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THE MEAN-SOLAR-TIME ORIGIN OF UNIVERSAL TIME AND UTC

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Universal Time is the measure of Earth rotation that serves as the astronomical basis of civil timekeeping. Since the end of the 19th century, Universal Time has been maintained to preserve continuity with Newcomb's mean solar time at Greenwich. Here, the concept of a fictitious mean sun is revisited and compared with UT1. Simulations affirm that Universal Time has separated from Newcomb's mean sun by approximately ($^1/_{365}$) ΔT as predicted by theory. The disparity is about 0.2^{s} presently, which is much less than the $\pm 0.9^{s}$ differences allowed between UT1 and UTC, the atomic realization of mean solar time used for civil timekeeping.

INTRODUCTION

The solar day is the elementary unit of all calendars.[1, 2] A natural solar day is measured as the duration between two culminations of the apparent Sun over a meridian, with *apparent solar time* being the hour angle of the Sun between culminations. However, the duration of apparent solar days and hours are irregular because of a non-uniform motion in the Sun's right ascension due to the obliquity of the ecliptic and the eccentricity of the Earth's orbit. The irregularity of such natural phenomena is mismatched with the behavior of man-made timekeeping devices, which tend to increment uniformly.

Mean solar time is the time scale by which the solar days of the calendar have been resolved with the uniformity of synthetic timekeepers. Its purpose was to establish a time scale of equal hours and days that kept pace with the Sun in the long term.[3] In the same way that successive culminations of the Sun on the celestial sphere define the apparent solar day, the mean solar day implies sequential culminations of a fictitious point along the celestial equator known as the *mean sun*, with mean time-of-day defined as the hour angle of this fictitious sun.[4]

As a mathematical abstraction traveling at uniform angular velocity, the mean sun cannot be observed directly. Rather, its imagined diurnal motion is a consequence of Earth's rotation relative to the mean longitude of the apparent Sun, at least in principle. Consider a *sidereal day*, $d_{\star}$, as the duration of the Earth's rotation between two successive culminations of the precessing vernal equinox (the point where the ascending Sun notionally crosses the celestial equator). Over one orbital revolution of the Earth about the Sun of duration P, the fractional number of sidereal days k can be counted, such that:

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$$d_{\star} = \frac{P}{k}, \tag{1}$$

with $k \sim 366\ ¼$. Because the Earth circuits the Sun during P, there is exactly one fewer Earth rotations relative to the Sun than to the equinox. Thus, the duration of a *mean solar day,* $d_{\odot}$ becomes:

$$d_{\odot} = \frac{P}{k-1}. \tag{2}$$

Combining equations (1) and (2) to eliminate k results in:

$$(d_{\odot})^{-1} = (d_{\star})^{-1} - P^{-1}. \tag{3}$$

Multiplying Eq. (3) by 2π, an expression for angular rate develops:

$$\omega_{\odot} = \omega_{\star} - n_{\oplus}, \tag{4}$$

where $\omega_{\odot}$ represents an angular rate for mean solar time, and where $\omega_{\star}$ is the angular rate of Earth rotation and $n_{\oplus}$ is the mean motion of Earth's orbit relative to the equinox. Consequently, the rate of mean solar time combines Earth's rotation rate and Earth's mean motion (Figure 1).

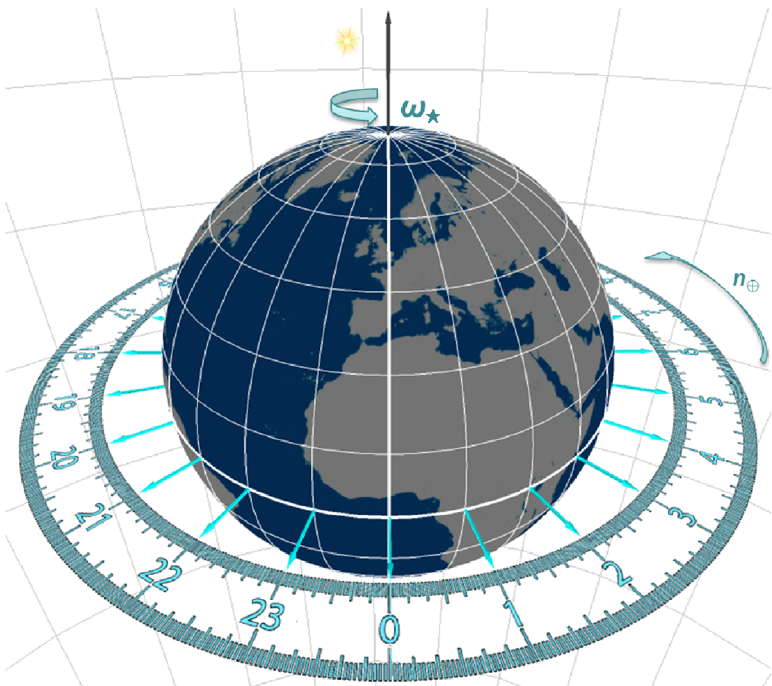

Figure 1. Earth as a Clock (0^h UT, February 29, 2000). The Earth spins at rate $\omega_{\star}$ while the mean-time "dial" rotates once per year with the fictitious mean sun at 12^h. Each meridian indicates its own mean solar time.

The concept of mean solar time has been used since antiquity as a specialized time scale for scientific and navigational purposes; for example, the relationship between mean solar time and apparent solar time was empirically determined by Ptolemy from lunar studies as early as the 2nd century A.D.[5] However, it was the proliferation of well-regulated mechanical clocks that ushered mean-solar timekeeping into general usage. Mean solar time on the meridian of Greenwich

(GMT) was eventually recommended as a universal standard by the 1884 International Meridian Conference at Washington, D.C. That Conference proposed "the adoption of the meridian passing through the centre of the transit instrument at the Observatory of Greenwich as the initial meridian for longitude." It also proposed "the adoption of a universal day for all purposes for which it may be found convenient," and that "this universal day is to be a mean solar day… to begin for all the world at the moment of midnight of the initial meridian."[6]

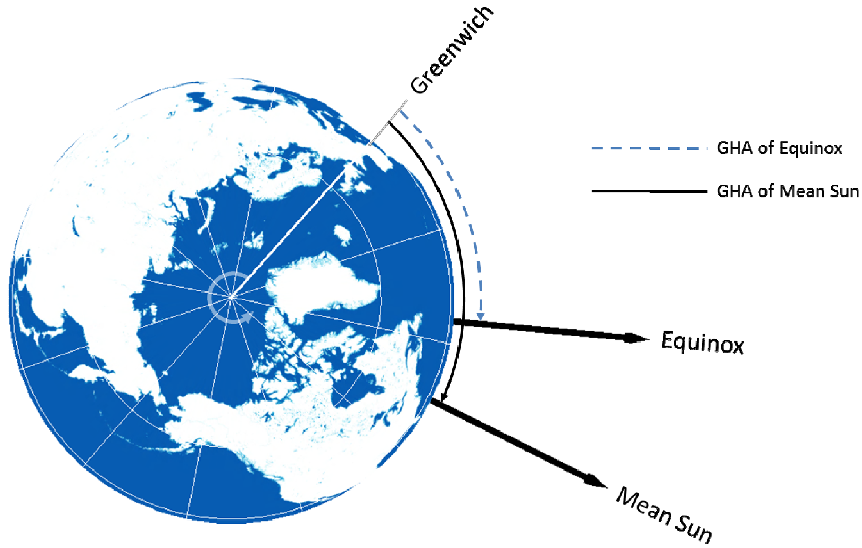

Figure 2. Greenwich Hour Angles of the Mean Equinox of Date and Fictitious Mean Sun (February 29, 2000, Noon, Eastern Standard Time).

Astronomical Convention from 1900 to 1983

Different theories for mean solar time were in use at the time of the 1884 meridian conference. Recognizing the problem of disparate astronomical standards, Newcomb strove to develop a self-consistent set of astronomical constants.[7] These became the basis of discussions at the Paris Conference of 1896 to further unify the calculation of astronomical ephemerides and phenomena internationally.[8] Newcomb's developments included a conventional expression for the right ascension of the fictitious mean sun, which was used through most of the 20th century:[9]

$$R_{\odot}(T) = 18^{h}38^{m}45.836^{s} + 86401845.42^{s}T + 9.29^{s}T^{2} , \tag{5}$$

where T is the number of Julian millennia of 365250 days elapsed since "1900, Jan. 0, Greenwich Mean noon."* Mean solar time at Greenwich, measured from midnight, was expressible as the hour angle of this fictitious mean sun + 12^{h}, or equivalently:[10]

$$\text{GMT} = 12^{h} + \text{Greenwich hour angle of the mean equinox of date} - R_{\odot}(T) , \tag{6}$$

Thus, mean solar noon (12^{h}) occurred at Greenwich whenever the Greenwich hour angle of the mean equinox of date equaled $R_{\odot}(T)$ (Figure 2). The significance of a fictitious mean sun with

* In contemporary parlance, this zero epoch is 31 December 1899, 12h UT. Newcomb also originally used units of Julian centuries instead of Julian millennia; the larger unit is adopted here to be consistent with modern conventions.

right ascension $R_{\odot}(T)$ was that its transit over any meridian of the celestial sphere defined, in principle, the moment of mean noon for that meridian.

