drag is related to the shape, but ultimately a difficult parameter to define. Gaposchkin (1994) discusses that the c_D is affected by a complex interaction of reflection, molecular content, attitude, etc. It will vary, but typically not very much as the satellite materials usually remain constant.

A The cross sectional area changes constantly (unless there is precise attitude control, or the satellite is a sphere). This variable can change by a factor of 10 or more depending on the specific satellite configuration. Macro models are often used for modeling solar pressure accelerations, but seldom if ever, for atmospheric drag.

m The mass is generally constant, but thrusting, ablation, etc., can change this quantity.

BC The ballistic coefficient (m/c_DA – a variation is the inverse of this in some systems) is generally used to lump the previous values together. It *will* vary, sometimes by a large factor. Several initiatives are examining the time-rate of change for this parameter, but not looking at the variable area, and its effect in this combined factor. It's probably best not to model this parameter because it includes several other time-varying parameters that are perhaps better modeled separately.

$\vec{v}_{rel}$ The velocity relative to the rotating atmosphere depends on the accuracy of the *a-priori* estimate, and the results of any differential correction processes. Because it's generally large, and squared, it becomes a *very* important factor in the calculation of the acceleration.

The primary inputs in any program are the atmospheric density (handled via a specified model), and the *BC*. The mass and cross-sectional area are usually well known, and an estimate of the drag coefficient permits reasonable approximations. The atmospheric models also vary depending on several factors, including the satellite orbit, intensity of the solar activity, and the geomagnetic activity.

Despite the simple expression, accurate modeling of atmospheric drag is quite challenging for several reasons. The major error sources are listed below (note that density, *BC*, etc., are not listed as they are derivative effects from the items listed below). This list is generally ordered in decreasing magnitude of effect, although the exact effect will differ over different orbital regimes.

- Using predicted values of $F_{10.7}$, K_p, a_p for real-time operations
- Not using the actual measurement time for the values ($F_{10.7}$ in particular at 2000 UTC)
- Using step functions for the atmospheric parameters vs interpolation
- Using the last 81-day average $F_{10.7}$ vs. the central 81-day average
- Using undocumented differences from the original atmospheric model definition
- Not accounting for [possibly] known dynamic effects – changing attitude, molecular interaction with the satellite materials, etc.
- Inherent limitations of the atmospheric models
- Use of differing interpolation techniques for the atmospheric parameters
- Using approximations for the satellite altitude, solar position, etc.
- Using a_p or K_p and converting between these values
- Use of $E_{10.7}$ vs $F_{10.7}$ in the atmospheric models (this is not well characterized yet)

Consider the variations in the input solar data (Fig. 9). "D Ap" represents the difference between the maximum individual 3 hourly values of a_p , vs the average of all 8 values. Notice this can vary almost as much as the individual spikes in the data. The "trend" (Vallado, 2007:560) and "Last F107" (last 81-day average) are shown to reference the particular location in the solar cycle. "DLast – Avg", the difference between the centered and the last 81-day averages, is perhaps the key point. Notice that this quantity can assume values of 30-50. From the sensitivity studies, this difference can cause hundreds of kilometers of difference between two different propagation runs. It would be useful to determine from post-processed data (POEs) if the centered average actually yields a more precise representation of the orbit than the last 81-day average during these times, but for now, the difference means that (at a minimum) this degree of variability should be expected in non-conservative force model evaluations.

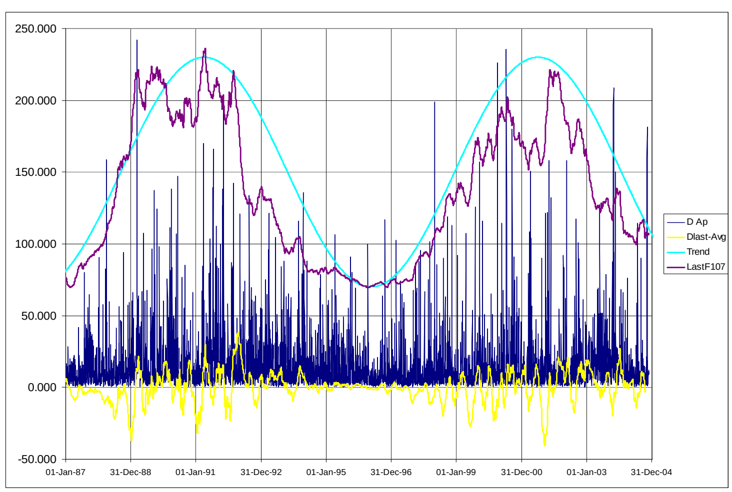


Figure 9: Variability of Solar Parameters: The variability in the a_p and $F_{10.7}$ parameters is shown for about the last two solar cycles. Note that the a_p difference ("D ap", daily vs the maximum 3 hourly value) is almost as large as the measurements themselves. Also note the $F_{10.7}$ difference ("Dlast – Avg", centered vs. last 81-days) can be 30-50 solar flux units.

The daily $F_{10.7}$ measurements have been made by the National Research Council of Canada since 1947. Until May 31, 1991, the observations were made at the Algonquin Radio Observatory, near Ottawa at 1700 UTC.

Since then, the observatory near Penticton, British Columbia has measured the data at 2000 UTC. Many programs simply use the $F_{10.7}$ value at 0000 UTC. Given the current measurement time, this places many programs potentially almost a full day off from when the data was taken. It would seem more appropriate to use the measured time of 2000 UTC, and use this time as the center point of any interpolation scheme. Note also that some atmospheric models suggest not interpolating the $F_{10.7}$ values. Coupled with the variability attributed to this particular parameter, it's easy to see that large variations are possible for drag comparisons. Compounding this problem is the fact that many programs were written so long ago that the original formulation has been lost or forgotten. This poses additional challenges when trying to align ephemerides, but even with this information, one is likely to still find differences due to the sensitive nature of the models. The key to remember is the overall result and its impact (if any) on an operational decision.

Atmospheric models also demonstrate the need to adhere to original formulations. With the advent of routine operations of the International Space Station (ISS), conjunction analysis and prediction have become common applications for numerical solutions. However, the dominant error source is still the atmospheric modeling. A great deal of interest centers on this topic, and numerous comparisons and studies have been performed, with few if any clear leaders. If we examine the Jacchia models (1965, 1970, 1971, 1977), we find four distinct sets of equations used to implement these atmospheric models. Jacchia spent a great deal of time preparing these models and trying to match the observational data. Aside from the original technical differences, if we examine existing computer code for these methods, we find many similarities because there are only so many ways to mechanize a given set of technical equations. However, we also find numerous omissions from the original papers, additional, or updated constants, and shortcuts such as loading tables, and creating splines and polynomials to better fit the observed data. Each of these modifications introduces potential differences in any solution.

Additionally, most models as implemented in computer code, do not follow the exact technical derivation as defined in the literature. In fact, I would state that none of the drag model implementations match the original technical definition. While gravitational models can match the definitions exactly, it is more difficult for non-conservative forces because the models are more complex (leading to a perceived need to simplify the approach), and there are more alternate ways to program the data. As a result, code contains numerous short cuts, and many additional features that may be the result of internal studies and information, but not the original work. This makes comparison of atmospheric models especially difficult.

Tangyin and Wright (2004) have already shown that the interpolation of a_p / K_p values can have a dramatic effect on the results of a differential correction. Further comments are required for the conversions between k_p and a_p. Figure 10 shows the values, and the obvious non-exact relationship between the quantities. Recall that the data exists only in discreet quantities. Because the a_p scale is larger, there are additional gradations to match the interpolated data. Thus, calculating an average K_p value and converting that result to a_p would be different from taking each k_p value and using the equivalent a_p value, and then finding the average ap. This process is likely implemented in a variety of ways in operational programs. Consider an example from March 18, 1989. The 3-hourly K_p and a_p values (including K_p sum (197) and a_p average (15)) are:

10	20	20	20	40	53	13	20	197	4	7	7	7	27	56	5	7	15

The average K_p would be 197/8 = 24.625 and the average ap is 15 (120/8). However, a K_p of 24 has no direct equivalent a_p value. The nearest pairs are K_p = 23: a_p = 9 and K_p = 27 : a_p = 12. If the individual values are converted, the process is consistent. It's only when averages are converted that difficulties arise. Thus, using the K_p or a_p value may give very different results, depending on how a program treats the transformation. Also recognize that some atmospheric models default to either K_p or a_p values for calculations, thus eliminating the need for conversions.

