SUN-MARS LIBRATION POINTS AND
MARS MISSION SIMULATIONS

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The equilibrium points of the Sun-Mars system bring some unique characteristics to the discussion of future inner solar system exploration missions, particularly an expedition to Mars itself. Existing research has identified potential utility and data for Sun-Mars libration point missions, particularly for satellites orbiting the L_1 and L_2 points serving as Earth-Mars communication relays. Regarding these Lissajous orbits, we address questions of "Why go there?" "How to get there?" and "How to stay there?" Namely, we address utility and usefulness, transfer and injection, and stationkeeping. The restricted 3-body problem involving a spacecraft in that system is reviewed; and past and present research and proposals involving the use of these orbits are summarized and discussed. Baseline historical stationkeeping concepts (ISEE-3, SOHO, ACE) are reviewed and applied to the Sun-Mars system. We use Satellite Tool Kit (STK)/*Astrogator* for simulation and analysis of Earth-Mars transfers, Lissajous orbit insertions, and station-keeping. The resulting data provides confirmation and insight for existing research and proposals, as well as new information on Mars transfer and Lissajous orbit insertion strategies to save ΔV, mission orbit amplitude dependencies on insertion method, and stationkeeping sensitivities. These data should prove useful to mission planners and concept developers for future Mars investigations.

INTRODUCTION

"NASA's vision is to…focus more of our energy on going to Mars and beyond." - Dan Goldin, AWST, Jan 01

"All the questions we have about Mars could now be answered…if we could just walk around on the planet for a few days." - Michael Malin, Malin Space Science Systems, National Geographic, Feb 01

As NASA and the space community renew their focus on Mars exploration, student researchers find several topics awaiting further study. From our work that originated in an advanced astrodynamics course at the US Naval Postgraduate School, we became interested in Mars, various aspects of the three-body problem, and the Lagrange or libration points, and we were eager to team with industry to conduct mission simulations and analysis. We examined several documented research efforts dealing with diverse aspects of these topics.[6,9,11,14,19,20] A concept that caught our interest was that introduced by Dr. Pernicka, et al, for a 2-satellite communications relay with one spacecraft in orbit about each of the co-linear, near Mars, Sun-Mars libration points, L1 and L2.[6] Further in-depth work by graduate researchers (Kok-Fai Tai and Danehy)

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refined this proposal and conducted extensive investigations into the technical and fiscal aspects of such a mission, including trade studies on communication relay constellation options.[15,16] This analysis resulted in some conclusions and rationale for a Mars communication relay system that utilizes 2-spacecraft in large amplitude Lissajous orbits, including system cost and performance measures comparable to a 3-spacecraft aerosynchronous system. A primary purpose of our study, then, was to re-examine the 2-vehicle system orbiting the libration points, including transfer orbits and stationkeeping, through desktop computer simulation using full-force models and the interplanetary propagation/targeting techniques of the STK/*Astrogator* module. Essentially, we wished to see how past studies and data compared to our full-force model targeting and propagation, and to generate new scenarios and data for future missions.

An additional purpose of the project was to demonstrate successful collaboration between military graduate researchers and industry professionals. Timely, affordable results from specific research can be obtained when diverse groups such as these can work, virtually and collaboratively, on pieces of a problem. These ideas flow into another purpose of the study: to show how commercial desktop computing can be used to easily create and analyze these types of missions and problems, again leading to faster and cheaper studies by more researchers.

As output for this study, we expand upon discussions of the usefulness of these orbits for Mars missions as well as re-examine the 2003 Earth-Mars transfers and L1 libration orbit insertions presented in the original Pernicka study and the follow-on work. We then expand that simulation and analysis to include the planning horizon of a 2016 transfer and mission orbit insertion, considering an L2 orbit insertion as well. We investigate the effects of orbit amplitude on insertion ΔV requirements and show some innovative mission orbit insertion techniques that may result in ΔV savings, namely using a Mars swingby and braking maneuver to assist in the insertion. All of the simulation and analysis presented here represents a first effort of utilizing full force models and current desktop tools to generate some data on the Sun-Mars libration point transfer and communication relay problem. The data does not represent optimized numerical solutions or proposed mission designs, but rather information and baseline data for follow-on researchers and mission planners.

MARS COMMUNICATION CONSTELLATIONS

Currently, spacecraft missions to Mars rely on their on-board equipment to provide faint transmissions directly to Earth or to a relay in Martian orbit. The addition of on-board communication equipment capable of reaching out through the interplanetary void between Mars and the Earth adds weight, cost, and risk to missions that operate within tight margins in these areas. To exacerbate this problem, once a lander has made it safely to the surface, it can only relay information to Earth when it is in direct line of sight. With a Martian day of just over 24 hours, there exist well over 12 hours of "blackout" where no signal can be sent to the Earth. With the addition of an orbiting

relay, these times are reduced substantially but significant blackouts will still exist. In order to provide continuous coverage for the entire Martian surface, a minimum of four satellites (in elliptical orbits) are required[6]. This, once again, raises the issues of cost and risk.