Newcomb did not specify the measure of time that T reckoned, although units of mean solar days are presumed.[11] In Newcomb's era, no distinction was drawn between the progressions of time indicated by the rotation of the Earth versus the independent variable of solar-system theory; these two concepts were not separated until the mid-20$^{\underline{\mathrm{th}}}$ century, after Earth rotation was concluded to be slightly non-uniform. Eventually, the theoretically uniform argument of solar-system theory became known as *Ephemeris Time*, ET, and was intended for scientific applications, while Earth rotation persisted as the basis of civil timekeeping under the name *Universal Time*, UT.

Accordingly, Newcomb's T of Eq. (5) was interpreted as the independent variable of the Earth's orbital motion, which became associated with Ephemeris Time (T_{E}). Hence, the "ephemeris mean sun" $R_{\odot}(T_{\mathrm{E}})$, was recognized as the right ascension of Newcomb's fictitious mean sun. Meanwhile, the practice continued of using observed Universal Time (T_{U}) in Eq. (5), and the "universal mean sun" $R_{\odot}(T_{\mathrm{U}})$, was deemed a conventional expression that related Universal Time and Greenwich mean sidereal time (GMST) at 12^{h} UT per Eq. (6). Thus, GMST was not rigorously attached to the right ascension of Newcomb's mean sun, and Universal Time was not technically definable as the hour angle of Newcomb's fictitious mean sun increased by 12^{h}. Rather, UT was practically defined by the operational procedures employing the conventional relationship of Eq. (6) with $R_{\odot}(T_{\mathrm{U}})$. The difference between $R_{\odot}(T_{\mathrm{E}})$ and $R_{\odot}(T_{\mathrm{U}})$ is:[12]

$$R_{\odot}(T_{\mathrm{E}}) - R_{\odot}(T_{\mathrm{U}}) \approx 0.002738\,\Delta T\,, \tag{7}$$

where ΔT, or "Delta-T", is the accumulated excess of the measure of Ephemeris Time over Universal Time: ET − UT.

Continuing operational improvements led to refined versions of Universal Time (UT0, UT1, UT2) having varying degrees of uniformity with periodic differences at the level of tens of milliseconds. A broadcast convention tying Universal Time to atomic frequency standards was further developed through the 1960's, which became known as *Coordinated Universal Time* (UTC).

By the 1970's, limitations with the definition and determination of Ephemeris Time led to the development of dynamical time scales based on the theory of general relativity. Ephemeris Time was superseded by the relativistic dynamical times, *Barycentric Coordinate Time* (TCB), *Geocentric Coordinate Time* (TCG), and *Terrestrial Time* (TT).[13] TT is a coordinate scale on the surface of the Earth (practically realized by removing 32.184^{s} from *International Atomic Time*, TAI). *Barycentric Dynamical Time* (TDB) is a scaled version of TCB that tracks TT on average and is operationally synonymous with the independent argument of JPL developmental ephemerides.

Astronomical Convention from 1984 to 1996

By the 1980's, traditional astrometry began to wane as technological advancements, such as Very Long Baseline Interferometry (VLBI) and Satellite / Lunar Laser Ranging (SLR, LLR), promised better accuracy in the measurement of Earth rotation. Two versions of Universal Time found widespread application: UT1 as the precise measure of rotation about Earth's observed rotational pole, and UTC as the atomic realization of Universal Time broadcast for precision work and civil-timekeeping. UT1 is made available to high accuracy by adding a correction to UTC:

$$\mathrm{UT1} = \mathrm{UTC} + (\mathrm{UT1} - \mathrm{UTC})\,. \tag{8}$$

UT1 − UTC is tabulated by the International Earth Rotation and Reference Systems Service (IERS), a service chartered in 1988 by the IAU and IUGG to supplant the *Bureau International de l'Heure* (BIH) and coordinate the results of astro-geodetic observing programs.

Newcomb's expression for $R_{\odot}(T_U)$ remained in effect until the relationship between GMST and UT was slightly revised effective January 1, 1984, in response to an improved system of astronomical constants adopted by the IAU in 1976 that included an update to the rate of precession. Upon adoption, it was recommended that the fundamental reference frame defined by the positions and rates of the fifth *Fundamental Katalog* (FK5) correspond as closely as possible to a dynamical reference frame derived from modern observations. This required a correction to the origin of right ascensions and the motion of the equinox of the former FK4. Changes to the positions and proper motions of the fundamental catalog would have affected the determination of sidereal time based on observations of cataloged stars, and consequently UT1, but the origin and rate of the relationship between GMST and UT was instead carefully redefined to maintain the same value of UT1 at the time of changeover.[14] Therefore, the expression for GMST at 0^h UT1 was slightly augmented to negate the effects of equinox adjustment, leaving:[15]

$$\mathrm{GMST}_{0\mathrm{h\,UT}} = 6^{\mathrm{h}}41^{\mathrm{m}}50.54841^{\mathrm{s}} + 86401848.12866^{\mathrm{s}}\, T_U + 9.3104^{\mathrm{s}}\, T_U^2 + 6.210\times10^{-3\mathrm{s}}\, T_U^3 \,. \qquad (9)$$

This action preserved numerical continuity within the UT1 time series, and this new expression effectively became a relationship that operationally defined Universal Time. Geocentric apparent coordinates of stars or planets, measured with respect to the earlier mean equator and mean equinox of date of the former FK4-based system, could then be placed on the FK5-based system by adding an equinox correction to the right ascension of date:[16]

$$\Delta\alpha = 0.0775^{\mathrm{s}} + 0.850^{\mathrm{s}} T = 1.1625'' + 12.75'' T \,, \qquad (10)$$

where T is the date expressed in units of Julian millennia TDB from epoch J2000.0.

Astronomical Convention from 1997 to 2002

Historically, the conventional expression for nutation in right ascension, also known as the *equation of the equinoxes*, was approximate. This resulted in periodic inaccuracies of a few mas in the kinematical relationship between Greenwich (apparent) sidereal time (GST) and GMST (and thus, UT1). Such effects were completely negligible considering the accuracy of traditional astrometric methods, but technological improvements motivated the development of more accurate expressions relating GST to GMST and UT1.[17] Consequently, effective January 1, 1997, the traditional expression for the equation of the equinoxes was amended to account for the largest periodic inaccuracy by adding:[18]

$$\mathrm{dUT1}(\Omega) = 0.00264''\sin(\Omega) + 0.000063''\sin(2\Omega) \,, \qquad (11)$$

where Ω is the mean longitude of the ascending note of the Moon's orbit.

Astronomical Convention from 2003 to Present

Effective January 1, 2003, the International Astronomical Union (IAU) redefined UT1 as having a linear relationship with the *Earth rotation angle*, θ_{UT1}, the angle between the Celestial Intermediate Origin (CIO) and the Terrestrial Intermediate Origin (TIO) of the latest IAU Precession-Nutation Theory:[19]

$$\theta_{UT1} = 2\pi\,(0.7790572732640 + 1.00273781191135448)\, T_{UT1} \qquad (12)$$

where T_{UT1} is the UT1 date measured in Julian days minus 2451545.0. An advantage of the CIO formulation is that the CIO is an origin with no precessional motion along the instantaneous plane of the equator; therefore, UT1 has a linear relationship with respect to the CIO. The values within Eq. (12) were carefully prescribed to maintain continuity with earlier definitions for Universal

Time insofar as possible, and to ensure that the time derivative of UT1 remained proportional to Earth rotation rate.[20]

GREENWICH MEAN SOLAR TIME TODAY

The constant of proportionality in the current definition of UT1 has traceability back to Newcomb's expression for the right ascension of the mean Sun. Consequently, UT1 is considered to be nominally equivalent to mean solar time reckoned from Greenwich.[21] However, from time to time the question arises as to how well the present realization of Universal Time represents mean solar time on the prime meridian. For example, Guinot (2011) suggests that all decisions thus far have preserved the role of UT1 as a representation of mean solar time at the Greenwich meridian "with a departure which may reach one or two seconds."[22] Proposed reasons for this estimated level of discrepancy include the following.