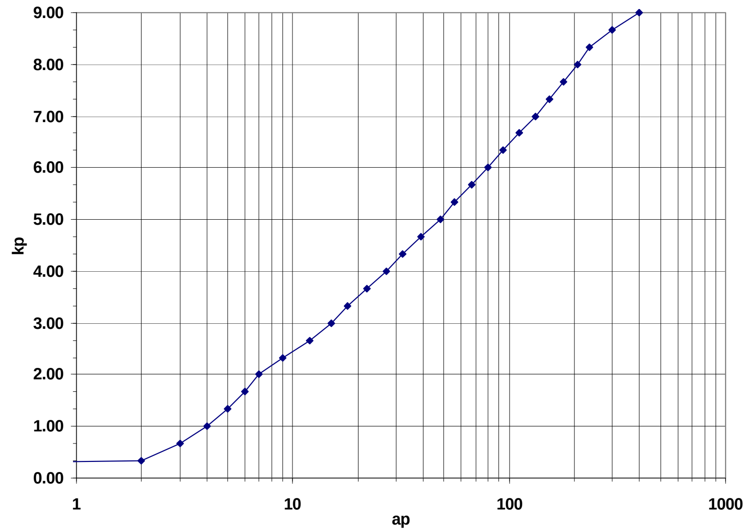
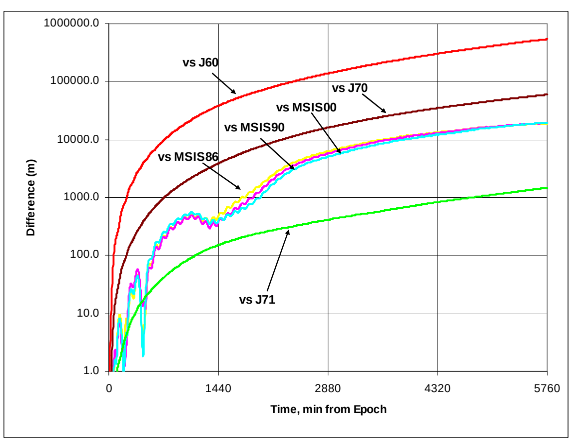


Figure 10: Relationship between a_p and K_p: The semi-logarithmic correlation between a_p and K_p is shown. Note that there are only distinct values that correspond between each scale and K_p is multiplied by 10.0.

A series of tests were run to determine the variability of different atmospheric models for a given satellite using a single flight dynamics program, and the differences resulting from the diverse treatment of the input solar weather data. The state vectors, epoch, *BC*, and solar radiation pressure coefficient ($m/c_r A_{sun}$) were held constant for all runs. The baseline used the Jacchia-Roberts atmospheric model. The simulations were run during a time of "average" solar flux (January 4, 2003, $F_{10.7} \sim 140$). Minimum solar flux periods ($F_{10.7} \sim 70$) will show little difference. Maximum periods ($F_{10.7} \sim 220$) will show much larger excursions. Figure 11 shows the results for satellite 21867. Additional runs were performed with different satellites and as expected, the results were larger for lower and more eccentric orbits.



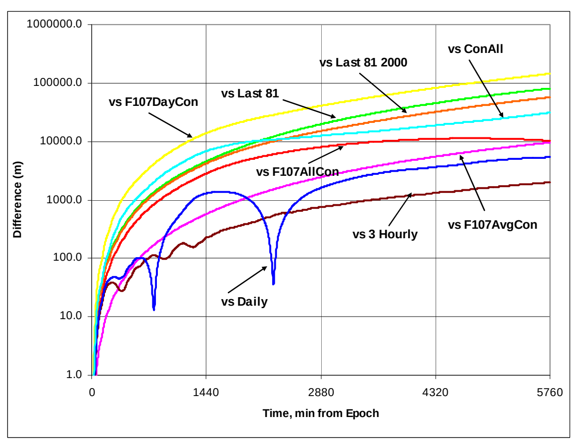


Figure 11: Sample Atmospheric Drag Sensitivity: Positional differences are shown for satellite 21867. Jacchia-Roberts is the baseline for all runs with 3-hourly interpolation. The left-hand graph shows the variations by simply selecting different atmospheric models. The right-hand graph shows the effect of various options for treating solar weather data. Specific options are discussed in the text. Note that the scales are the same, the relative effect of different models and solar data options are about the same, and any transient effects quickly disappear as the effect of drag overwhelms the contributions.

For most of the simulations, the MSIS-86 and MSIS-90 models were quite close, as expected by the model descriptions. The J60 model appeared to be significantly different in all cases from the other models and J70 seemed to differ most from the J71 and JRob models. Because this paper did not extensively examine comparisons with POEs, it's most important to come away with the overall level of variability within the different models. Essentially, if varying atmospheric models show differences that are significantly larger than differences between flight dynamics programs using the "same" models, which is right? After examining these data, I conclude that neither are right. Primarily, this is due to the results shown on the right-hand side of Fig. 11 which is discussed next. Although each atmospheric model is carefully designed, the treatment of solar weather data by each program adds so much variability, coupled with the lack of independent references and availability of observational data for comprehensive evaluation makes it highly unlikely that one approach is definitive for all cases.

The second point to notice is the variability induced by the treatment of the atmospheric data parameters, and how they are processed within an application. I have examined several common approximations that are used with the data. In the following, $F_{avg10.7}$ is assumed to be the centered 81-day average unless noted.

Daily	The daily average value for a_p (or the daily k_p sum divided by 8), and the daily $F_{10.7}$, $F_{avg10.7}$ values are used without interpolation.
3-hourly	The 3-hourly values for a_p (or the 3-hourly k_p values), and the daily $F_{10.7}$, $F_{avg10.7}$ values are used without interpolation.
3-hourly int	The 3-hourly values for a_p (or the 3-hourly k_p values), and the daily $F_{10.7}$, $F_{avg10.7}$ values are used with interpolation.
Last 81d 2000	The 3-hourly values for a_p (or the 3-hourly k_p values), and the daily $F_{10.7}$, $F_{avg10.7}$ values are used with interpolation. All $F_{10.7}$, $F_{avg10.7}$ values are taken at 2000 UTC.
F107DayCon	The daily average value for a_p (or the daily k_p sum divided by 8), a single daily $F_{10.7}$ value for the entire propagation, and the daily $F_{avg10.7}$ values are used without interpolation.
F107AvgCon	The daily average value for a_p (or the daily k_p sum divided by 8), the daily $F_{10.7}$ values, and a single daily $F_{avg10.7}$ value for the entire propagation are used without interpolation.
F107AllCon	The daily average value for a_p (or the daily k_p sum divided by 8), a single daily $F_{10.7}$ value, and a single daily $F_{avg10.7}$ value are used for the entire propagation without interpolation.
Last 81d	The daily average value for a_p (or the daily k_p sum divided by 8), the daily $F_{10.7}$ value, and the daily last 81-day $F_{avg10.7}$ values are used without interpolation.
ConAll	A single daily average value for a_p (or the single daily k_p sum divided by 8), and a single daily $F_{10.7}$, $F_{avg10.7}$ value are used without interpolation.

There was no need to examine an interpolation of the daily a_p (or K_p) values because the 3-hourly values provide the additional interim data points. The default was taken as “Last 81d 2000” as this should best approximate the actual dynamics of the atmosphere. From the graphs, it was somewhat unexpected that holding the daily values of $F_{10.7}$ constant would produce such large variations – in fact, this difference was always larger than even selecting different atmospheric models. Holding the average $F_{10.7}$ value constant had a smaller effect, but in all cases, the variations were smaller when the average and daily values of $F_{10.7}$ were either both constant, or both actual. These results are dependent on the particular atmospheric model, and how heavily it “weights” the $F_{10.7}$ input. Finally, the difference in using the centered $F_{10.7}$ versus the last $F_{10.7}$ was shown in the simulation with a different $F_{10.7}$ value. This difference can be quite large (Fig. 11) and although many operational centers use the last 81-day averages, the atmospheric models are usually designed for operation with a centered 81-day average.

For atmospheric drag, the variability in treatment of $F_{10.7}$, and the a_p / k_p values, had a greater effect than the model comparisons between the programs. These large differences in atmospheric drag suggest the need for a recommended approach to minimize differences between programs. Although these options refer primarily to the computer code, the code itself would still not be the standard. In fact, the approach would merely be a recommended practice because none of the atmospheric models indicate precisely how to treat the incoming data. The following recommendations are set forth.

1. There should be an option to use either the last $F_{10.7}$ 81-day average, or the centered 81-day average. Atmospheric model descriptions generally cite a centered average, but this is impractical for many operational systems, and a trailing 81-day average is often used. I’ve seen many uses of both sides of these approaches, and it’s a simple flag in the computer code.

2. Using K_p or a_p should be seamless, but I think there is the possibility of difficulties for certain conversions of average values. There are discreet values for which a_p and K_p exist in the daily data. Thus, a program needs to be

careful not to input a derived value that doesn't exist in the other scale. Inside a program, however, conversions may proceed without restriction to value. Consistency should be maintained with the atmospheric model.

3. The lack of test cases for the MSIS models using the array of back a_p values (SW(9) option) highlight a need for the community to adopt the recommendations here, and provide documented test cases to ensure the code is implemented properly.

4. The codes should treat all $F_{10.7}$ measurements at the time the measurement is actually taken. The offset (2000 UTC) should be used with all $F_{10.7}$ and average $F_{10.7}$ values. Any model specific "day before", "6.7 hours before", etc., should be done with this offset in mind. There is not an established approach, yet it's a *big* factor (sometimes km level) in the comparisons!

5. The options for using a_p (or K_p) should be
   a. daily – just the daily values are interpolated. All 3-hourly values are ignored.
   b. 3-hourly – just the 3-hourly values are used. The daily values are ignored and there is no interpolation. This will produce step function discontinuities, but that could be how some programs work.
   c. 3-hourly interp – this should be the interpolation and rationale discussed in Tanygin and Wright (2004). It should produce the smoothest transitions from one time to the next. The measurements should reproduce exactly at the measurement times (0000, 0300, 0600, etc. UTC), and be smooth in between.

6. The lag time for a_p (K_p) values is somewhat fixed to 6.7 hours, but others have been proposed. Since it's a variable option, it would be prudent to have a means to change it, without recompiling the entire program.