Some of these problems can be solved by the use of a communication network around Mars that takes advantage of the geometry provided by placement at the Sun-Mars Lagrange points. A minimum of two satellites located at the Sun-Mars L1 and L2 points could provide near continuous coverage for multiple vehicles on the surface and in orbit[6]. Lagrange points are equilibrium points in a three-body orbital system, consisting of two primaries (Sun and Mars) and the much smaller satellite body, whose mass is sufficiently small that the system can essentially be described with two-body equations.[16] The Lagrange points remain at the same location as the two primary bodies rotate about their center of mass (see Figure 1). The communication satellites would be inserted into large amplitude orbits about the L1 and L2 points, circling their respective Lagrange points and the Sun-Mars line. These satellites could communicate with landers anywhere on the Martian surface, any spacecraft in Martian orbit, and provide the critical communications link between the Earth and Mars.[6]
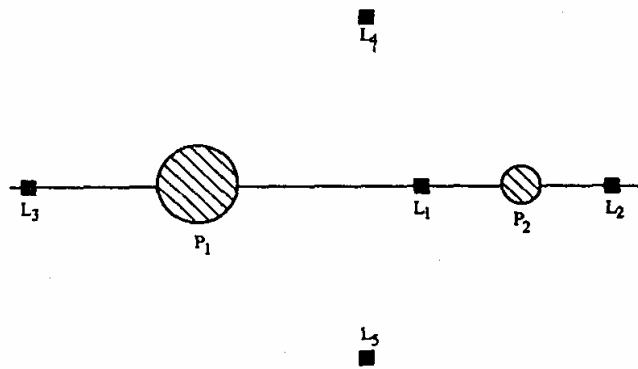

Figure 1   Geometry of the Lagrange Points of Two Primary Masses P$_1$ and P$_2$[7]

Other constellations could be used for a Mars communications network, but each has disadvantages that outweigh the advantages.[16] A group (four to six) of low to medium orbiting relay satellites would ensure that every satellite would cover the entire planet at some point, but the cost and risk of inserting so many satellites and the limited instantaneous field of view the satellites can offer do not make it an attractive option. Four satellites in common-period, inclined orbits, or a Draim constellation, could cover the entire surface of Mars, but again require twice as many satellites as the Lissajous orbit concept, as well as the added complexity of a ground station continuously switching from one satellite to another. An aerosynchronous constellation (Earth geosynchronous transferred to Mars, approximately 20,462 km altitude) requires three or four satellites, and works well with ground stations that can simply point to one spot in the sky.

However, in addition to the fact more than two satellites are required, there is virtually no polar coverage. Another proposal places communication landers on the Martian moons of Phobos and/or Deimos, but this constellation has the same inefficiencies as the aerosynchronous satellites with large gaps of polar coverage.

The L1 and L2 orbit constellation requires only two satellites for a fully operational constellation (each spacecraft sees almost half of Mars at all times), making it the most attractive option. The Sun is always visible to both satellites, greatly simplifying power requirements for that spacecraft. Lander pointing requirements are simple, given the spacecraft is always the same relative distance from the Sun-Mars line, and the spacecraft stationkeeping budget is relatively small. Disadvantages are overcoming the approximate one million kilometer distance from the Lagrange points to the Martian surface. This distance requires a large, high frequency antenna, which could complicate solar panel design to minimize antenna shadow and may require a more complex lander communication system to interact with the high frequency signals. Interference from constant solar radiation along the Sun-Mars line and for certain Earth viewing geometries may also have to be considered, and the loss of one satellite means half the planet loses communications coverage for approximately 12 hours.
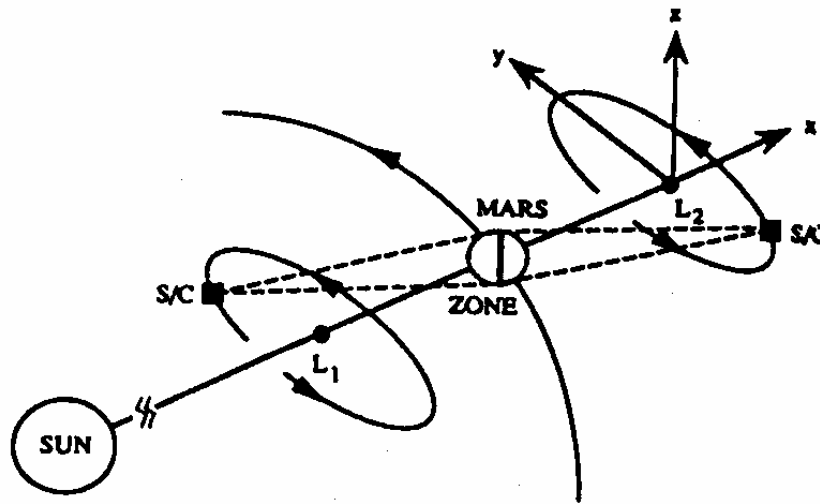
SUN-MARS LIBRATION POINT ORBITS