- The timing of solar transits cannot be precisely measured with respect to the background stars, such that historical determinations may be biased on the order of 0.1^s.
- The BIH did not define 0° longitude with the Royal Observatory at Greenwich, but referred its estimate of Universal Time to the reference meridian of a "mean observatory". This estimate was determined through a weighted average of Universal Times from radio signals of contributing observatories relative to their adopted longitudes. Changes to the averaging algorithms over the years may have preserved continuity at the order tens of milliseconds.
- The effects of polar motion cause declination-dependent non-uniformities in the time observed (on the order of 30 milliseconds at Greenwich latitude), requiring the IAU to place the terrestrial fiducial direction for Universal Time at zero latitude.
- Plate tectonics cause the relative longitudes of stations to change on the order of centimeters (microseconds) per annum.
- To keep the UT1 time series continuous, the BIH effectively altered the origin of global longitude when introducing systemic changes into its terrestrial system.[23] Adjustments and changes to conventions over time perturbed the International Reference Meridian 0.089' (5.34") to the east from the Airy transit circle room, or, advanced it by 0.356^s in time.[24, 25] A shift of approximately 100 meters is easily noticed today by a visitor at Greenwich equipped with a handheld GNSS receiver.

An error budget of these combined effects does not sum effortlessly into "one or two seconds," and indeed, there may be some debate as to what the phrase "mean solar time at Greenwich" implies in any analysis of the question. Nevertheless, this situation does not prevent comparison of the IERS Universal Time series with a specific mathematical model for the motion of a "mean sun". Toward this end, Newcomb's expression for mean solar time has the advantage of being the only convention globally adopted for such purposes; therefore, it seems justifiable to give strong preference to Eq. (5) as "a representation of mean time at the Greenwich meridian." Comparison with an expression based on more modern solar-system theory presents a further opportunity to assess both Newcomb's expression and the UT1 series from the IERS.

TRANSIT ANALYSIS OF THE SATELLITE MODEL

Systems Tool Kit (STK) is commercial software which specializes in the dynamical analysis and visualization of vehicle- and sensor-behaviors in terrestrial and extra-terrestrial environs. New analysis capabilities in the most recent version can trigger the times of when special values occur within custom calculations. This allows for a relatively expedient assessment of when simulated events happen that was inconvenient or impossible with previous software versions. These features are known under the STK option of *Analysis Workbench*, which is available to educational instructors without cost from Analytical Graphics, Inc. (AGI).

Newcomb imagined the fictitious mean sun as a point on the celestial sphere having uniform sidereal motion (μ) in the plane of the Earth's equator. A circular two-body orbit also has constant angular velocity, such that the mean anomaly increases in a theoretically uniform way. Therefore, the approach of this research is to model the fictitious mean sun as an artificial Earth satellite, and employ the new analysis features of STK to calculate and report the times of transit at the prime meridian relative to the UT1 time scale. If the UT1 scale represents the model of the fictitious mean sun, then the transits at the prime meridian will occur precisely at noon UT1; otherwise, transit times will indicate the difference in right ascension between the model of the fictitious mean sun and that implied by UT1 from the IERS.

(a) Sunrise at Greenwich (6:52 UT) (b) Greenwich Mean Noon

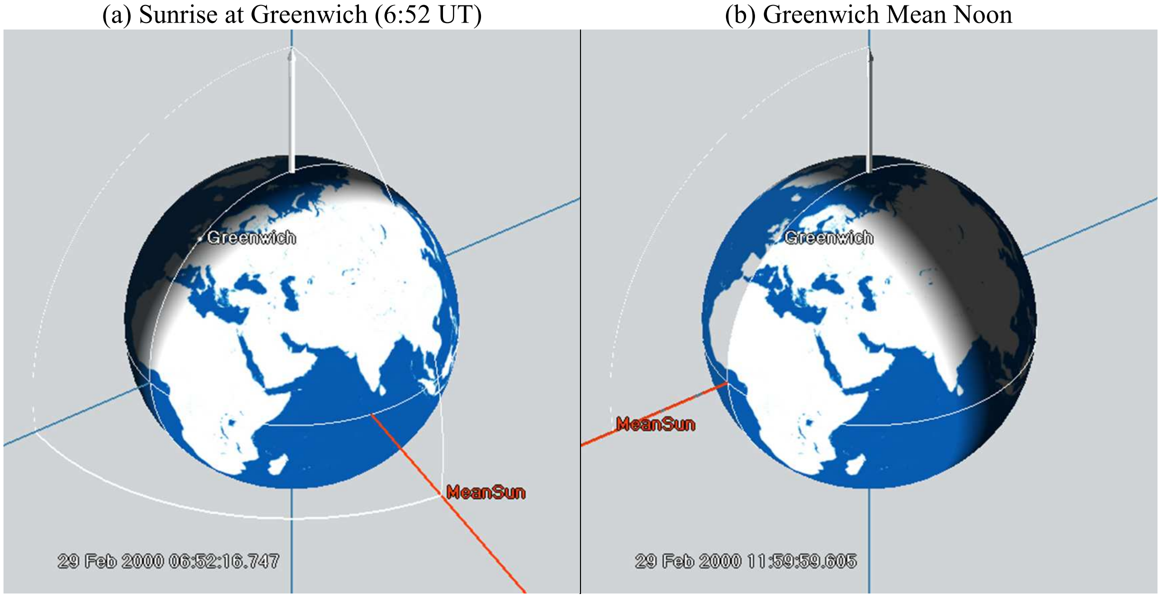

Figure 3. Dihedral Angle Between the Mean-Sun Satellite Direction and the Prime Meridian Plane, February 29, 2000. Mean solar transit is reached when this angle is zero.

Terrestrial Frame of Reference

For this analysis, each transit is modeled as the time at which the dihedral angle becomes zero between the "mean-sun" satellite position (measured from the geocenter) and a meridian plane that includes the ITRF Z-axis as the pole and ITRF X-axis as the longitude origin (Figure 3).*

Celestial Frame of Reference

One constraint for a mean sun is that the motion always resides within the plane of the mean equator. If the proposed two-body orbit is defined with zero inclination, then the out-of-plane (Z) positions and velocities are exactly zero and the motion is equatorial. Only two non-zero orbital

* The Celestial Intermediate Pole (CIP), which approximates the spin axis of the Earth (known in STK as the *pseudo-fixed* Z axis), could have been used as well, but the choice of pole does not materially affect the results, because the effects of polar motion are proportional to the tangent of latitude and transits are modeled at the equator.

elements are then required: the origin of right ascension at some initial epoch, and the orbital period. However, an inertial two-body orbit will not remain aligned with the plane of the Earth's equator, because the Earth's pole and equator will precess over time.

The requirement for equatorial motion is met in STK by exporting the modeled two-body motion to an ephemeris, changing the reference frame to a precessing *MeanOfDate* frame, and then importing the altered ephemeris as the basis of modeled motion (Figure 4). After this keyword edit, the software presumes that the ephemeris represents motion relative to the precessing equinox and celestial pole of the IAU 1976 Theory of Precession. Indeed, Newcomb's expression for right ascension of the fictitious mean sun, Eq. (5), is with respect to the mean equinox of date. However, declaring the mean-solar ephemeris to be relative to a precessing frame introduces subtle complications.

(a) before (b) after

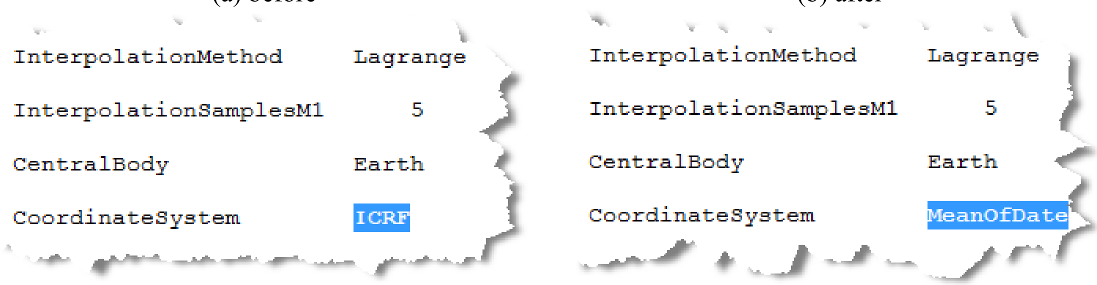

Figure 4. Editing the STK Ephemeris to Declare a Precessing Mean-of-Date Frame.