The bottom line for drag (and to a lesser extent solar radiation pressure, as we'll see shortly) is to have as many options and choices as possible. While the programming task becomes more complicated, this non-conservative force is often the most difficult to match in ephemeris comparisons and having these options provides the user with a much greater ability to minimize differences with other programs.

SOLAR RADIATION PRESSURE SENSITIVITY ANALYSIS

The other primary non-conservative force is solar radiation pressure. Although not studied as extensively in the literature, it poses many of the same challenges as atmospheric drag, but has a significantly smaller effect than the other forces. Consider the basic equation.

$$\vec{a}_{srp} = -\rho_{SR} \frac{c_R A_{Sun}}{m} \frac{\vec{r}_{sat-Sun}}{\left|\vec{r}_{sat-Sun}\right|}$$

ρ_{SR} The incoming solar pressure depends on the time of year, and the intensity of the solar output. It's derived from the incoming solar flux (Vallado, 2004, 547-548) and values of about 1358-1373 W/m^2 are common.

c_R The coefficient of reflectivity indicates the absorptive and reflective properties of the material, and thus the susceptibility to incoming solar radiation.

A_{Sun} The cross-sectional area changes constantly (unless there is precise attitude control, or it's spherical). This variable can change by a factor of 10 or more depending on the specific satellite configuration. Macro models are often used for geosynchronous satellites. This area is generally *not* the same as the cross-sectional area for drag.

m The mass is generally constant, but thrusting, ablation, etc., can change this quantity.

$r_{\text{sat-Sun}}$ The orientation of the force depends on the satellite-Sun vector – again a difference with atmospheric drag.

Despite the simple expression, accurate modeling of solar radiation pressure is challenging for several reasons. The major error sources are:

- Use of macro models/attitude – this is perhaps the largest difference between programs
- Use of differing shadow models (umbral / penumbral regions, cylindrical, none, etc.)
- Using a single value for the incoming solar luminosity, or equivalent flux at 1 AU
- Use of an effective Earth radius for shadow calculations (23 km additional altitude is common) – this approximates the effect of attenuation from the atmosphere

- Using different methods to account for seasonal variations in the solar pressure
- Not integrating to the exact points of arrival and departure at the shadow boundary
- Use of simplified treatment for the light-time travel from the Sun to the satellite (instantaneous (true), light delay to central body accounted for (app to true), light delay to satellite (default))

A series of runs were made to determine the impact of each of these items on the results for a few selected satellites. Results are shown in Fig. 12 for a nominal GPS satellite.

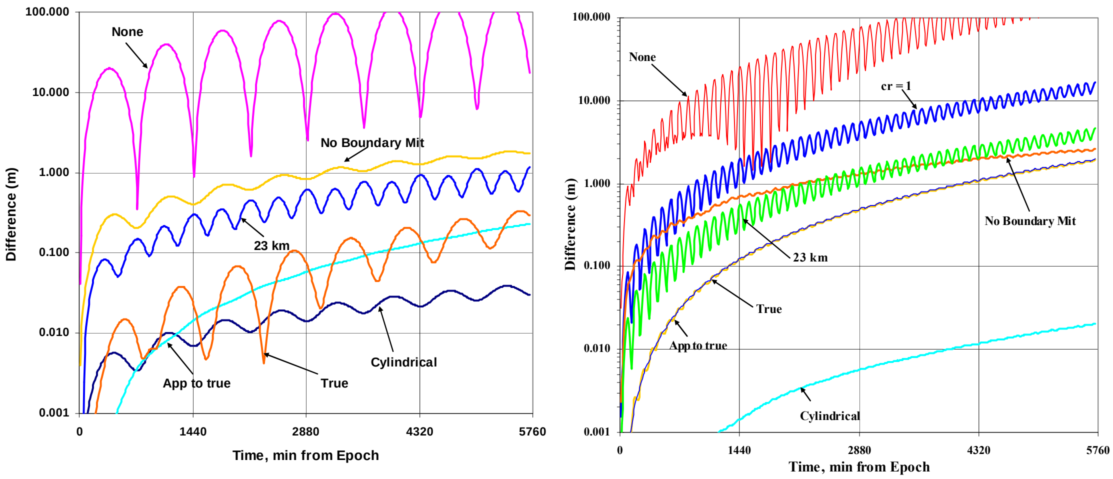


Figure 12: Sample Solar Radiation Pressure Sensitivity: Positional differences are shown for a GPS satellite which is in eclipse (left), and Starlette (right) at about 800x1100 km altitude. The baseline is a dual-cone (umbra/penumbra) shadow model. Using no shadow model (none) produces the largest differences. A simple cylindrical model introduces modest differences. Shadow boundary mitigation (no boundary) and the effective Earth size (80) contribute noticeable differences. The treatment of light travel time between the Sun and central body (app to true) and instantaneous travel (true) produce smaller, but still detectable results.

III. EPHEMERIS COMPARISON RESULTS AND DISCUSSION

Given the background of the previous two sections, we can begin the main task. Although we had AFSPC ephemeris data available from Joint Astrodynamics Working Group tests (Vallado, 2000), it was not analyzed because the output was calculated in a true-equator, mean-equinox (TEME) coordinate system. TEME is not recommended for any precise computations because it's a non-standard "system". No official public documentation exists for TEME, and it is not internationally recognized. In addition, this author believes that its definition has changed over time (nutation calculations). Results are roughly presented by perturbing force because it was felt this would best show the ability to align various programs.

CENTRAL-BODY GRAVITATIONAL MODELING

The first force considered was the gravitational force between the satellite and the central body. Conservative forces showed remarkable agreement between programs, despite being the largest perturbation force. Data for several satellites were made available for many of the programs, including some challenging cases. In the cases of GTDS and Raytheon TRACE, the force models were built up in a step-wise manner. This was extremely helpful, and it didn't add very much analysis time. Note that the position and velocity curves essentially "mirror" each other – something that requires equal significant digits in each ephemeris.

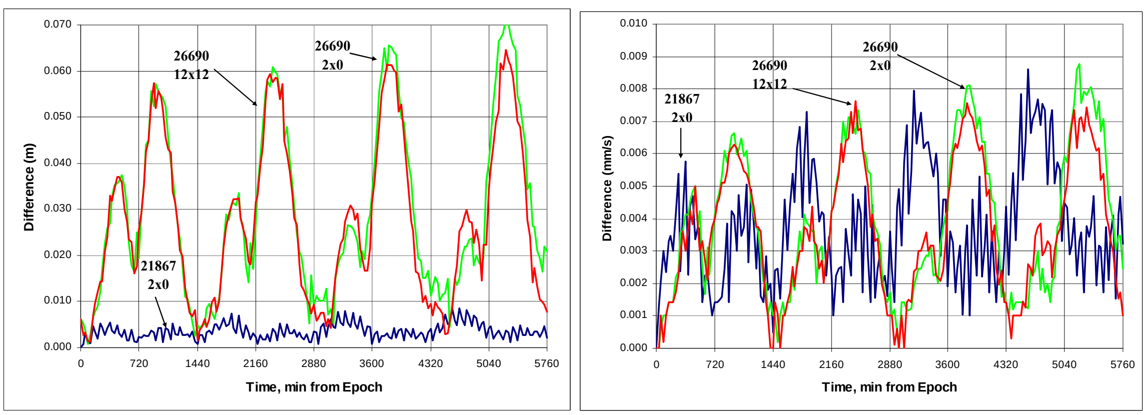


Figure 13: GTDS Gravitational Comparisons to STK/HPOP: Two satellites are shown with varying gravity field sizes. The positional differences are on the left and the velocity differences are on the right. Note that similar shapes between the two plots and that all the results are less than 0.07 m (0.009 mm/s) after 4 days of propagation.

Analyzing the satellite normal, tangential, and orbit plane normal (NTW) components (Vallado, 2007:65) for the initial 2×0 cases for 21867 and 26690, we find the following components. The along-track component is the largest.

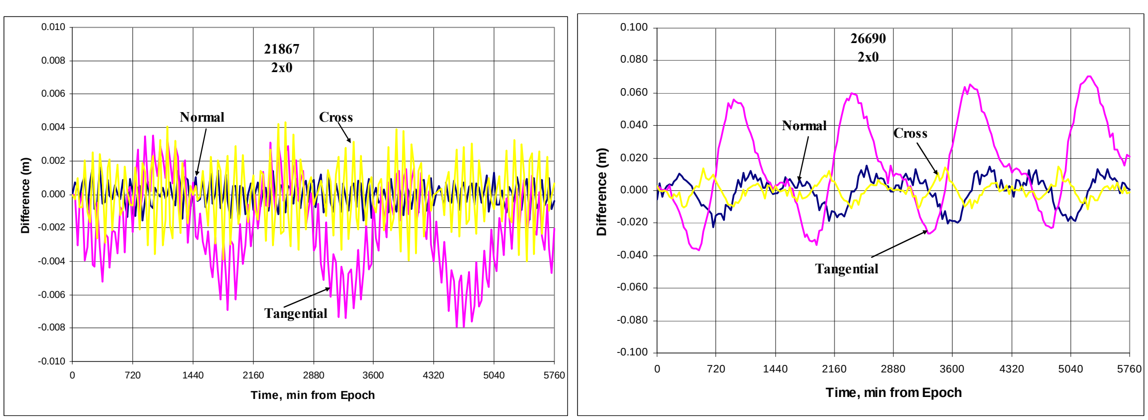


Figure 14: GTDS Component Differences to STK/HPOP: NTW components are given for the 2×0 21867 case, and the 2×0 26690 case. Note that the primary effect is along-track – aligned with the velocity vector direction. A signature appears with the GPS (26690) propagation that was not resolved.