Figure 2   Sun-Mars L1 and L2 Halo Orbit Constellation[6]

The design of the halo orbit communication network around Mars is a simple but elegant one, as depicted in Figure 2.[6] In order to prevent an unneeded overlap of coverage, the orbits of each satellite at L1 and L2 would be opposed by 180 degrees but moving in the same orbital direction. Herein lies a minor problem with this configuration: because L1 and L2 are at finite distances from Mars ($1x10^6$ km), the actual view of Mars is slightly less than hemispherical. Despite this geometry, the network will

be able to view 99.81% of the planet at all times. The "down time" in this scenario would be minimal: a vehicle caught in this band would have to wait a mere 1.5 minutes before coverage would be switched over and reestablished with the other satellite.

In designing the proper orbits in which to place the two satellites, the most important consideration is that they permit efficient maintenance of the 180 degree offset.[6] An additional consideration is that of avoiding having the satellite cross what is known as the "solar exclusion zone", the line between Mars and the Sun. Passing through this zone, communications would be disrupted due to intense solar interference. To avoid this problem the orbit must be large enough to avoid this crossing; an orbit of period greater than 0.9 years should suffice. Another obvious consideration is the choice of geometry and size of the orbit that reduces the required insertion maneuvers, and thus cost, from Earth.

As an aside, one might wonder if the L4 and L5 points could play some role in the design of a communication network around Mars. The L4 and L5 Lagrange points lead and trail Mars by 60 degrees in its orbit, thus form equilateral triangles with Mars and the Sun (see Figure 1). The distance from Mars to either of these two Lagrange points is the same as the distance from Mars to the Sun (227.9×10^6 km). To communicate over these distances, current interplanetary missions use very large dishes, such as the Goldstone Deep Space Network (DSN) facility in California, in order to eliminate the need for large, powerful transceivers on the spacecraft itself. Links over this 230 million kilometer range would require space borne communications elements whose size, weight, and power would be on the order of a DSN ground station. The size and power of the needed equipment for these distances make the L4 and L5 unrealistic as locations for the network.[8]

There is one other interesting aspect of the L4 and L5 points worth mentioning here. While their stability can be exploited for use in missions that require minimal stationkeeping, this same stability also attracts a multitude of interplanetary bodies that populate this region of the solar system. These special bodies are known as "Trojans" because the first few such objects discovered happened to be named for several heroes from the Trojan War. By convention established by the International Astronomical Union, all similar objects must be named after Trojan War heroes, Greeks ahead of the planet and Trojans trailing the planet. Two of the larger Martian Trojans (in the 1-2 km range), 5261 Eureka and 1998 VF31, represent what could be thousands of other bodies that reside at the L4 and L5 points making these fairly dangerous places indeed. Consideration must be made as to whether the benefits of the inherent stability of the L4 and L5 points outweigh the risks of residing there.[8]

Mars L1 & L2 Constellation Advantages

Probably the most significant advantage to using the large amplitude Lissajous orbit constellation is minimum cost associated with only 2 spacecraft required, when

compared to other constellation options.[15,16] Additionally, spacecraft orbiting about L1 and L2 can readily see the Sun and Earth, potentially simplifying spacecraft solar cell placement and communication antenna design.

The vehicles circling the Sun-Mars L1 and L2 points will orbit the Sun-Mars line with periods on the order of 1 year. The long period dynamics of such orbits may make them attractive for interplanetary missions with significant communication time delays. Perhaps even more significantly, on a given day (or series of days) the relay vehicle will appear motionless and remain in a fixed position relative to the sun (or local midnight vector, for the L2 relay). Thus, a communication relay tracking system for explorers on the surface could be simplified and automated for tracking of this position in the sky. From the surface, there would be only one switch between relay vehicles each day, a distinct advantage over Draim or low orbiting concepts. The Sun-line geometry may also allow for simplified and robust safe-modes for the vehicles based on the sun vector. Since both spacecraft orbit about the Sun-Mars line in large amplitude Lissajous orbits and avoid eclipsing, the relays would always have access to the sun for their solar cells, thus allowing reduced battery sizes.

An additional benefit of the unique geometry offered by Lissajous orbit missions is a secondary mission for these communications relay vehicles as observation platforms. The L1 satellite is able to perform continuous solar activity monitoring via a secondary payload on the vehicle, and thus provide advance warning of activity to Mars surface missions. This could be done using low power, simple instruments for simple early warning of solar storms/flares, perhaps derived from legacy missions. Regular monitoring of the sun is thus possible, and can additionally be compared to solar data from sensors closer to the sun. This secondary mission for solar activity monitoring increases in importance when the Earth is on the opposite side of the solar system from Mars. Another observation mission for both spacecraft could include Martian weather sensing and relay to Mars expeditions and Earth. The L2 vehicle offers the opportunity of a secondary payload for asteroid and outer solar system observations.