The orbital period specifies the duration through which the mean anomaly returns to its directional origin. The precessional rate of the mean equinox, to which the mean anomaly is referenced, is not strictly constant; thus, strictly uniform equatorial motion cannot be specified using a purely linear expression relative to a mean equinox of date.* However, the simulated two-body satellite motion, declared to be with respect to a mean-of-date equinox per Figure 4, can only be expressed linearly using two terms: mean anomaly and mean motion. Thus, the non-uniform precessional motion introduced by the moving frame of reference cannot be completely represented through the proposed two-body orbit model. A simple way to mitigate the non-linear effects of precession is to limit consideration only to times when the argument of precession theory, T, is relatively small, such that precession terms of order T^2 and higher have a negligible contribution. Another way is to remove these effects after the fact, because general precession of the conventional equinox is well known and simply modeled as a polynomial in right ascension. This approach is employed in the sequel.

Finally, there is a choice of methods in determining the equinox from numerical integrated ephemerides.[26] Newcomb considered the equatorial plane to be moving uniformly, or "rotating", which was the standard method of definition until 1998. Before him, LeVerrier took an instantaneous, or "inertial", plane for the equator.[27] This is the method used now for the definitions in the International Celestial Reference System (ICRS). When deducing the equinoctial direction dynamically, the difference between the two conventions creates an offset of approximately a tenth of an arcsecond. This discrepancy needs to be accounted for when comparing results across different conventions.

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* This is why Eq. (5) includes a non-linear term of $9.29^{s} T^2$.

THE TROPICAL YEAR

Long before the development of gravitational theories, the average duration of the Earth's orbital period was determined by observing the annual progression of the Sun over a large number of orbital cycles. Each year was historically measured from the solstices, when the latitude of the Sun approached the *tropics* (meaning "turn") and thus changed direction.[28] Yet, the term *tropical year* has also traditionally described the duration between successive solar passages of the vernal-equinox.[29] This *vernal-equinox year* has been critical to establishing the Gregorian calendar for determining the date of Easter.[30]

Unfortunately, successive passages of the solstices or equinoxes are not strictly uniform, because the orbital motion of the Earth's line of apses relative to the equinox introduces tiny rate differences, which differ depending on the starting location within the orbit.[31] Also, the intersection of the planes of the equator and ecliptic, which traditionally defines the equinox direction, has secular motion relative to inertial space due to precession. This motion is further subject to periodic nutation of the orientation of the Earth's spin axis and equator, primarily caused by gravitational torqueing of the Earth's figure by the Moon. Finally, the direction of the Earth-Moon barycenter relative to the Sun, which is located about 4700 km from the Earth's center of mass, differs from that of the Earth's center, depending on the location of the Moon. Thus, the time between successive annual passages of the Sun can vary on the order of several minutes, because of natural perturbations to the Earth's orbit, and depending on the points of reference used (Table 1).

Table 1. Estimated Duration Between Vernal Equinoxes (TDB Seconds)

Interval	Meeus & Savoie[31] (1992)	*Earth w.r.t. True Equinox	*Earth w.r.t. Mean Equinox	*E.-M. Barycenter w.r.t. Mean Equinox
1985-86	$365^d\ 5^h\ 48^m\ 58^s$	$365^d\ 5^h\ 48^m\ 58^s$	$365^d\ 5^h\ 50^m\ 42^s$	$365^d\ 5^h\ 53^m\ 33^s$
1986-87	$365^d\ 5^h\ 49^m\ 15^s$	$365^d\ 5^h\ 49^m\ 17^s$	$365^d\ 5^h\ 51^m\ 16^s$	$365^d\ 5^h\ 46^m\ 29^s$
1987-88	$365^d\ 5^h\ 46^m\ 38^s$	$365^d\ 5^h\ 46^m\ 39^s$	$365^d\ 5^h\ 48^m\ 57^s$	$365^d\ 5^h\ 52^m\ 39^s$
1988-89	$365^d\ 5^h\ 49^m\ 42^s$	$365^d\ 5^h\ 49^m\ 39^s$	$365^d\ 5^h\ 51^m\ 59^s$	$365^d\ 5^h\ 51^m\ 37^s$
1989-90	$365^d\ 5^h\ 51^m\ 06^s$	$365^d\ 5^h\ 51^m\ 01^s$	$365^d\ 5^h\ 52^m\ 50^s$	$365^d\ 5^h\ 49^m\ 19^s$
Interval Average	$365^d\ 5^h\ 49^m\ 08^s$ 365.2425^d	$365^d\ 5^h\ 49^m\ 06^s$ 365.2424^d	$365^d\ 5^h\ 51^m\ 09^s$ 365.2439^d	$365^d\ 5^h\ 50^m\ 43^s$ 365.2436^d

*as modeled by STK using IAU 1976 Precession, 1980 Nutation, and JPL DE421 Earth ephemeris

Today, the duration known as a *tropical year* is most accurately defined as the period by which it takes the mean orbital longitude of Earth, measured with respect to a precessing equinox, to advance 360º (one revolution or cycle). This modern definition results from the practice of relating the non-uniform motion of the Earth to the independent argument of a solar-system theory. Thus, the period of the Earth's orbit is still determined, in some average sense, from a finite number of orbital cycles, but in terms of the independent variable of celestial mechanics.

Since the 19th century it has become customary to express the duration of the tropical year as the inverse of the mean motion of the Earth-Moon barycenter from a precise solar-system theory. Specifically, if the mean longitude of the Sun (relative to a precessing mean equinox of date) can be generally expressed in the time-polynomial form:

$$L = L_0 + L_1 t + L_2 t^2 + L_3 t^3 + \dots , \tag{13}$$

then the (precessing) mean motion can be expressed as the time rate of change of L:

$$dL/dt = L_1 + 2L_2t + 3L_3t^2 + \ldots \, . \tag{14}$$

To express the duration of the tropical year Y in units of TDB seconds, one takes the reciprocal of Eq. (14) and applies appropriate scale factors to convert from units of arcseconds and Julian millennia:[32]

$$Y = \frac{(360 \times 60 \times 60) \times 325250 \times 86400}{L_1 + 2L_2t + 3L_3t^2 \ + \ldots} \ . \tag{15}$$

Borkowski (1991) reminds that if $L_1 >> L_2$, L_3, …, then Eq. (15) can be conveniently expressed as:[33]

$$Y = \frac{(360 \times 60 \times 60) \times 365250 \times 86400}{L_1} \left(1 - 2\frac{L_2}{L_1} T - 3\frac{L_3}{L_1} T^2 - \ldots \right) . \tag{16}$$

Using Eq. (5) as an example, Newcomb’s tropical year is $31556925.9747^{\mathrm{s}} - 5.3032^{\mathrm{s}} T$, or $365.242198782^{\mathrm{d}}$ at epoch 1900 and decreasing one-half second (TDB) per century.

RIGHT ASCENSION OF THE FICTITIOUS MEAN SUN

The mean longitude of the apparent Sun, referred to a precessing mean equinox of date, and displaced by aberration, may be expressed more precisely in the time-polynomial form:[34]

$$L = (\lambda_0 + \lambda_1 t + \lambda_2 t^2 + \ldots) + (h_1 t + h_2 t^2 + \ldots) - \kappa \, . \tag{17}$$

Here, terms of λ_i represent ecliptic mean motion, terms of h_i represent general precession in longitude (along the ecliptic), and κ represents the constant of aberration. Comparing Eq. (17) with Eq. (13), one notices that $L_0 = (\lambda_0 - \kappa)$ and $L_i = (\lambda_i + h_i)$; $i > 0$.