A run was made for satellite 25054 in which regularized time was used. The results were off a little more than the fixed time-step cases, but time limited further investigation and resolution of this small anomaly.

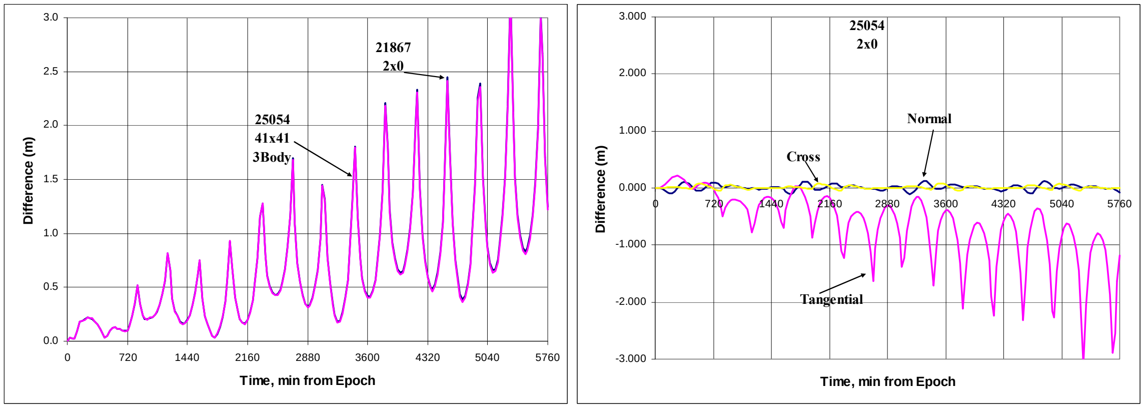


Figure 15 : GTDS Regularized Time Gravitational Comparisons to STK/HPOP: Data is shown for 25054 using regularized time for two force model combinations including NTW differences. Note the differences are somewhat larger than the previous examples, but still more than an order-of-magnitude less than the effect of simply truncating from 70×70 to a 60×60 field.

Using Raytheon TRACE, several runs were also made (Fig. 10). Special-K had one run with 21867 including just gravity. There appears to be a signature here for several runs suggesting something in the gravity field constants that may be different. There may also be an integrator difference, but the results are very small. Note that the 0x0 case can be different depending on how individual programs examine the "two-body" case. This can be done analytically, or via the numerical integration equations. See also Fig. 16 ahead. I did not explore various step-size options in detail, nor was there an examination of regularized time. However, the results show rather clearly that even for highly eccentric orbits, the gravitational forces can be very accurately aligned. Centimeter-level (even meter-level) agreement is remarkable considering the variety of programs, programming languages, platforms, data inputs, etc.

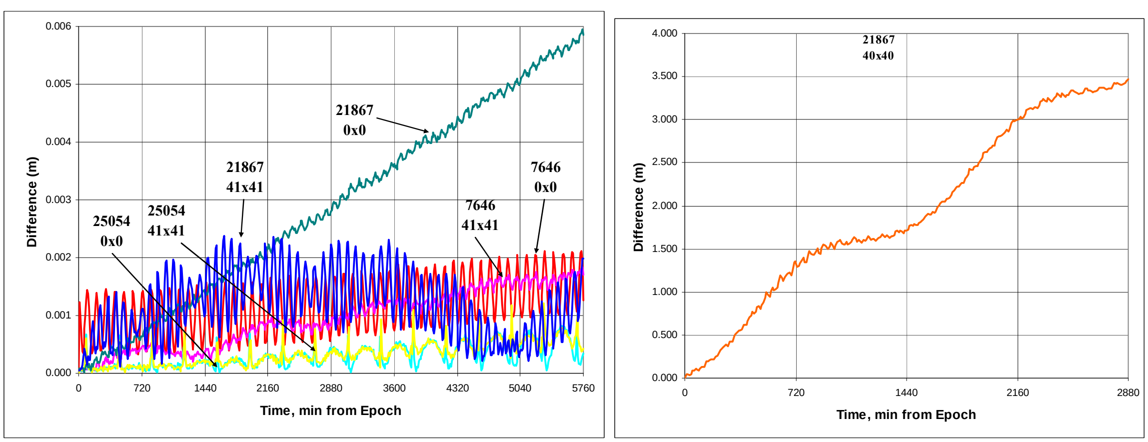
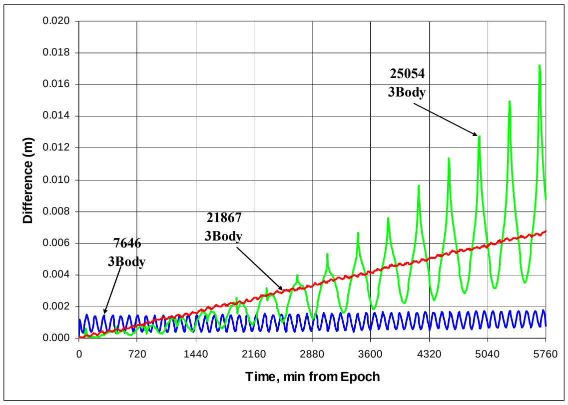


Figure 16:TRACE and Special-K Gravitational Comparisons to STK/HPOP: Three satellites are shown with various gravity fields for TRACE. Note the close comparisons for all cases (< 1 cm) in 4 days. The Special-K run for satellite 21867 showed a slight drift over 2 days. This was not examined extensively but is probably a gravitational constant or integrator issue.

THIRD-BODY MODELING

The results of third-body gravitational force modeling paralleled those of central body gravitational forces. GTDS and Raytheon TRACE provided the essential runs, shown in Fig. 17.



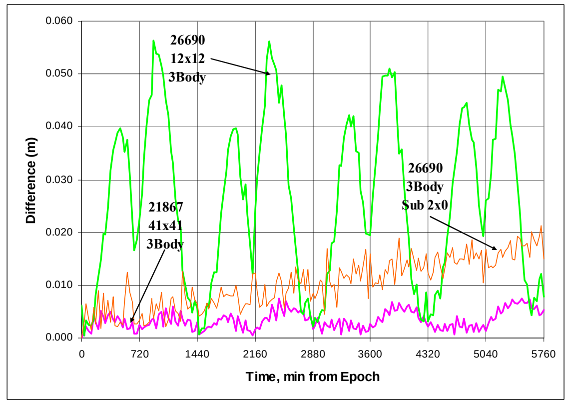


Figure 17:TRACE and GTDS Third-body Comparisons to STK/HPOP: Positional differences of several satellites are shown for third-body forces. The "sub" line in the GTDS graph represents the difference of runs of combined forces to better show the effect of just third-body forces. The higher line contains the original signature from the gravitational forces.

The GTDS runs included gravitational forces, which as we saw earlier, included about a 0.06 m difference at the end of 4 days. Thus, the added contribution due to third-body modeling for GTDS is really the difference in the two runs, or about 2 cm, which is comparable to all the other runs for third-body perturbations. The "3b sub" line shows these differences. The Raytheon TRACE runs have just the effect of third-body forces, and are comparable with the differences seen with GTDS. The 21867 case for Raytheon TRACE shows a very similar signature to the 0×0 case in Fig. 14. Also note that I've only provided the position differences as the velocity differences mirrored the positional differences.

Special-K had one run that could be configured to show the approximate behavior of third–body perturbations. Because the result comes from the difference of two runs, it is only an estimate, and although the results are slightly higher than the TRACE and GTDS runs, it is within the same order of magnitude.

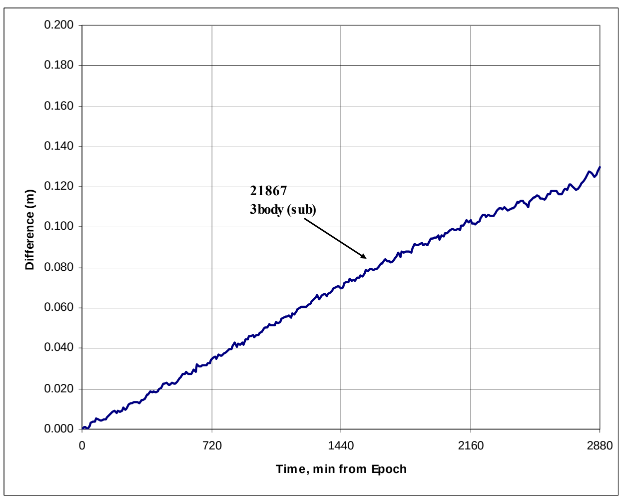


Figure 18: Special-K Third-body Comparisons to STK/HPOP: Positional differences for a single satellite are shown. Note that these results are obtained by subtracting two runs in which only third body perturbations were different.

SOLAR RADIATION PRESSURE MODELING

Next, I examined the results for solar radiation pressure. Although a smaller magnitude effect, it was a non-conservative force that could "easily" be examined and "tweaked" to analyze potential variations. Note that although the GTDS run includes some differences that were present in the gravitational and third-body force modeling, those differences were well below the differences seen in the solar radiation pressure results. Also as before, the TRACE runs are for just solar radiation pressure, and are still comparable. All runs used dual-cone shadow models. Note that some of these orbits experienced no shadowing during the propagation interval. This makes analysis more difficult as many of the options had no effect because they treat the eclipse periods differently.