One of the fundamental advantages of Lissajous orbit is, of course, the relatively small ΔV maneuvers required for stationkeeping, when compared to other mission orbits. Annual ΔVs for each vehicle could to be on the order of 2 m/s [14] and studies have produced data showing annual vehicle stationkeeping estimates of 50 m/s for low orbits, almost 200 m/s for aerosynchronous, and 30 m/s for the inclined common period missions.[15] We provide more discussion of stationkeeping in a later section.

MARS MISSION SIMULATIONS AND ANALYSIS

2003 Direct Insertion into L1 Large Amplitude Lissajous Orbit

The original study by Pernicka, et al, a system level analysis, used co-planer transfer trajectories and circular orbits to determine C3 energy and orbit insertion ΔV

requirements for a 2003 Sun-Mars Lissajous communications relay concept. That data was generated with simplified force models and direct transfer to an orbit about L1. Our study used STK/*Astrogator* and the targeting process described in detail in the subsequent section to reproduce a subset of this data for three different times of flight (TOF) for closer analysis. No Z amplitude specifications were considered for these scenarios. A screen capture of the trajectory from the STK output is shown in Figure 3, where the view is of the XY plane looking in the –Z direction, using a Sun-Mars rotating coordinate frame. We found some agreement between the resulting data, which is presented in the table that follows.
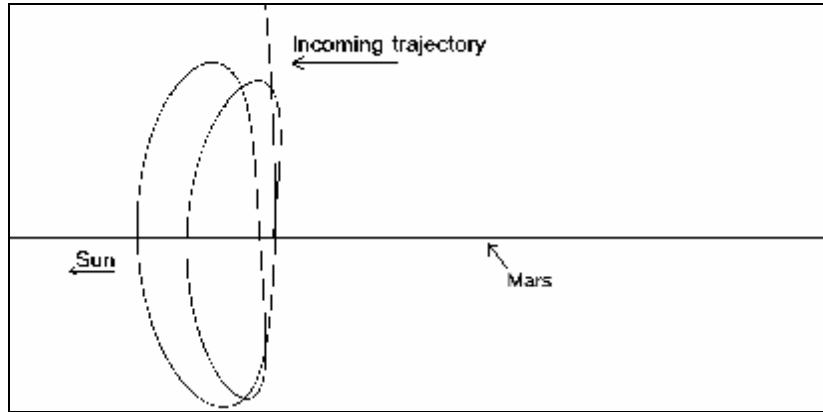

Figure 3   L1 Orbit Direct Insertion

**Table 1   Summary of Direct Transfer to L1 Lissajous; 13 Jun 03 Departure**

TOF (days)	C3 Energy (km^2/sec^2) *Original Study*	C3 Energy (km^2/sec^2) *Full Force Model*	Orbit Insertion Δv (km/sec) *Original Study*	Orbit Insertion Δv (km/sec) *Full Force Model*
170	8.561	9.553	1.604	2.807
200	7.910	8.883	1.196	2.425
240	7.986	14.218	1.747	3.400

The differences in C3 energy and ΔV are most likely due to the differences in the model parameters of the studies. This study used full force models, non-circular and non-coplaner transfers, and direct computation of the orbit insertion ΔV. Based on inspection of the Mars arrival trajectories, use of actual non-coplaner, eccentric planetary orbits seems to be a major factor. The data trends are still evident, however: the 200 day TOF case provides the minimum C3 energy and orbit insertion values (for the cases studied) and the longer and shorter duration flights require more energy and velocity change. Thus, for a 2003 mission (a baseline comparison year to be compatible with the original study) we provide this refined transfer data from our simulations with full-force models

and non-coplaner, eccentric orbits. With these mission scenarios developed, more extensive simulation and analysis for various transfer parameters could be undertaken.