Table 2. Terms Contributing to the Parameterization of the Fictitious Mean Sun

Degree	*Coefficient.*	*Components*	*Description*
t^0	α_0	λ_0	origin of mean longitude of true Sun
		$-\kappa$	annual aberration
t^1	α_1	λ_1	mean motion of true Sun in the ecliptic
		$+h_1$	precession rate of equinox in longitude
		μ	uniform sidereal motion along the celestial equator
		$+m_1$	precession rate of equinox in right ascension
t^2	α_2	m_2	higher-order precession of equinox in right ascension

To create a uniform scale from solar time, Newcomb defined his fictitious mean sun to have strict uniform motion μ along the celestial equator in the reference frame defined by precessional constants of the early 1890's, with a right ascension α as nearly as possible to the Sun’s mean longitude L. Thus:[35]

$$L \approx \alpha = \alpha_0 + \mu t + (m_1 t + m_2 t^2 + \ldots) \, . \tag{18}$$

Here, terms of m_i express the motion of general precession of the equinox in right ascension (along the equator) as the reference point of right ascension. Equating the coefficients of like terms between Eq. (13) and Eq. (18), one notices that $\alpha_0 = L_0 = (\lambda_0 - \kappa)$, and $L_1 = (\lambda_1 + h_1) = (\mu + m_1)$. Therefore, the right ascension of the mean sun is expressible as:

$$\alpha = L_0 + L_1 t + (m_2 t^2 + \ldots) = (\lambda_0 - \kappa) + (\lambda_1 + h_1)t + (m_2 t^2 + \ldots) \ . \tag{19}$$

The various terms contributing to the parameterization of the fictitious mean sun are summarized in Table 2. When modeling the right ascension of the mean sun as a two-body orbit in STK, the origin α_0 specifies the mean anomaly at $t = 0$, and the rate coefficient α_1 defines the mean motion or period. The higher-order terms of precession m_i cannot be specified using a two-body orbit model, which are negligible only near the time origin, when t is relatively small.

SIMULATION USING NEWCOMB'S EXPRESSION

Restating Newcomb's expression, Eq. (5), in units of arcseconds and Julian millennia yields:

$$R_\odot = 1006887.54'' + 1296027681.3''T + 139.4''T^2 \text{ [epoch 1900]} \ . \tag{20}$$

Updating the epoch from 1900 to 2000 (by substituting $T + 0.1$ for T) yields:

$$R_\odot = 1009657.0635'' + 1296027709.18''T + 139.4''T^2 \text{ [epoch 2000]} \ . \tag{21}$$

Applying the FK4 equinox adjustment of Eq. (10) further updates Newcomb's right ascension to be with respect to the dynamical equinox at J2000:

$$R_\odot = 1009658.2260'' + 1296027721.93''T + 139.4''T^2 \text{ [epoch 2000]} \ . \tag{22}$$

STK models the motion of the mean equinox according to the IAU 1976 Precession theory. A mean sun, whose equinox is affected by general precession in right ascension of IAU 1976 Precession, would have its right ascension expressed as:[36, 37, 38]

$$\alpha_{\text{IAU1976}} = \alpha_0 + \mu T + (46124.362''T + 139.656''T^2 - 0.0927''T^3) \text{ [epoch 2000]} \ , \tag{23}$$

where T is in units of Julian millennia TDB from epoch J2000.0. Subtracting Eq. (22) from Eq. (23) suggests Newcomb's $\alpha_0 = 1009658.2260''$ and $\mu = 1295981597.568''$. However, because the rate term of the mean sun (L_1) is already explicitly determined by $(\lambda_1 + h_1)$, the exact values of μ and m_1 contributing to Eq. (5) are not critical, so long as their combination provides the desired rate (in this case, Newcomb's expression updated to the dynamical equinox at J2000).

Table 3. Orbital Elements for Artificial Satellite Model of Newcomb's Fictitious Mean Sun

Frame:	Mean Equator & Equinox of Date	Period:	31556924.98544332^s	Arg. of Perigee:	0°	Eccentricity:	0°
Epoch:	1 Jan 2000 12^h (TDB)	Mean Anomaly:	1009658.2260"	R.A. Asc. Node:	0°	Inclination:	0°

Results of the Simulated Transits

STK simulated and reported the UTC times of 14,975 meridional crossings from January 1, 1972 to December 31, 2012 using the origin and rate of Eq. (22) as the updated expression for Newcomb's mean sun, where the rate term corresponds to a period of 31556924.98544332^s (Table 3). At each transit time, STK interpolated the daily (UT1−UTC) time series available from the IERS C04 Earth-orientation parameter solution and reported the interpolated result. STK does not natively output time on the UT1 time scale, so the interpolated value of (UT1−UTC) was added to UTC per Eq. (8) to determine the UT1 time of meridional transit. It is most convenient to consider the resultant times of transit relative to noon UT1, for if UT1 is represented by the modeled fictitious mean sun, then the mean sun would transit at exactly noon UT1. If the transit time of the mean-sun model occurs after noon UT1, then its right ascension is increased (east) of UT1. Conversely, if the transit time of our mean-sun model occurs before noon UT1, then its right ascension is decreased (west) of UT1.

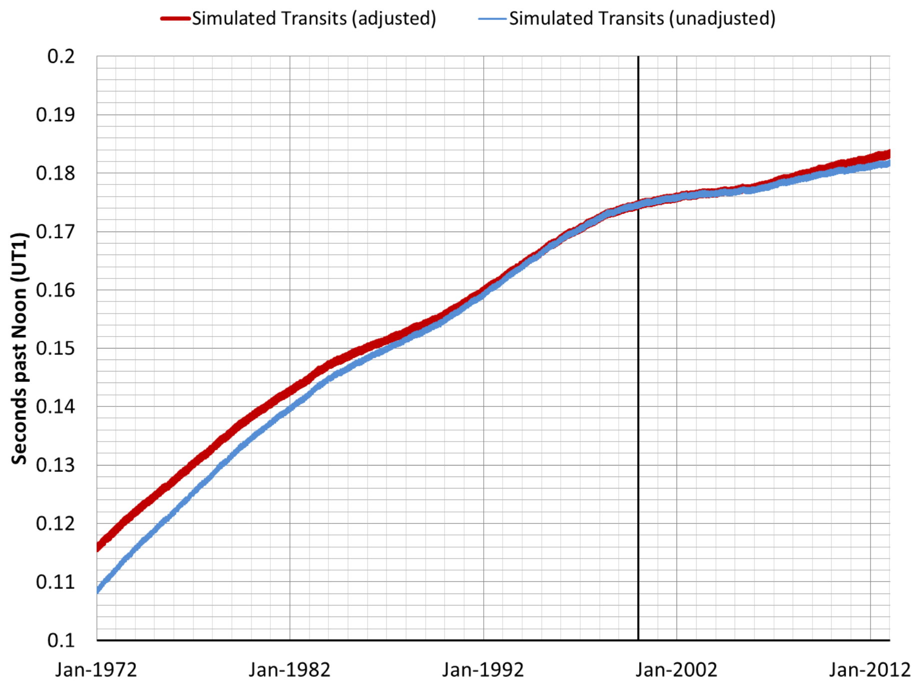

Figure 5. Times of simulated transits since noon UT1 for Newcomb's mean sun updated to epoch J2000, and also adjusted for non-linear motion of the equinox.

Two results of the simulation are illustrated with Figure 5. One curve indicates the "raw" unadjusted transit timings from the simulation, which are affected by the slight non-uniform precessional motion of the mean equinox of date previously described. Because there is no mechanism to explicitly account for non-uniform precessional motion when modeling two-body motion with respect to the mean equinox of date, a posterior adjustment of $139.656''T^2 - 0.0927''T^3$, per Eq. (23), was applied to the transit timings. Both curves in Figure 5 show that an object, moving according to Newcomb's expression for right ascension in the equator, transits the prime meridian at $12^h00^m00.156^s$ UT1 on average over the previous four decades, with some noticeable trending and decadal variation relative to UT1. Or, said another way, the right ascension of Newcomb's mean sun is 0.156^s ahead of a right-ascension point implied by 12^h UT1. It is also apparent that the non-linear precessional motion is negligibly small near the epoch of J2000.

The varying difference between simulated transits and UT1 indicate that the rate of separation between UT1 and simulated mean-time is not constant. Eq. (7) predicts that the difference in right ascension implied by UT1 versus Newcomb's mean sun is proportional to ΔT, which explains the discordance. Plotting Eq. (7) against the adjusted simulation affirms an almost perfect correlation (Figure 6).