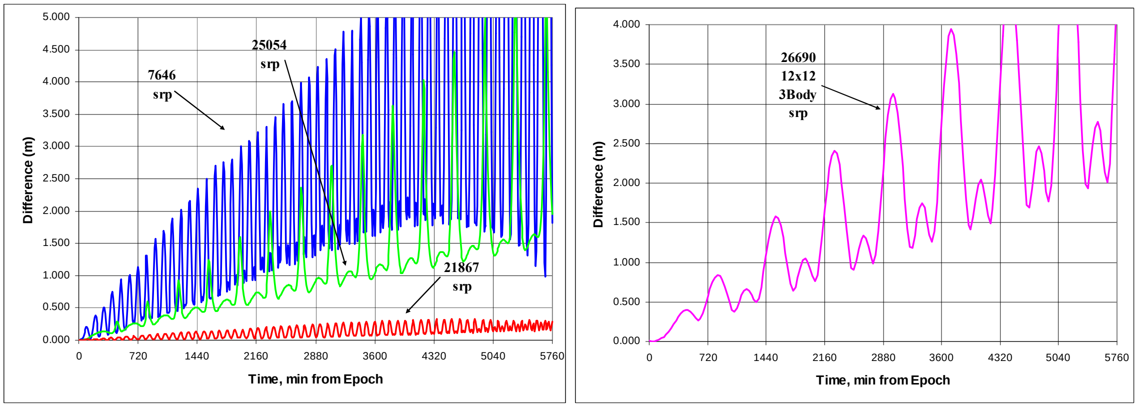


Figure 19: TRACE and GTDS Solar Radiation Pressure Modeling Comparisons to STK/HPOP: Several satellite positional differences are shown for solar radiation pressure. Although the results show meter-level differences, these are orders of magnitude below the effects of the dominant other forces.

It appeared that the value used for the solar irradiance strongly affects the results. A change, for example, from 1367.7 to 1363 W/m^2 usually allowed the programs to match more closely. This quantity, including how it is adjusted during program operation, is generally not documented, despite its' large impact on subsequent propagation.

ATMOSPHERIC DRAG MODELING

Atmospheric drag was analyzed last. Despite only a few test cases, significant analysis was performed to gain an understanding of the sensitivity to this perturbing force. The results are in Fig. 20.

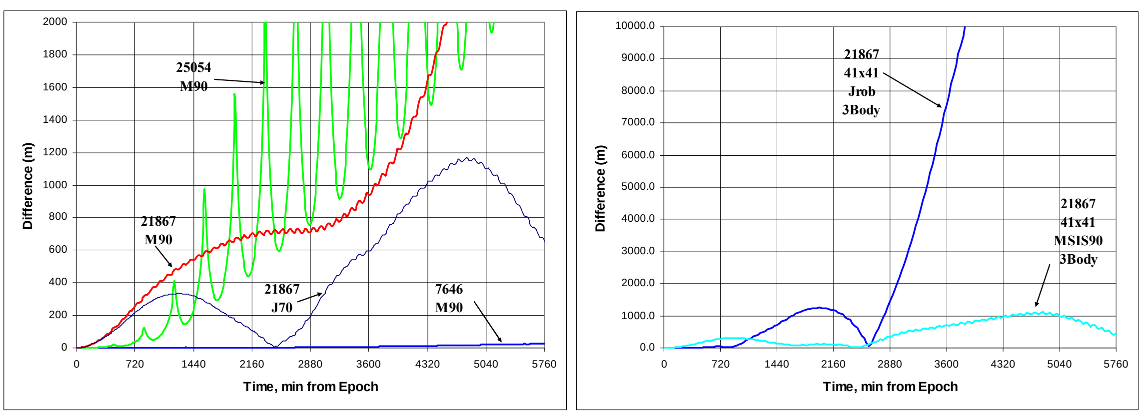


Figure 20: TRACE and GTDS Atmospheric Drag Comparisons to STK/HPOP: Several satellites are shown using the MSIS-90 and Jacchia-Roberts atmospheric models.

The GTDS test cases showed similar differences to the TRACE runs. A detailed analysis was not performed, but from initial discussions, it appears the treatment of the solar weather data (Fig. 11xx) contributes most, if not all these differences. Atmospheric drag produced slightly larger differences than the other forces, but the variability resulting from the treatment of solar weather parameters contributes an order of magnitude larger difference. Likewise, simply changing the atmospheric models within a given flight dynamics program resulted in larger variations than those seen between any two programs – as we will see later.

A test showed the approximate behavior against Special-K. As in Fig. 21, it is only approximate, but since the drag perturbation caused the primary portion of the difference, the results are reasonably valid. Notice the similar results to Fig 20, even accounting for the 2 day propagation instead of 4.

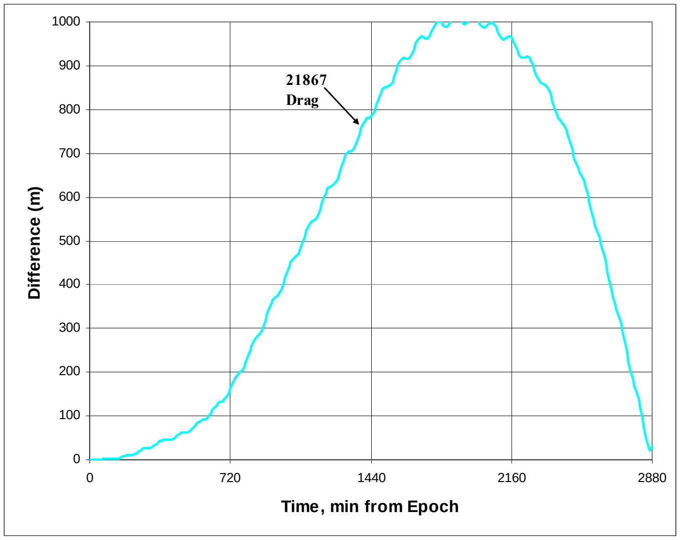


Figure 21: Special-K Atmospheric Drag Comparisons to STK/HPOP: A single test for atmospheric drag was conducted by comparing two runs in which only atmospheric drag was different.

COMBINED FORCES MODELING

Several ephemerides were available that lacked the initial stepwise comparisons to understand the specifics of each program. Therefore, these results showed larger variations, but were still indicative of the initial success in aligning numerical programs with limited analysis and contact. There were runs from GEODYN and Special-K.

For the GEODYN cases, we had different atmospheric models for the same initial configuration of Starlette (07646) and JERS (21867) shown in Fig. 22. This was important because the different atmospheric models showed much larger differences (60-250 m) than the differing flight dynamics programs (10-60 m). Note that these differences would typically be mitigated because the force model parameters would be adjusted via differential correction fits with each atmospheric model, thereby improving the propagation performance with each model. However, the difference does show the sensitive nature of the model chosen. The results were for 4 days. Note that the J70/J71 model in GEODYN is a hybrid not matched exactly with STK/HPOP as only versions of J70 and J71 and Jacchia-Roberts are available in the Jacchia class. Nevertheless, Jacchia-71 showed just about 10-20 m difference at 4-days.

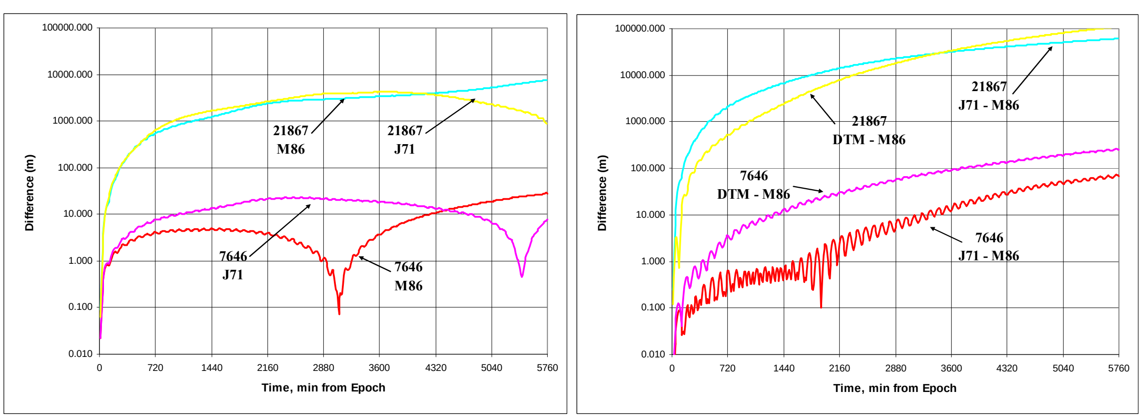


Figure 22: GEODYN Ephemeris Comparisons to STK/HPOP: Results are given for Starlette (07646) and JERS (21867). The left plot shows the comparisons while the right plot shows the difference from simply switching atmospheric models. The scales are the same. Note the comparison differences are smaller than the differences resulting from simply changing the atmospheric model.

The results for JERS (21867) are also in Fig. 22. While the results are not as good as 07646, the variability is again significantly less than the drag model differences within a single flight dynamics program. In addition, the relative differences between STK/HPOP J71 and MSIS are about the same order of magnitude (or less) as the difference from GEODYN using different atmospheric models. From the drag sensitivity tests, this is expected. The GEODYN runs again provided the opportunity to gauge the differences resulting from changing the atmospheric models within a single program. GEODYN showed almost 20 km differences at 2 days, and 60-110 km at 4 days just by switching the atmospheric model within its suite of available models.