2003 Transfer Braking Maneuver at Mars Periapses

Based on a helpful suggestion by Chauncey Uphoff, we investigated the use of a braking maneuver at close approach to Mars to lower the ΔV required for the Lissajous orbit insertion maneuver. This added a segment to the trajectory design and required some careful targeting for a close swingby and braking maneuver around Mars (targeting details discussed in the next section of the paper). We modeled and simulated the 200 day TOF case for the 2003 mission to L1 with the braking maneuver, shown in Figures 4a (looking edge-on at the XZ plane) and 4b (looking down on the XY plane). The data and a comparison to the direct insertion case are presented in the table below.
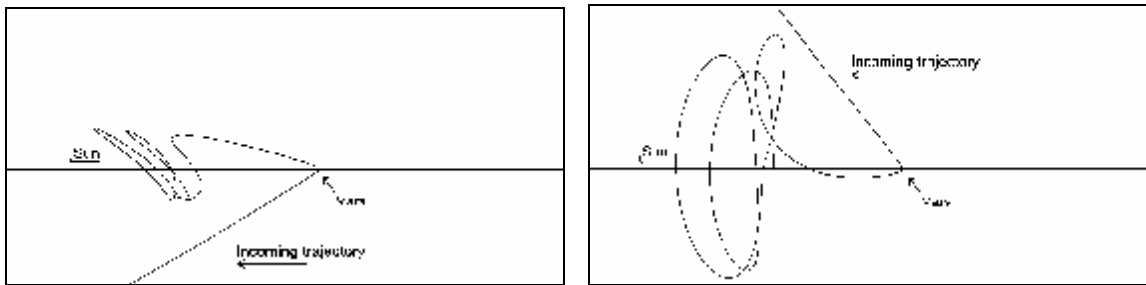
Figures 4a and 4b   L1 Orbit Insertion with Braking Maneuver

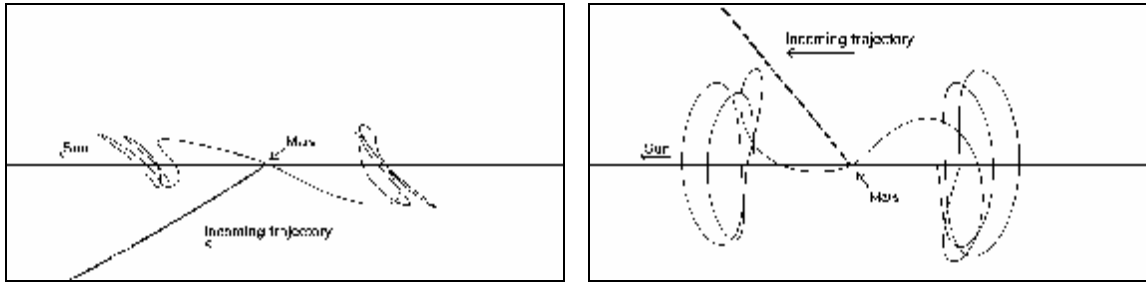**Table 2   Comparison of 2003 Transfers to L1 Orbit; 200 Day TOF**

Scenario	C3 Energy (km^2/sec^2)	Braking Δv (km/sec)	Orbit Insertion Δv (km/sec)	Total Δv (km/sec)
Direct Injection	8.883	0	2.425	2.425
Braking Maneuver	9.056	0.856	0.104	0.960

The braking maneuver resulted in a ΔV savings of 1.465 km/sec, which would lead to fuel mass savings and/or increase in payload capacity. This type of maneuver seems promising as a ΔV conserving technique, and so we adopted it for the other mission simulations that follow. However, there is plainly room for future investigations into the applicability of this trajectory for various mission profiles.

2016 Transfer Braking Maneuver

In order to provide data from this study that may aid future mission planners or lead to further research, we modeled a 2016 mission to place two vehicles in orbit about L1 and L2. We simulated a 200 day TOF as a baseline, as well as a 181 day TOF which,

along with the departure date of 20 Feb 2016, was inspired by a JPL Ballistic Earth-Mars Trajectory study.[17] These trajectories are depicted in Figures 5a and 5b and relevant data is shown in the two tables which follow.

Figures 5a and 5b L1 and L2 Orbit Insertion with Braking Maneuver

Table 3   Comparison of 2016 Transfers to L1 Orbit for Different TOF

TOF (days)	C3 Energy (km^2/sec^2)	Braking Δv (km/sec)	Orbit Insertion Δv (km/sec)	Total Δv (km/sec)
181	8.847	2.314	0.047	2.360
200	10.377	1.710	0.047	1.757

Table 4   2016 Transfers to L1 & L2 Orbits for 200 Day TOF

Orbit	C3 Energy (km^2/sec^2)	Mid-course Δv (km/sec)	Braking Δv (km/sec)	Orbit Insertion Δv (km/sec)	Total Δv (km/sec)
L1	10.377	0	1.710	0.047	1.757
L2	10.377	0.001	1.708	0.085	1.795

Table 3 shows that a shorter TOF to Mars can be achieved with a lower C3 energy value, but that trajectory requires a larger braking maneuver than the longer transfer, to achieve the same mission orbit. This indicates that with these types of missions the lower energy transfer may not yield a lower braking and insertion ΔV specification.

Table 4 shows how two vehicles could start on the same transfer trajectory initially (as with a simultaneous launch) and the L2 vehicle targeted for it's close approach via a small mid-course correction. The simulation method is explained further in the next section. The *total* TOF to Lissajous orbit insertion is different for each vehicle, which would assist in the phasing of the vehicles that is required for the communications relay system to maintain adequate coverage of Mars.