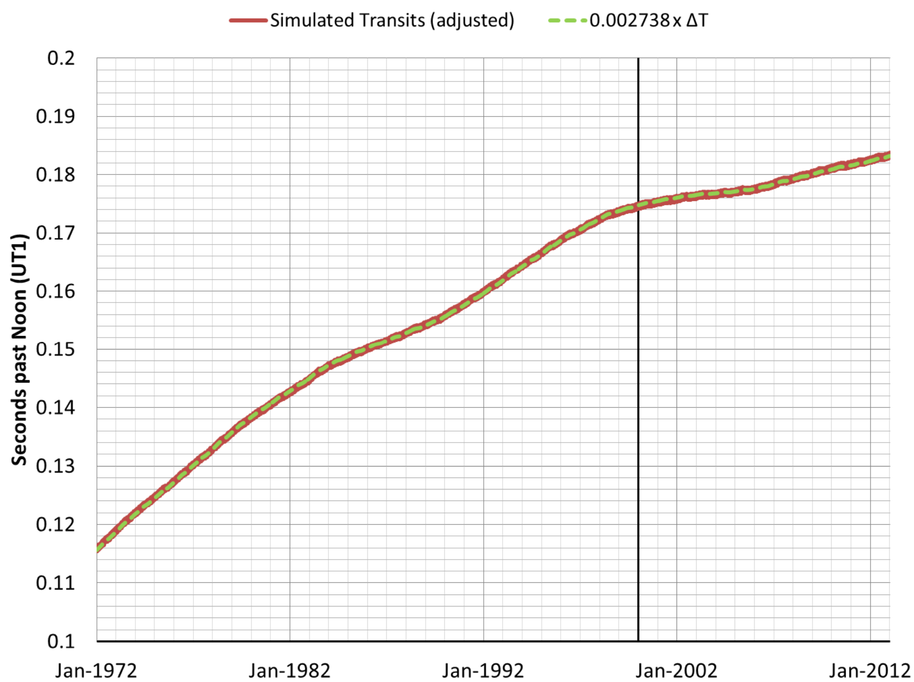

Figure 6. Adjusted simulated transits times for Newcomb's mean sun compared to the theoretical expression for the difference in R.A. of UT1 and ET.

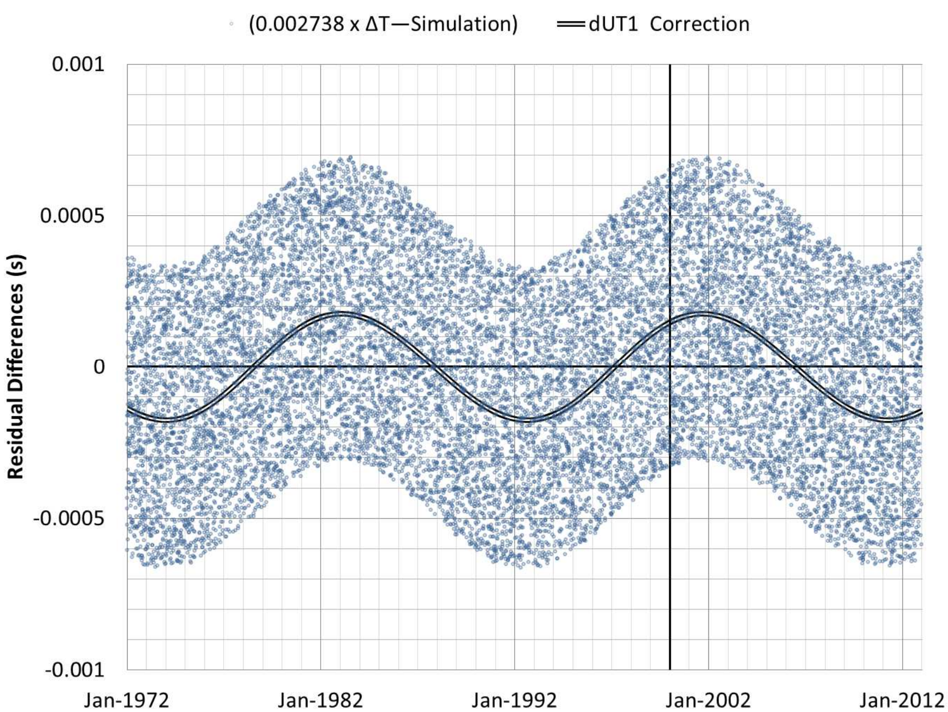

Figure 7. Timing residuals of theoretical expression for R.A. differences minus Newcomb's simulated mean sun, along with the post-1996 adjustment to the equation of the equinox.

Subtracting Eq. (7) from the adjusted simulated transit times reveals a familiar pattern affecting the residuals (Figure 7), illustrated per Figure 2 of Capitaine & Gontier (1993).[39] Such a pattern seems consistent with the latest UT1 time series having been retroactively corrected by Eq. (11) prior to 1997. Removing this periodic signature[*] results in residual noise, uniformly distributed between ±0.5 ms. This noise is an artifact of numerical round-off in the simulation process as implemented, which did not seek to preserve precision below 0.5 ms (as Newcomb's original expression was only precise to 1 ms). The sample mean of the residual noise was +13.5 μs over the time period analyzed—a value which could have been made arbitrarily close to zero by adding significant digits to the proportionality constant 0.002738 of Eq. (7). Thus, to better than a millisecond, the simulation of Newcomb's mean-solar-time is represented by:

$$\mathrm{GMT}_{(\mathrm{Newcomb})} \approx \mathrm{UT1} + 0.002738\ \Delta T - \mathrm{dUT1}(\Omega) \tag{24}$$

A "CONTEMPORARY" MEAN-SOLAR LONGITUDE

Any measure of time based on an average of apparent solar time is essentially dependent upon the particular theory adopted for the Sun.[40] Newcomb's convention for the fictitious mean sun was based on a pre-relativistic solar-system theory fitted to telescopic observations of the 18th and 19th centuries. In principle, a more modern convention for mean-solar origin and rate should match the true Sun more accurately.

As late as 1974, in a proposed update of mean planetary elements to take advantage of the most recent observational analyses, Seidelmann *et al*. continued to prefer Newcomb's expression for the mean longitude of the Earth-Moon barycenter without alteration.[41] Since then, the most contemporary expressions for mean elements of the Earth, of which the authors are aware, are those of Simon *et al*. (1994).[42] These elements were developed from an analytical theory adjusted to the Jet Propulsion Laboratory (JPL) Development Ephemerides. Simon *et al*. expressed the mean longitude of the Earth as:

$$\begin{aligned} L_{\oplus} &= 361679.244588'' + 1296027711.03429''T + 109.15809''T^2 \\ &+ 0.07207''T^3 - 0.23530''T^4 - 0.00180''T^5 + 0.00020''T^6 \text{ [epoch 2000]}, \end{aligned} \tag{25}$$

where T is in units of Julian millennia TDB.

Table 4. Orbital Elements for Artificial Satellite Model of Simon *et al*. Fictitious Mean Sun w.r.t. a "Rotating" Equinox

Frame:	Mean Equator & Equinox of Date	Period:	31556925.1888328^{s}	Arg. of Perigee:	0	Eccentricity:	0
Epoch:	1 Jan 2000 12^{h} (TDB)	Mean Anomaly:	1009658.6554"	R.A. Asc. Node:	0	Inclination:	0

To define the origin of right ascension for a fictitious mean sun, one must add 180 degrees (648000") to the mean longitude of the Earth $L_{\oplus}$ and then adjust for annual aberration to reflect a geocentric frame of reference. A modern value for κ at epoch J2000 is 20.49551" or 1.36637^{s}.[43]

[*] STK is able to remove this sinusoidal pattern directly by setting the parameter `UseUpdatedEquationOfEquinox` to `No` in the file …\STKData\CentralBodies\Earth\Earth.cb, a file which defines certain geophysical constants and astrodynamical calculation options used for the Earth.

Additionally, the dynamical ecliptic and equinox of Simon *et al*. (1994) is "inertial" in the sense defined by Standish (1981);[44] an additional adjustment should be applied to compare with Newcomb's "rotating" equinox convention, which Standish estimated to be at epoch J2000:

$$\alpha_{\text{rotating}} = \alpha_{\text{inertial}} - 0.09366''. \tag{26}$$

Thus, 361679.244588" + 648000" − 20.49551" − 0.09366" updates α_0 to 1009658.65542".