The Special-K tests appeared varied until plotted together. There is a slight signature present in each of the ephemeris propagations. The cause was not investigated due to time, but is most likely in a gravitational constant or coefficient.

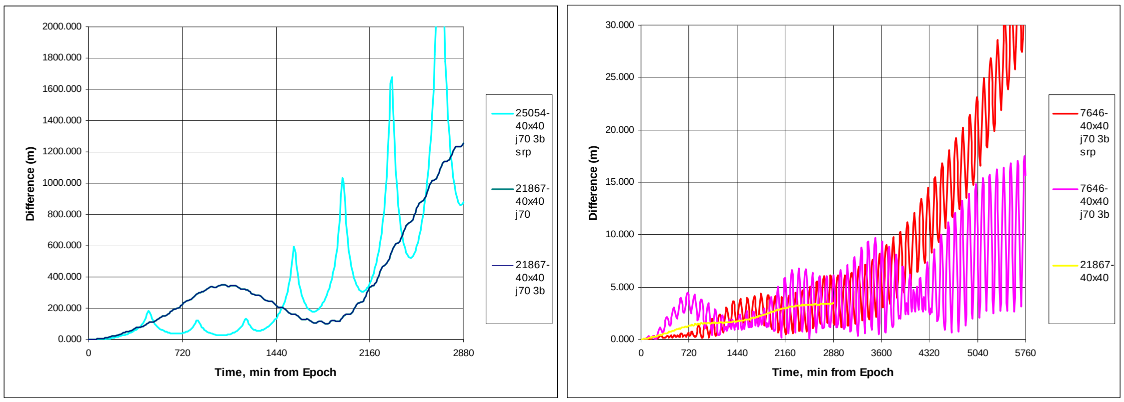


Figure 23: Special-K Ephemeris Comparisons to STK/HPOP: Results are given for several satellites. Notice that the scales are not the same. Also recognize the drifting signature present in all the comparisons. The 21867 40x40 gravity field case is shown to highlight the "signature".

A series of tests were run against GEODYN for geosynchronous orbits, TDRS-4 and TDRS-6. A constant acceleration term was added throughout the ephemeris generation of each. For TDRS-6, two additional runs were made starting from a different epoch. One of those runs did not include solar radiation pressure. The TDRS runs are for an entire month, although Fig. 24 also shows 4-day performance.

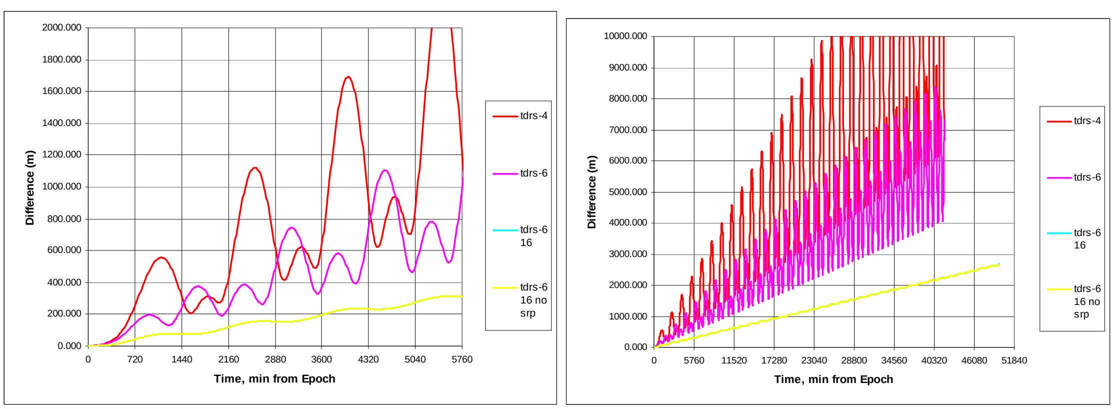


Figure 24: GEODYN Ephemeris Comparisons to STK/HPOP : Results are given for TDRS-4 and TDRS-6. The scales are different to show 4-day and ~30-day performance. The differences show the importance of obtaining a good initial state.

Figure 24 indicates an important consideration for the operational planner – that of obtaining an accurate initial state. Clearly, the TDRS ephemeris comparisons should have produced much smaller differences, but the results are indicative of the effect of varying initial accuracies and the resulting ephemeris generation. This led to the last analysis for this paper.

SUMMARY RESULTS

The results show that the gravitational (conservative) forces were remarkably similar between all flight-dynamics programs. Agreement at the cm and mm-level indicates that essentially no differences exist between the programs, despite the fact that they represent a wide spectrum of programming languages, sources, history, application, etc. Third-body forces introduced slightly more error, but still well below a meter. This level of accuracy is clearly acceptable for many, if not all operations. For operational considerations, it is doubtful that any planner would make a different operational decision based on a few mm difference in a predicted position.

Non-conservative forces showed larger variations, but upon examination, they yield the same conclusions as the conservative forces. The majority of the differences for non-conservative forces arose from interpretations of how to treat the input data for each force model (solar weather data and solar irradiance). None of these represent "errors" in the sense of right or wrong answers. Rather, they represent opinions on how the data should be used. Other problems result from the implementation. Specifically, the actual computer code seldom follows the original technical standard defined in the publication, report, etc. While the need to adopt certain recommended practices for computer code can be made, forcing a single computer code use can cause unwanted negative consequences. Specifically, mathematical documentation and peer-reviewed validation and verification, are often unavailable for these codes. Clearly this is simply not practical, and adopting a single code in this environment would surely stifle innovation and progress in astrodynamics.

Given the differences in drag and solar radiation results, it may seem that mm-level comparisons are not possible. However, informal discussions with engineers in the community reveal that when the computer code, technical documentation, precise treatment of input data, etc. is examined, it is possible to align even the non-conservative forces to this level. The conclusion is two-fold. First, comparisons that result in the "first-cut" results demonstrated in this paper are sufficient for many, if not all, operational applications. Special applications that require additional accuracy can be made to match as long as the code and inputs are flexible to accommodate any specific implementations.

Table 4 : Summary Ephemeris Comparisons: Approximate differences at the end of the propagation.

Program	Satellite	Force Models	Span (days)	Δr (m)	Comments
GTDS	26690 – GPS	G2×0	4	0.060	
GTDS	26690 – GPS	G12×12	4	0.060	
GTDS	21867 – JERS	G2×0	4	0.030	
GTDS	25054 – SL-12R/B	G2×0	4	2.000	Regularized time
Ray TRACE	7646-Starlette	2B	4	0.001	
Ray TRACE	7646-Starlette	G41×41	4	0.002	
Ray TRACE	25054 – SL-12R/B	2B	4	0.001	
Ray TRACE	25054 – SL-12R/B	G41×41	4	0.001	
Ray TRACE	21867-JERS	2B	4	0.006	
Ray TRACE	21867-JERS	G41×41	4	0.010	
Special-K	21867-JERS	G40×40	2	3.400	
GTDS	21867 – JERS	G41×41 3B	4	0.030	Just 3B is 0.007
GTDS	25054 – SL-12R/B	G41×41 3B	4	2.000	Regularized time
GTDS	26690 – GPS	G12×12 3B	4	0.060	Just 3B is 0.006
Ray TRACE	7646-Starlette	3B	4	0.001	
Ray TRACE	25054 – SL-12R/B	3B	4	0.007	
Ray TRACE	21867-JERS	3B	4	0.006	
GTDS	26690 – GPS	G12×12 3B SRP	4	2.500	
Ray TRACE	7646-Starlette	SR	4	3.000	
Ray TRACE	25054 – SL-12R/B	SR	4	1.000	
Ray TRACE	21867 – JERS	SR	4	0.540	
Ray TRACE	7646-Starlette	M90	4	18.000	
Ray TRACE	25054 – SL-12R/B	M90	4	1280.000	
Ray TRACE	21867 – JERS	M90	4	2200.000	
GTDS	21867 – JERS	G41×41 JRob 3B	4	20000.000	

GTDS	21867 – JERS	G41×41 M90 3B	4	53000.000	
GEODYN	7646-Starlette	G41×41 M86 3B	4	53.000	
GEODYN	7646-Starlette	G41×41 J71 3B	4	7.000	
GEODYN	21867-JERS	G41×41 M86 3B	4	40000.000	
GEODYN	21867-JERS	G41×41 J71 3B	4	600.000	
GEODYN	Tdrs4	G40×40 3B SR ST	4	1200.000	At 30 days, 8000
GEODYN	Tdrs6	G40×40 3B SR ST	4	800.000	At 30 days, 6000
GEODYN	Tdrs6	G40×40 3B ST	4	300.000	At 34.5 days, 2600
GEODYN	Tdrs6	G40×40 3B SR ST alt srp	4	300.000	At 34.5 days, 2600
Special-K	7646-Starlette	G40×40 J70 3B	4	12.000	
Special-K	7646-Starlette	G40×40 J70 3B SR	4	30.000	Diff srp
Special-K	21867-JERS	G40×40 J70	2	1200.000	
Special-K	21867-JERS	G40×40 J70 3B	2	1200.000	
Special-K	25054 – SL-12R/B	G40×40 J70 3B SR	2	1400.000	

COMMUNITY STANDARD EPHEMERIS BASELINE

There is great interest in numerical operations today, but insufficient data with which to compare, contrast, and baseline. Thus, I decided to provide several ephemerides under different force model conditions to act as an interactive astrodynamic community forum for the testing and development of numerical programs. There is no assumption of right or wrong – just a community forum to discuss the relative merits of different approaches and implementations. It is hoped that this will foster better communication between organizations using different flight dynamics programs and increase productivity and research. Unlike similar efforts attempted in the past, this is not intended to force compliance, but rather to stimulate collaboration. CSSI is willing to act as a clearinghouse for this operation, and the sample ephemerides are located on the CSSI website for your use (http://www.centerforspace.com/downloads/). The website contains ephemeris information for many of the runs. STK/HPOP provides a unique platform to tailor many parameters including coordinate systems, EOP data, solar weather data, etc., and as such, many comparisons can be conducted by a STK user. For users with access to STK, you can also download the scenarios that will reproduce these runs.