Z Amplitude and ΔV Analysis

Since specific mission performance specifications will drive the Lissajous orbit shaping requirements, we performed some basic investigations into the relationship between the orbit Z amplitudes and the ΔV needed for the braking and mission orbit insertion maneuvers. We examined the 2016 transfer to an L1 orbit for 200 day TOF, targeting various amplitude values. The trajectories were very sensitive to small changes in the maneuvers (see Figure 6, looking towards the Sun), and the results are summarized in Table 5 below. The C3 energy for all cases was kept constant at 10.377 km^2/sec^2.

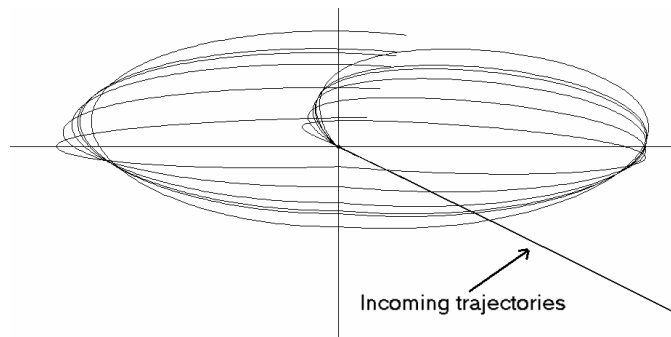Incoming trajectories

Figure 6   Orbits about L1 with Different Z Amplitudes

Table 5   2016 Transfer to L1 Orbit with Varying Z Amplitude

Z Amplitude (km)	Periapsis elevation (degrees)	Mid-course Δv (km/sec)	Braking Δv (km/sec)	Orbit Insertion Δv (km/sec)	Total Δv (km/sec)
50000	-4.3	0.00017	1.71040	0.04181	1.75238
100000	-8.7	0.00009	1.71034	0.04283	1.75327
140000	-12.3	0.00003	1.71025	0.04463	1.75491
160000	-14.1	0	1.71019	0.04595	1.75614
167447	-14.7	0	1.71017	0.04653	1.75670
200000	-17.6	0.00006	1.71004	0.04653	1.75980

The data from Table 5 demonstrate a correlation of the geometry of periapsis with the Z-amplitude. As a measure of the geometry, the elevation angle of the periapsis measured with respect to Mars' orbit plane was used. Very slight changes in the elevation angle caused dramatic changes in the Z amplitude. (It was also noticed that the class of the Lissajous orbit could be changed by large variation of elevation angle, however this was not thoroughly investigated for this study.) As shown in the table, the mid-course correction ΔV to change the elevation angle at periapsis is insignificant. Additionally, there is no significant change in the braking maneuver, leading to the conclusion that a wide range of Z amplitudes can be achieved with no fuel penalty.

TARGETING METHODS USING STK/*ASTROGATOR*[1]

The transfer from the Earth to a Mars Lagrange orbit was targeted in a series of steps. The purpose of the targeting was to determine the control variables necessary to achieve this transfer. The initial orbit state represented the post launch Earth-centered hyperbolic trajectory. This was specified in target vector form, in the Earth-centered mean ecliptic and equinox of J2000 coordinate system. The seven parameters of the target vector are: epoch, radius of periapsis, C3 energy, right ascension (RA) and declination (Dec) of the outgoing hyperbolic asymptote, the velocity azimuth at periapsis, and the true anomaly. (Note: C3 is defined as negative the gravitational parameter of the central body divided by the semimajor axis. For hyperbolic orbits this is the square of the hyperbolic excess velocity.)

For this study, the epoch was chosen to match previous work, the true anomaly was set to zero, the velocity azimuth set to 90 degrees, and the radius of periapsis set to 6678.0 km. This represents a satellite near the Earth at perigee. The remaining parameters, C3 energy and the direction of the trajectory (RA and Dec of the asymptote) were used as control parameters. Two methods of insertion into Lagrange orbits were utilized and are discussed separately below.

Direct transfer to L1 Lagrange orbit

For the direct transfer from Earth to the L1 Lagrange orbit, the control parameters were adjusted using a differential corrector technique to achieve three constraints at the point the trajectory crossed the ZX plane of Sun-Mars rotating libration-point coordinate system (this is the plane containing the Sun-Mars line and perpendicular to Mar's orbit plane). The three constraints are the desired epoch of arrival, and the X and Z positions in the Sun-Mars rotating libration-point coordinates.

Once the desired time and position was achieved, the three components of the Lagrange-orbit insertion maneuver (LOI) was targeted as an impulsive ΔV maneuver in four steps. First, LOI was targeted to achieve somewhat ideal velocity components for the Lissajous orbit at this point. Velocity in the X and Z rotating libration point directions were targeted to zero. Velocity in the Y direction was targeted to -0.16 km/sec (a representative value from previous analysis). Second, the LOI maneuver was corrected so that after propagating the trajectory a half revolution to the first ZX plane crossing, the X component of velocity (Vx) would be zero (this represents a perpendicular plane crossing when projected into the XY plane, and is the same energy balancing technique mentioned by Dunham and Roberts [14]). After achieving the first ZX plane crossing, the third and fourth steps were to correct the LOI maneuver to achieve Vx of zero at the second, and then the third ZX plane crossings.