Simon *et al*. also used a precession theory that improved upon that of IAU 1976. Their accumulated general precession in right ascension can be deduced by summing their expressions for the traditional precession angles ζ_A and z_A:[*]

$$\begin{aligned}\alpha_{\text{Simon}} = \alpha_0 + \mu T + (&46121.8194''T + 139.7496''T^2 + 36.2850''T^3 \\ &- 0.3404''T^4 - 0.0586''T^5 - 0.0003''T^6)\ \text{[epoch 2000]},\end{aligned} \tag{27}$$

where T is in units of Julian millennia TDB. Understanding that the rate coefficient of Eq. (25), L_1, equals $(\mu + m_1)$ of Eq. (27), the Simon *et al*. coefficient for μ equals 1295981589.2149". To be compatible with the IAU 1976 precession theory used for this analysis, the expression for right ascension of the mean sun should account for the precession difference to first order. This is done by updating Eq. (23) to include the Simon *et al*. values of α_0 and μ. Thus, the expression based on Simon *et al*., compatible with IAU 1976 precession, becomes:

$$\alpha_{\text{Simon}} = 1009658.65542'' + 1296027713.5769''T + \ldots\ \text{[epoch 2000]}. \tag{28}$$

This expression has a corresponding annual period of 31556925.1888328^{s} (365.24218969^{d}) at epoch J2000. The difference between the expression of Simon *et al*., and Newcomb's expression at J2000, is then:

$$\alpha_{\text{Simon}} = \alpha_{\text{Newcomb}} + 0.42942'' - 8.35311''T + \ldots\ \text{[epoch 2000]}. \tag{29}$$

Thus, to a fair degree of approximation, an expression for mean solar time based the conventions of Simon *et al*. may be represented as:

$$\text{GMT}_{(\text{Simon } et.\ al)} \approx \text{UT1} + 0.002738\ \Delta T + (\Delta\alpha_0 + \Delta\mu\ T)\ \text{[epoch 2000]} \tag{30}$$

where $\Delta\alpha_0 \approx 0.0286^{s}$ and $\Delta\mu \approx -0.55687^{s}$.

The general result is illustrated in Figure 8, with Newcomb's expression and UTC included for scale. Both expressions for the mean sun differ from Universal Time by an amount which is much less than the variation between UTC and UT1. The discontinuities in the graph of UTC versus UT1 result from the introduction of *leap seconds* which maintain UTC's proximity to UT1.[45] Of the three realizations illustrated, Newcomb's mean sun is closest to UT1 currently, but the mean sun of Simon *et al*. is expected to be closest after the year 2051.

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[*] $\zeta_A = 23060.9097''T + 30.2226''T^2 + 18.0183''T^3 - 0.0583''T^4 - 0.0285''T^5 - 0.0002''T^6$
$z_A = 23060.9097''T + 109.5270''T^2 + 18.2667''T^3 - 0.2821''T^4 - 0.0301''T^5 - 0.0001''T^6$

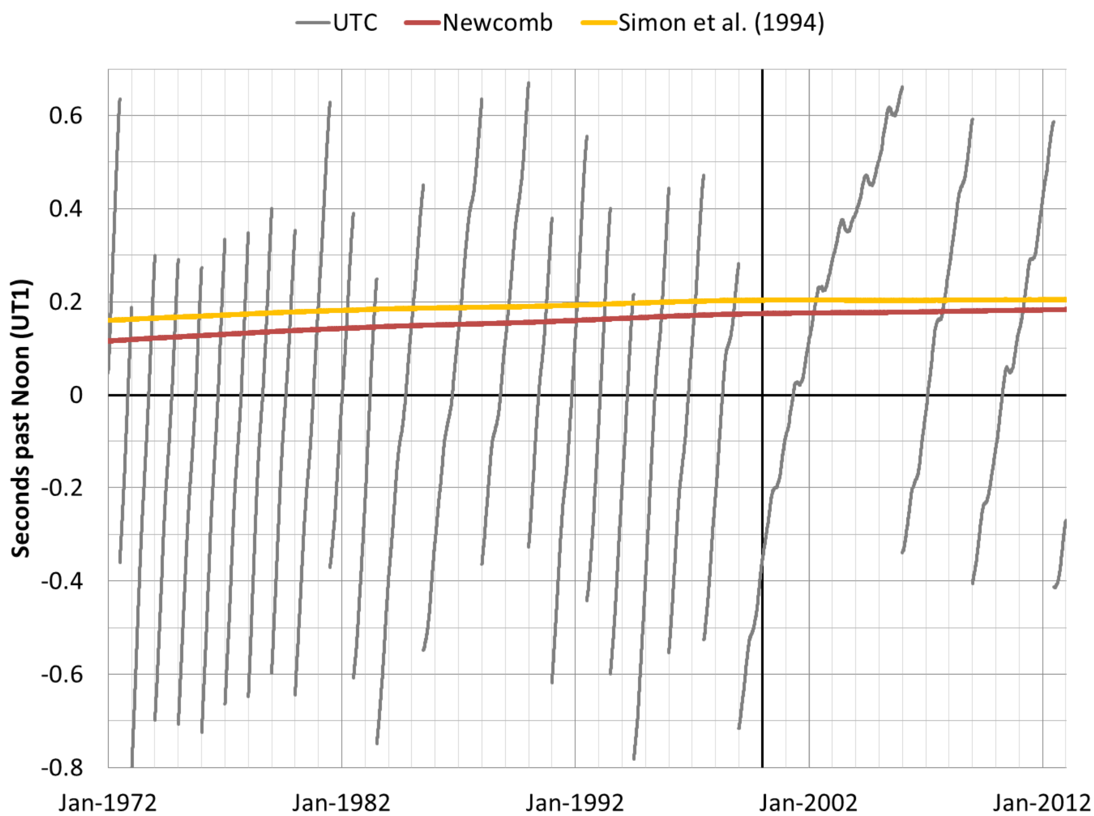

Figure 8. Times of simulated transit after noon UT1 for different mean-solar-time conventions.

CONCLUDING SUMMARY

The authors have expressed the right ascension of Newcomb's fictitious mean sun with respect to the dynamical equinox of epoch J2000, and compared its behavior with UT1. To make the comparison, commercial software was used to simulate the supposed motion of the mean sun as an artificial satellite, moving uniformly in the plane of the mean equator, with a period of one tropical year. The transit times at the prime meridian were then reported relative to noon UT1 over four decades. Restricting the motion to the equatorial plane required the satellite motion to be expressed with respect to the mean equinox of date. This origin has some non-uniform precessional motion in right ascension which could not be explicitly modeled by the right ascension of a uniformly moving two-body satellite model; therefore, a small (mostly quadratic) adjustment from the IAU 1976 precession theory was applied to the simulated transit results retroactively.

The simulation results affirmed that the difference between Newcomb's mean solar time at the prime meridian and UT1 is approximated by:

$$\mathrm{GMT}_{(\mathrm{Newcomb})} - \mathrm{UT1} \approx 0.002738\ \Delta T \tag{31}$$

where ΔT = Terrestrial Time minus UT1. The expectation that the right ascension of Newcomb's mean sun diverges from the Universal Time by approximately $(^1/_{365}) \times \Delta T$ was already expressed numerically in the mid-20$^{\text{th}}$ century, when ephemeris time was recognized as its own separate scale. An additional periodic correction, Eq. (11), may be further subtracted to eliminate a spurious sub-millisecond signature; this correction was introduced after 1996 when the traditional relationship between Greenwich sidereal time and UT1 was refined. The present separation between Newcomb's mean solar time and Universal Time is about 0.18^s.

Generally, expressions for Greenwich mean time that are different from Newcomb's may be approximated as:

$$\mathrm{GMT} - \mathrm{UT1} \approx 0.002738\ \Delta T + (\Delta\alpha_0 + \Delta\mu\ T)\ [\text{epoch 2000}]. \tag{32}$$

where $\Delta\alpha_0$ and $\Delta\mu$ represent offsets from the origin and rate of the right ascension of Newcomb's mean sun, and T being in Julian millennia TDB. To compare Newcomb's expression and the UT1 series with a more modern solar-system theory, the mean solar longitude of Simon *et al*. (1994) was analyzed.[40] This expression differs from Newcomb's by ($0.0286^s - 0.55687^s T$) after accounting for some differences between their conventional reference-frames. The difference of 0.0286^s in origin is small relative to the currently estimated divergence of 0.18^s from Universal Time, and the rate difference of 0.55687^s is practically negligible relative to the variable duration of the tropical year (which decreases by one-half second per century). Either expression for the mean sun differs from Universal Time by an amount which is far less than the allowable difference between UTC and UT1.

Accordingly, UT1 still appears synonymous with "mean solar time at the prime meridian" to within a fraction of a second, despite the definition and maintenance of UT1 having evolved significantly over the past century. The "prime meridian" in this case implies the International Reference Meridian that includes the terrestrial origin from which Universal Time is measured, and which has advanced approximately 0.36^s to the east of the internationally recommended origin of longitude—the Airy transit instrument at Greenwich. This translation resulted from the changing conventions and methods by which the relationship between Universal Time and the terrestrial reference were maintained over time by the *Bureau International de l'Heure*. Evolution of the terrestrial reference frame is a subject perhaps unto itself.

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REFERENCES

[1] Urban, S.E., P.K. Seidelmann (eds., 2012), *Explanatory Supplement to the Astronomical Almanac*, 3rd ed.. University Science Books, Mill Valley, CA, p. 14.