CONCLUSIONS

There were many conclusions from the paper, so I simply list them in bullet form:

- Standards are required to ensure efficient, interoperable operations.
- Standards provide a state-of-the-art forum upon which new theories can be designed, tested, and introduced.
- Computer code is not a standard.
- Periodic checks are needed to independently validate and verify results and approaches.
- Examination of any differences ensures proper operation, and should not be used to judge “right” or “wrong”.
- Simply re-running old test cases is *not sufficient* to ensure trust and confidence in the final answer.
- No one force model selection suffices for all orbital altitudes, locations within a solar cycle, types of satellites, and accuracy requirements.
- Orbital classes may not the best way to determine orbit force model setups, depending on the desired accuracy.
- Precise applications require passing the state vectors and detailed information about the force models.
- Organizations sharing state vector data must pass all the relevant information to effectively operate independent systems while still achieving the same answers (the proposed format at the end of this paper is intended to stimulate discussion and action to develop and implement a new format).
- Analyses of perturbation contributions should use dynamic, time-varying representations, and not single averaged values over a period of time.

- Examining the force model effects on a particular satellite reveals important ordering of the forces. This is critical to proper alignment between programs as the largest differences need to be aligned first.
- Gravity fields usually contribute the largest single effect on satellites.
- Gravity fields should not be truncated for precise operations and zonals should be retained for additional accuracy.
- Atmospheric drag is generally the second largest effect, although third-body forces are sometimes larger at higher altitudes.
- Even for high-altitude orbits, solar radiation pressure is a “smaller” force.
- Tides, albedo, and other forces contribute very small effects to orbits, and should generally be considered only for the most precise operations.
- Because the programs compared closely with a central program, they would also compare at the same level between themselves – a significant existing interoperability statement about all the programs in this study.
- It’s important to ensure the implementation of a particular technique is the same, and doesn’t have just the same title. This is especially apparent in the drag models.
- Conservative forces (gravity and third-body) matched to cm and mm-level with little additional analysis.
- Atmospheric drag showed the largest variations, but also included the largest number of approaches from which assumptions and variations could be taken.
- Atmospheric drag comparisons need to process the solar weather data similarly ($F_{10.7}$, a_p, interpolation, daily, 3-hourly, etc.) The variability among programs was shown and a recommended practice was proposed.
- Comparison to POEs is recommended because the distinction between post-processing and prediction is important. Operations inherently use prediction – something not often examined (See my 2007 paper on www.CenterForSpace.com).
- Even with accurate comparisons between programs, prediction will always result in disparate results due to un-modeled forces, and imprecise initial state vectors.
- The programs considered can operate and yield answers that will give the same operational decision.
- Millimeter-level comparisons are possible with additional study, technical documentation, and cooperation.

In summary for our initial question and objective, identical code is ***not*** needed to align programs, but attention to detail is. The combination of the initial state uncertainties, the lack of a standardized approach to transfer all the input data parameters, and the error growth with respect to “truth” (POE comparisons), make answers that agree from a single code meaningless.

There are several follow-on studies that could extend the results of this paper. Most of these would examine atmospheric drag because it showed the largest variations. A covariance study to compare the resulting atmospheric density process noise (Wright and Woodburn, 2004) resulting from different atmospheric models, and treatment of input solar data could indicate if one approach better models the dynamics. Comparing legacy orbit determination methods and approaches is also needed as it encompasses additional techniques necessary to answer the fundamental space surveillance question on processing observational data. Ultimately, the covariance should be used to determine the alignment of flight dynamics programs. If the program differences are less than the uncertainty, they are unimportant.

ACKNOWLEDGMENTS

This project was very gratifying not only to work with these highly respected individuals and organizations, but for the results and collaborative approach we accomplished. I am humbled and very grateful to all the individuals who assisted this effort by providing ephemerides for comparison – Paul Cefola, Shannon Coffey, Steve Crumpton, Mark Davis, Aaron Trask, and Michael Zedd. They, and several others are to be commended for their many insightful comments for this paper – Matthew Berry, Doug Cather, Vince Coppola, David Finkleman, Michael Gabor, Dick Hujsak, Jay Middour, Justin Register, TS Kelso, and Jim Woodburn. It was a pleasure to be able to work with this group to discover how to line the various programs up. I believe it also provided valuable insight for each group to re-look at their specific implementation. Interoperability and commonality between these groups is significantly enhanced.

REFERENCES

1. AIAA Standards. 1995. Recommended Practice, Part. AIAA BSR-AIAA, R-064-1995, Reston VA.
2. Barker, W. N., et al. 1996. Earth Gravitational Error Budget. Paper AAS 96-124 presented at the AAS/AIAA Space Flight Mechanics Conference. Austin, TX.
3. Cefola, Paul J., and D. J. Fonte. 1996. Extension of the Naval Space Command Satellite Theory to include a General Tesseral m-daily Model. Paper AIAA-96-3606 presented at the AIAA/ AAS Astrodynamics Conference. San Diego, CA.
4. Cefola, Paul J., et al. 2003. Extension Comparison of the DSST and the USM Semi-analytical Orbit Propagators. Paper AAS-03-236 presented at the AAS/AIAA Space Flight Mechanics Conference. Ponce, Puerto Rico.
5. Chao, C. C., et al. 2000. *Independent Verification and Validation for Analytical Graphics Inc. of Three Astrodynamic Functions of the Satellite Toolkit, Ver 4.1.0*. Aerospace TR 2000 (7605)-1. El Segundo, CA.
6. Coffey, S. L., and H. L. Neal. 1998. An Operational Special-Perturbations-Based Catalog. Paper AAS 98-113 presented at the AAS/AIAA Space Flight Mechanics Conference. Monterey, CA.
7. Gaposchkin, E. M. 1994. Calculation of Satellite Drag Coefficients. Technical Report 998. MIT Lincoln Laboratory, MA.
8. Hoots, Felix R., and Ronald L. Roehrich. 1980. Models for Propagation of NORAD Element Sets. Spacetrack Report #3. U.S. Air Force: Aerospace Defense Command.
9. Hoots, Felix R., and Richard G. France and Richard H. Smith. 1986. Transformations between Element Sets**.** Spacetrack Report #5. U.S. Air Force: Aerospace Defense Command.
10. Jacchia, L. G. 1965. Static Diffusion Models of the Upper Atmosphere with Empirical Temperature Profiles. Smithsonian Contributions to Astrophysics. Vol. 8. pp. 215-257.
11. Jacchia, L. G. 1970. New Static Models for the Thermosphere and Exosphere with Empirical Temperature Profiles. SAO Special Report No. 313. Cambridge, MA: Smithsonian Institution Astrophysical Observatory.
12. Jacchia, L. G. 1971. Revised Static Models for the Thermosphere and Exosphere with Empirical Temperature Profiles. SAO Special Report No. 332. Cambridge, MA: Smithsonian Institution Astrophysical Observatory.
13. Jacchia, L. G. 1977 Thermospheric Temperature, Density, and Composition: New Models. SAO Special Report 375. Cambridge, MA.
14. Kaya, Denise, et al. 2004. AFSPC Astrodynamics Standard Software. Paper AAS 04-124 presented at the AAS/AIAA Space Flight Mechanics Conference. Maui, HI.
15. McCarthy, Dennis D., and Gerard Petit. 2003. IERS Technical Note #32 – IERS Conventions (2003). U.S. Naval Observatory.
16. Register, Justin. 2003. Contributions of Individual Forces to Orbit Determination Accuracy. Paper AAS 03-227 presented at the AAS/AIAA Astrodynamics Specialist Conference. Big Sky, MT.
17. Rim, Hyung-Jin., et el. 2000. Comparison of GPS-based Precision Orbit Determination Approaches for ICESat. Paper AAS 00-114 presented at the AAS/AIAA Space Flight Mechanics Conference. Clearwater, FL.
18. Schutz, B. E., B. D. Tapley, R. J. Eanes, J. G. Marsh, R. G. Williamson, and T. V. Martin, Precision Orbit Determination Software Validation Experiment. The *Journal of the Astronautical Sciences*, Volume XXVIII, No. 4, pp. 327-343, October-December 1980.
19. Seago, John and David Vallado. 2000. Coordinate Frames of the U.S. Space Object Catalogs. Paper AIAA 2000-4025 presented at the AIAA/AAS Astrodynamics Specialist Conference. Denver CO.
20. Tanygin, Sergei, and James R. Wright. 2004. Removal of Arbitrary Discontinuities in Atmospheric Density Modeling. Paper AAS 04-176 presented at the AAS/AIAA Space Flight Mechanics Conference. Maui, HI.
21. Vallado, David A., and Salvatore Alfano. 1999. A Future Look at Space Surveillance and Operations. Paper AAS 99-113 presented at the AAS/AIAA Space Flight Mechanics Conference, February 7-10, 1999. Breckenridge, CO.
22. Vallado, David A., and Scott S. Carter. 1997. Accurate Orbit Determination from Short-arc Dense Observational Data. Paper AAS 97-204 presented at the AIAA/AAS Astrodynamics Specialist Conference. Sun Valley, Idaho.
23. Vallado, David A. 1999. Joint Astrodynamic Working Group Meeting Minutes, September 20, 1999. USSPACECOM/AN, CO.