Transfer using braking maneuver at Mars periapsis

The transfer to a Mars Lagrange point orbit using a braking maneuver at the close approach at Mars before the LOI maneuver was also targeted in stages. First, the target vector control parameters were adjusted by the differential corrector to achieve an epoch at periapsis Mars, and B-Plane components to place the trajectory on the anti-Sun side of Mars. Since this stage is just a first guess, the values used were B-dot-T of -10,000 km, and B-dot-R of 0.0 km.

The second step refined this to the desired close approach conditions. Using the same control parameters, the radius of close approach was used instead of B-dot-T, and was targeted to a radius of 3,600 km (about 200 kilometers altitude).

After the constraints at periapsis were met, the magnitude of a retrograde braking maneuver (anti-velocity direction) was used at periapsis to shape the trajectory until the trajectory crossed the XZ plane at the desired X distance in the Sun-Mars rotating libration-point coordinate system (X_{RLP}). After the retrograde maneuver was calculated, the LOI maneuver was planned using the same 4-step method previously described for the direct transfer.

The transfer to the L2 Lagrange orbit was planned in a similar manner, except that the trajectory must pass on the Sunward side of Mars at the close approach. This was done using a mid-course correction (MCC) maneuver as a control parameter, which also allowed the initial transfer parameters to be the same for both the L1 and L2 vehicles.

Transfer Using Braking Maneuver to Achieve Desired Z Amplitude

The concept for targeting a desired Z amplitude for the Lagrange orbit is analogous to the technique using Earth's moon as described by Sharer, et al.[21] The Z amplitude can be directly controlled as a function of the position of the trajectory as it passes through its close approach to Mars.

The initial targeting is the same B-Plane targeting described above: first B-dot-T and B-dot-R, and then B-dot-R and Radius of periapsis. The initial target vector parameters were not used at this step because the corrections to the parameters were too small, being on the order of a double precision number. Instead, a MCC maneuver was used 30 days after Earth departure.

After the epoch, B-plane, and radius of periapsis constraints were achieved, the retrograde braking maneuver was targeted to achieve the as described above. Then the Z distance (amplitude) was checked, and if it was significantly far from the desired value,

the previous step was repeated with a different B-dot-R value. (B-dot-R is directly related to the elevation of periapsis with respect to the Mars' orbit plane.)

The next stage involved targeting the four constraints that must be simultaneously met: the epoch at periapsis, the radius of periapsis (to prevent the trajectory from hitting Mars), the X position at LOI, and the Z amplitude. In addition to the three components of the MCC maneuver, the magnitude of the braking maneuver was also used as a control.

Once this step converged, the LOI maneuver was targeted using the same 4-step method described above for the direct insertion.

STATIONKEEPING

Due to the precarious nature of the Lissajous orbit, precise and continuous stationkeeping (SK) techniques must be employed. Additionally, the precision required for certain missions located around the Sun-Mars Lagrange points requires the fidelity of such SK maneuvers to be extremely high. SK ΔVs as little as 1 mm/sec could be required.

Stationkeeping techniques fall into two major categories.[14] The first, referred to as a "tight" control technique, attempts to target the vehicle back to a nominal three-dimensional path. The second is the "loose" control technique that uses a simpler "orbital energy balancing" strategy to closely mirror a Lissajous orbit. The two control techniques differ only in the number of ΔV components that are varied. The loose technique will simply vary one component of ΔV while the tight technique varies two or more to achieve a nominal Lissajous orbit.

History[14,18]

The third International Sun-Earth Explorer (ISEE-3) flown to the Sun-Earth L1 point in 1978 used the tight control technique in an attempt to maintain its trajectory as close to a nominal halo orbit as possible. This mission, being the first to orbit a Sun-Earth libration point, had the luxury of a large supply of fuel to allow for uncertainties in the insertion to and maintenance of the new orbit. The relatively small errors encountered during insertion into the halo orbit left a large amount of fuel that could be used specifically for stationkeeping. Over the four years that ISEE-3 was established at the L1 point, 15 SK maneuvers were performed totaling 30.06 m/sec at an average of 2.00 m/sec per maneuver. The time between the maneuvers averaged 82 days.

While the large amount of fuel planned for the ISEE-3 mission allowed for very tight control of its halo orbit, a more optimal SK method was planned for the Solar Heliospheric Observatory (SOHO). Prior to its establishment at the Sun-Earth L1 point in 1996, SOHO mission planners sought ways in which to decrease its SK costs. If the

complexity of SK maneuvers for SOHO could be dramatically reduced, or "loosely" controlled, the fuel load, and therefore costs, could be also be reduced. In the "orbital energy balancing" technique that evolved, only one component of ΔV, in this case the x-component, would be varied. The result of this simplification achieved a threefold reduction in SK costs from roughly 7.5 m/sec per year for ISEE-3 to less than 2.3 m/sec per year for SOHO.