[2] Newcomb, S. (1906), *A Compendium of Spherical Astronomy*. Macmillan Company, p. 114.

[3] Newcomb, S. & E.S. Holden (1880), *Astronomy for students and general readers*, Revised 2nd ed., p. 260.

[4] Woodhouse (1812), *An Elementary Treatise on Astronomy,* Cambridge University, pp. 222-24.

[5] Neugebauer, O. (1975), *A History of Ancient Mathematical Astronomy, Vol. 1*. Springer, Berlin, pp. 61-66.

[6] US Government (1884), International Conference held at Washington for the purpose of fixing a prime meridian and a universal day, October 1884: Protocol of the Proceedings. Gibson Bros., Washington, D.C., p. 199.

[7] Newcomb, S. (1883), "On the Present State of the Theories of the Celestial Motions." *Sidereal Messenger*, Vol. 2, No. 2, p.39.

[8] *Explanatory Supplement to The Astronomical Ephemeris and The American Ephemeris and Nautical Almanac* (1961). Her Majesty's Stationery Office, London, pp. 174, 185.

[9] Newcomb, S. (1898), "Tables of the Motion of the Earth on its Axis and around the Sun.", from *Tables of the Four Inner Planets. Astronomical papers prepared for the use of the American ephemeris and Nautical Almanac*, 2nd. ed., Vol. VI, Part I, p. 9

[10] *Explanatory Supplement to The Astronomical Ephemeris and The American Ephemeris and Nautical Almanac* (1961). Her Majesty's Stationery Office, London, p. 74.

[11] Seidelmann, P.K. (ed.), *Explanatory Supplement to the Astronomical Almanac*, University Science Books, Mill Valley, CA, 1992, p. 78.

[12] *Explanatory Supplement to The Astronomical Ephemeris and The American Ephemeris and Nautical Almanac* (1961). Her Majesty's Stationery Office, London, p. 78.

[13] McCarthy D.D., P.K. Seidelmann (2009), *Time—from Earth Rotation to Atomic Physics*. Wiley –VCH, pp. 113-27.

[14] Wayman, P.A. (ed., 1980), *Proceedings of the Seventeenth General Assembly, Montreal 1979*, Transactions of the International Astronomical Union, Volume XVIIB. D. Reidel Publishing, Dordrecht, Holland. p. 66.

[15] Aoki, S., B. Guinot, G.H. Kaplan, H. Kinoshita, D.D. McCarthy, P.K. Seidelmann (1982), "The New Definition of Universal Time." *Astronomy and Astrophysics*, Vol. 105, No. 2, pp. 359-361.

[16] Kaplan, G.H. (ed., 1981), "The IAU Resolutions on Astronomical Constants, Time Scales, and the Fundamental Reference Frame." *USNO Circular No. 163*, U.S. Naval Observatory, Washington DC, December 10, 1981, p. B5.

[17] Aoki, S. & H. Kinoshita (1983), "Note on the relation between the equinox and Guinot's non-rotating origin." Celestial Mechanics, Vol. 29, p. 356.

[18] McCarthy, D.D. (ed., 1996), *IERS Conventions (1996)*. (IERS Technical Note 21), Paris: Central Bureau of IERS - Observatoire de Paris, p. 21.

[19] Capitaine, N., B. Guinot, D.D. McCarthy (2000), "Definition of the Celestial Ephemeris Origin and of UT1 in the International Celestial Reference Frame." *Astronomy and Astrophysics*, Vol. 355, p. 403.

[20] Capitaine, N., B. Guinot, J. Souchay (1986), "A Non-Rotating Origin on the Instantaneous Equator: Definition, Properties, and Use." *Celestial Mechanics*, Vol. 39, pp. 283-307

[21] Urban, S.E., P.K. Seidelmann (eds., 2012), *Explanatory Supplement to the Astronomical Almanac*, 3rd ed.. University Science Books, Mill Valley, CA, p. 81.

[22] Guinot, B. (2011) "Solar time, legal time, time in use." *Metrologia*, Vol. 48, pp. S181-S185.

[23] Feissel, M. (1980), "Determination of the Earth Rotation Parameters by the Bureau International de l'Heure, 1962-1979." *Bulletin Géodésique*, Vol. 20, p. 94.

[24] Gebel, G. & B. Matthews (1971), "Navigation at the Prime Meridian." *Navigation*: *Journal of the Institute of Navigation*, Vol. 18, No. 2, pp. 141-46.

[25] Dillon, S.C., Gebel, G. & L.L. Pryor (1977), "Navigation at the Prime Meridian Revisited." *Navigation: Journal of the Institute of Navigation*, Vol. 24, No. 3, pp. 264-66.

[26] Standish, E.M. (1981), "Two Differing Definitions of the Dynamical Equinox and the Mean Obliquity." *Astronomy and Astrophysics*, Vol. 101, pp. L17-L18.

[27] LeVerrier, U.J.J. (1858), "Théorie et Tables du Mouvement Apparent du Soleil." *Annales de l'Observatoire Impérial de Paris*, Vol 4.

[28] Woodhouse (1812), *An Elementary Treatise on Astronomy,* Cambridge University, p. 66.

[29] Gummere (1842), *An Elementary Treatise on Astronomy in Two Parts, 3rd ed*. Kimber & Sharpless, Philadelphia, p. 50.

[30] Steel, D. (2000), *Marking Time – The Epic Quest to Invent the Perfect Calendar*. John Wiley & Sons, New York, pp. 381-84.

[31] Meeus, J., D. Savoie (1992), "The history of the tropical year." *Journal of the British Astronomical Association*, Vol. 102, No. 1, pp. 40-42.

[32] Smith, C.A., G.H. Kaplan, J.A. Hughes, P.K. Seidelmann, B.D. Yallop, C.Y. Hohenkerk (1989), "Mean and Apparent Place Computations in the New IAU System. I. The Transformation of Astrometric Catalog Systems to the Equinox J2000.0." *The Astronomical Journal*, Vol. 97, No. 1, January 1989, p. 266.

[33] Borkowski, K.M. (1991), "The Tropical Year and the Solar Calendar." *The Journal of the Royal Astronomical Society of Canada*, Vol. 85, No. 3 (Whole No. 630), p. 127.

[34] Woolard, E.W., G.M. Clemence (1966), *Spherical Astronomy*. Academic Press, New York. p. 346-7.

[35] Newcomb, S. (1895), "The Elements of the Four Inner Planets and the Fundamental Constants of Astronomy." *Supplement to The American Ephemeris for 1897*, p. 188.

[36] Lieske, J.H., T. Lederle, W. Fricke, B. Morando (1977), "Expressions for the precession quantities based upon the IAU (1976) system of astronomical constants." *Astronomy and Astrophysics*, Vol. 58, No. 1-2, June 1977, p. 15.

[37] Kaplan, G.H. (ed., 1981), "The IAU Resolutions on Astronomical Constants, Time Scales, and the Fundamental Reference Frame." *USNO Circular No. 163*, U.S. Naval Observatory, Washington DC, December 10, 1981.

[38] Seidelmann, P.K. (ed.), *Explanatory Supplement to the Astronomical Almanac*, University Science Books, Mill Valley, CA, 1992, p. 104.

[39] Capitaine, N. & Gontier, A.-M. (1993), "Accurate procedure for deriving UTI at a submilliarcsecond accuracy from Greenwich Sidereal Time or from the stellar angle." *Astronomy and Astrophysics*, Vol. 275, p. 648-9.

[40] *Explanatory Supplement to The Astronomical Ephemeris and The American Ephemeris and Nautical Almanac* (1961). Her Majesty's Stationery Office, London, p. 92.

[41] Seidelmann, P.K., L.E. Doggett, M.R. DeLuccia (1974), "Mean elements of the principal planets", *The Astronomical Journal*, Vol. 79, No. 1 (January), p. 58.

[42] Simon, J. L.; P. Bretagnon, J. Chapront, M. Chapront-Touze, G. Francou, J. Laskar, (1994), "Numerical expressions for precession formulae and mean elements for the Moon and the planets." *Astronomy and Astrophysics*, Vol. 282, No. 2, pp. 663-683.

[43] *Astronomical Almanac for the Year 2009*, U.S. Gov't Printing Office, Washington. p. K7.

[44] Standish, E.M. (1981), "Two Differing Definitions of the Dynamical Equinox and the Mean Obliquity." *Astronomy and Astrophysics*, Vol. 101, p. L17-L18.

[45] Seidelmann, P.K., J.H. Seago (2011), "Time Scales, Their Users, and Leap Seconds." *Metrologia*, Vol. 48, pp. S186–S194.