24. Vallado, David A. 2000. Joint Astrodynamic Working Group Meeting Minutes, February 3, 2000. USSPACECOM/AN, CO. Note the data from the JAWG March 1999 meeting was referenced and distributed at this meeting.

25. Vallado, David A. 2001. A Summary of the AIAA Astrodynamic Standards Effort. Paper AAS 01-429 presented at the AIAA/AAS Astrodynamics Specialist Conference. Quebec City, Canada.

26. Vallado, David A. 2007. *Fundamentals of Astrodynamics and Applications*. Third Edition. Microcosm, Hawthorne, CA.

27. Vallado, David A. and T. S. Kelso. 2005. Using EOP and Space Weather data for Satellite Operations. Paper USR 05-S7.3 presented at the 6th US/Russian Space Surveillance Workshop, St. Petersburg, Russia.

28. Wright, James R. and James Woodburn. 2004. Simultaneous Real-time Estimation of Atmospheric Density and Ballistic Coefficient. Paper AAS-04-175 presented at the AAS/AIAA Space Flight Mechanics Conference. Maui, HI.

APPENDIX: PROPOSED STATE VECTOR FORMAT

The importance of having a standard method to transfer data is clearly indicated, and at present, this is incomplete within the astrodynamics community – hence this format for transferring state vector information. This constitutes an initial set of information necessary to align numerical integration programs. Depending on the level of agreement desired, more or less information will be required. Experience from the various comparisons in this paper suggests that these parameters are sufficient to gain a rough comparison between programs. These formats are not intended to be forced upon the community. Rather, they are intended to stimulate discussion, change, addition, etc., so they can become a standard vehicle through which we operate. If additional tests, integrators, force models, etc., are desired, please contact me for additional assistance. Samples are included on the web – http://centerforspace.com/downloads/.

```
STATE VECTOR DATA
SATELLITE NUM  : xxxxxxxxx  COMMON NAME: ____________________ INT DES: _________
ORIGIN         : _____________
EPOCH (UTC)    : yyyy mmm ddd hh:mm:ss.sssssss
COORD SYS      : _________________
POS KM         : ±xxxxxxx.xxxxxxxx   ±yyyyyyy.yyyyyyyy   ±zzzzzzz.zzzzzzzz
VEL KM/S       : ±xxx.xxxxxxxxxxxx   ±yyy.yyyyyyyyyyyy   ±zzz.zzzzzzzzzzzz
ACCEL KM/S2    : ±xx.xxxxxxxxxxxxx   ±yy.yyyyyyyyyyyyy   ±zz.zzzzzzzzzzzzz

GEOPOTENTIAL   : __________________________ DEGREE/ORDER: ____ x ____
ATMOS DRAG     : ___  MODEL : ________________________
THIRD BODY     : ___  SOURCE: ___________  BODIES: __ __ __ __ __ __ __ __ __ __
SOLAR PRESS    : ___                               SU MN ME VE MR JP ST UR NP PL
SOLID TIDES    : ___  MODEL : ________  TERMS : ________
OCEAN TIDES    : ___  MODEL : ________  TERMS : ________
EARTH ALBEDO   : ___  GRID SIZE: ____
ACCELERATIONS  : ___  DIRECTION: ____________________ MAGNITUDE : ______________
MANEUVERS      : ___

1/BC (M2/KG)   : ±________________ CD: ____  AREA(M2): _________ MASS (KG): _____
1/SRPC (M2/KG) : ±________________ CR: ____  AREA(M2): _________
THR ACC (M/S2) : ±________________ CM OFFSET(M)      : ±________
ATTITUDE       : _________________

EOP DATA       : _________________
SOLAR WEATHER  : _________________
INTERPOLATION  : _________________
SOLAR F10.7    : _________________ AVG F10.7: ____  AVG AP: ____
SHADOW MODEL   : _________________
PREC/NUT UP (S): _________________

INTEGRATOR     : _________________ STEP MODE      : _________________
INIT STEP (S)  : _________________ ERROR CONTROL  : _________________
REG TIME EXP   : _________________ REG TIME STEPS : _________________

COV COORD SYS  : ___
POS SIGMA (KM) : ±nnnn.nnnnnn       ±tttt.tttttt       ±wwww.wwwwww
VEL SIGMA(KM/S): ±nn.nnnnnn         ±tt.tttttt         ±ww.wwwwww
COV COORDINATES: __________________________ DIMENSION: ____
COV SOLVE-FORS : __________________________
UPPER DIAGONAL COVARIANCE
±x.xxxxxE±xx ±x.xxxxxE±xx ±x.xxxxxE±xx ±x.xxxxxE±xx ±x.xxxxxE±xx ±x.xxxxxE±xx
±x.xxxxxE±xx ±x.xxxxxE±xx ±x.xxxxxE±xx ±x.xxxxxE±xx ±x.xxxxxE±xx ±x.xxxxxE±xx
±x.xxxxxE±xx ±x.xxxxxE±xx ±x.xxxxxE±xx ±x.xxxxxE±xx ±x.xxxxxE±xx ±x.xxxxxE±xx
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±x.xxxxxE±xx ±x.xxxxxE±xx ±x.xxxxxE±xx ±x.xxxxxE±xx ±x.xxxxxE±xx ±x.xxxxxE±xx
±x.xxxxxE±xx ±x.xxxxxE±xx ±x.xxxxxE±xx ±x.xxxxxE±xx ±x.xxxxxE±xx ±x.xxxxxE±xx
     :            :            :            :            :            :
```

Many of the parameters would be optional. The format is shown as a static form, but the best implementation would be via XML so that a common form would be exchanged between organizations, but each organization could tailor the specific inputs for their programs from the XML data. In the above format, additional blank lines have been inserted to aid seeing each grouping within the data. The groupings include satellite location data, force models, physical satellite characteristics, input data, integrator controls, and covariance information. The file contains enough information to recreate the ephemeris generation in multiple programs. Also notice that the fields are separated by spaces to assist free-form reading. There may be concern about the size. Using a catalog of 20,000 satellites, each of which have an 8x8 covariance, the total file (static) size is about 50 Mb using the format above. This could easily be compressed. Even though an XML file format would be larger, comparing the uncompressed size to the file size of a recent Microsoft update for XP (95Mb), this seems pretty reasonable for accurate positional information on the entire satellite catalog. Notes for some of the fields are as follows:

Basic satellite information	
ORIGIN	Text field for the location of processing GEODYN, GTDS, NAVSPACOM, RayTRACE, STK, etc.
COORD SYS	Coordinate system and designator (both are needed) B1950, J2000, IAU2000 ECI, MOD, TOD, PEF, ECEF, etc.
Force model information	
GEOPOTENTIAL	Gravitational model – EGM-96, WGS-84/EGM-96, WGS-84, GGM-01, TEG-4, etc. Although not listed, there should be a one-time transfer of the gravitational parameter, radius of the Earth, angular rotation, and possibly the gravitational coefficients themselves to ensure the same gravity model is in use
ATMOS DRAG	Atmospheric models – MSISE90, NRLMSIS00, J70, J71, JRob, DTM, etc.
TIDES	Models – IERS 2003, IERS 1996, UT, Other Terms – nutation dependent, other Notes about what Solar/Lunar ephemeris used – DE/LE, analytical, other
ACCELERATIONS	Duration, orientation, method, etc. for empirical accelerations
MANEUVERS	Number, duration, orientation, method, etc. for maneuvers and thrusting profiles. This particular field may need to be expanded to include mass flow rates, engine models, I_{sp}, etc.
Satellite detailed information	
BALLISTIC COEFF / SOLAR RAD COEFF	
	Reciprocal values of the coefficients entered as a combined value, or as component values. Note that the attitude and macro models may be used, and that the area for drag and solar radiation pressure are likely different.
ATTITUDE	The attitude may not be known, or may have a file of quaternions, or something else
External Data	
EOP / SOLAR WEATHER DATA	
	ACTUAL, CONSTANT, etc.
INTERPOLATION	Used for the EOP and solar weather data HERMITE, LAGRANGE, etc.
SHADOW MODEL	Shadow modeling for SRP. Dual cone uses both umbra and penumbra regions NONE, CYLINDRICAL, DUAL CONE
PREC/NUT UP	Update interval for precession nutation values
Integrator information	
INTEGRATOR	Integration scheme – RKF78, GAUSSJACK, ADAMSB, other
STEPMODE	Type of integration – FIXED, RELATIVE ERROR, REGTIME
INIT STEP	Step sizes, not used if relative error is selected
ERROR CONTROL	Error control if needed by the integrator, e.g. 1.0 e-15, other
Covariance information	
COV COORD SYS	Coordinates for sigma values – RSW, NTW, ECI, other
COV COORDINATES	Format of the covariance matrix – J2000 ECI, CARTESIAN, EQUINOCTIAL
DIMENSION	Size of covariance matrix
COV SOLVE-FORS	Parameters included as solve-fors in the covariance