The major drawback with the loosely controlled technique used on SOHO was that it did not maintain a periodic halo orbit *precisely*. The resultant orbit was, however, a Lissajous path that mirrored the nominal halo orbit so closely that for all practical purposes it could be considered equivalent. The loose control technique was therefore proven as an effective means of achieving lower SK costs when precise orbit mapping was not necessary.

Stationkeeping for the Sun-Mars Lissajous Orbits

For future missions to Mars using the Sun-Mars Lagrange points, mission planners will have to consider several factors prior to making a decision on the SK technique to be used. Obviously mission requirements will dictate whether the loose control technique can be used to optimize SK costs or if the higher precision of the tight technique is necessary. In our example of a communication system in orbits about the L1 and L2 points, the loose technique should be sufficient as a communication system's global nature does not depend on a precise halo orbit. Furthermore, the success of ACE and SOHO with using the loose technique has served to prove its utility and make it a preferred approach for scheduled missions to Sun-Earth Lagrange points.

The frequency of such SK maneuvers depends mainly on two factors. The first, and most crucial, is the accuracy of the insertion maneuver. If this maneuver is performed with a minimal ΔV error, the SK that follows is also minimized. If, however, the insertion ΔV error is greater than that expected the magnitude of the SK maneuvers will increase and the SK costs will rise.

The second factor in SK frequency determination is the effect that subsequent SK burns have on the overall orbit error. This can be further broken down to the actual magnitude of the orbit error at the end of the last burn, the time since that burn, and the accuracy of the burn itself as executed. Obviously, this orbit error will increase with time and the larger the error, the sooner a subsequent burn will need to be performed. The key here is to minimize the magnitudes of the burns.[14]

Once the frequency and magnitude of the required SK maneuvers are determined, the optimal timing of such maneuvers will need to be considered. For the communications system example, the timing of SK burns is critical so as to prevent unexpected and inconvenient losses in communications coverage to the users on the Martian surface. One solution to such a problem is to overlap the SK maneuvers with the

spacecraft's preplanned attitude and momentum adjustments. This allows the attitude control, momentum management, and SK maneuvers to complement one another and minimizes the down time of the system.

Dunham and Roberts have shown that small ΔV errors on order of 0.1 mm/sec for the Sun-Earth/Moon system cause noticeable deviation from the nominal after about 3 revolutions in the Lissajous orbit. The same error was applied to the Sun-Mars L1 Lissajous orbit as shown in Figure 7. This error caused noticeable deviations after only 1 and a half revolutions. However, because the period of the Mars Lissajous is about twice that of the Earth Lissajous, the deviations occur approximately after the same duration. This is an indicator that the stationkeeping requirements for the Mars Lissajous will be on the same order as seen for the Earth missions, in terms of fuel used per year. Of course, a thorough error analysis study could be made later to prove this, accounting for the errors and uncertainties.
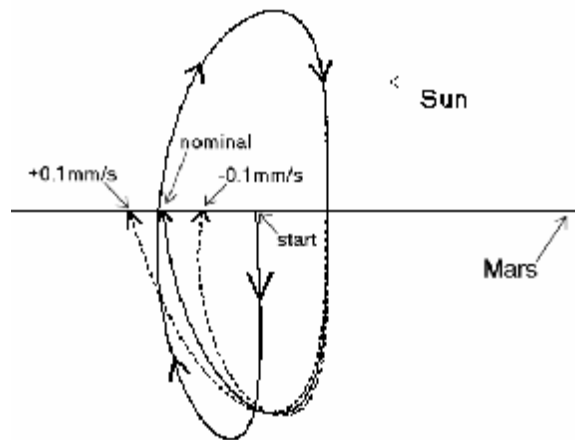

Figure 7 – Effect of small errors on Lissajous orbit

CONCLUSION

The trends from the previous studies are still valid when using full force models, however the actual magnitudes of the maneuvers can be significantly increased. This current work also highlights the fact that the minimum departure C3 energy does not always correspond to the minimum LOI maneuver. Additionally, the use of a braking maneuver at a low altitude (200 km) Mars periapsis prior to LOI saves significant spacecraft on-board fuel. The geometry of this close approach can be taken advantage of to control the Z amplitude and class of the Lissajous orbit.

The loose control technique for stationkeeping would be appropriate for the L1 and L2 communication relay concept. The stability of these orbits are on the same order as the Sun-Earth orbits in terms of deviations from nominal as a function of time. One must remember, however, that because the period of these Sun-Mars Lissajous orbits are twice as long as Sun-Earth orbits, errors cause deviations more quickly with respect to

position in the orbit. Therefore, it is anticipated that annual stationkeeping costs should be similar to Sun-Earth orbits.

ACKNOWLEDGMENTS

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