OPTIMAL TIME TRANSFER

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Abstract

We have designed an optimal time transfer algorithm using GPS carrier-phase and pseudo-range measurements. This will enable the near-real-time estimation of the difference between the phase of a stable real-time UTC ground clock attached to a GPS ground receiver and the phase of each NAVSTAR spacecraft clock in the ensemble of GPS spacecraft. We propose a master filter (MF) to drive multiple instances of spawned filters (SF) and fixed-epoch smoothers (FES). The MF is designed to map all GPS carrier-phase measurement information of each GPS ground-station/NAVSTAR pass into the complete MF state estimate of all NAVSTAR parameters and all station parameters, with all serial correlations and all cross-correlations accounted for. Each SF runs forward with time while the FES linearly maps measurement information backwards to fixed epochs for individual NAVSTAR spacecraft. FES estimates are then folded into the MF as pseudo measurements. The resulting time-transfer algorithm is an autonomous forward running sequential estimator, with no state-sized matrix inverse calculations required[1].

INTRODUCTION

We propose optimal time-transfer in *near-real-time* using undifferenced GPS carrier-phase and pseudo-range measurements. We will estimate the difference between the phase of a stable real-time UTC ground clock attached to a GPS ground receiver and the phase of each NAVSTAR spacecraft clock in the ensemble of GPS spacecraft. Our new time-transfer algorithm consists of a master filter (MF), and multiple filter-smoothers. The MF will not directly process the GPS measurements. Rather, the MF will create and initialize a spawned filter (SF) to process the GPS carrier-phase and pseudo-range measurements for each ground-station/NAVSTAR pass. The time-tag for the first carrier-phase measurement in each ground-station/NAVSTAR pass will be called a fixed epoch. The MF will initialize a SF and a fixed-epoch smoother (FES) at each fixed epoch, and will attach a fixed-epoch smoothing window forward of each fixed epoch. The length of the smoothing window is bounded above by 5 hours, the maximum time-length of a ground-station/NAVSTAR pass. The operator may choose to terminate each smoothing window according to programmed options. The SF moves forward through each smoothing window processing GPS carrier-phase and pseudo-range measurements while the FES moves the new measurement information, linearly and simultaneously, back to the fixed epoch. The SF-FES filter-smoother is applied to each pass to autonomously estimate a carrier-phase-as-range bias (ambiguity) at the fixed epoch, with carrier-phase tracking time interval of maximum length.

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[1] Notice that the extensively used fixed-interval smoother (following a sequential filter) runs backwards with time and requires the calculation of a state-sized matrix inverse at each step.

MASTER FILTER

The master filter (MF) will simultaneously estimate orbit, clock, and solar photon pressure parameters for every GPS NAVSTAR spacecraft, and station coordinates and wet troposphere component for every GPS ground station. Let N denote the size of the MF state estimate. Then $300 < N < 500$ for the MF state estimate, with variations of N that are dependent on the number of NAVSTAR spacecraft and number of ground stations. Time-transfer will be realized in estimates of clock phase difference between a real-time UTC clock attached to a GPS ground receiver and each NAVSTAR spacecraft clock. The MF epoch is paused at the fixed epoch until the SF and FES have completed their work. Then the FES state estimate at the fixed epoch is converted to a multidimensional pseudo-measurement, and the pseudo-measurement is processed by the MF, using the FES covariance matrix as the pseudo-measurement covariance. The SF and FES are then destroyed by the MF, and the MF state estimate and error covariance are propagated forward to the next fixed epoch.

When one or more station/NAVSTAR passes are being processed by SF-FES filter-smoothers, and a new station/NAVSTAR pass is realized with its first measurement time-tag (new fixed-epoch) forward of the paused MF epoch, a *copy* of the MF is propagated forward to the new fixed-epoch to spawn the new SF and FES. The new SF covariance matrix is checked for numerical negative eigenvalues and corrected if necessary.

Alternatively one could spawn the new SF from the MF at the paused MF epoch and propagate the new SF forward to the new fixed-epoch, and there create the new FES by the new SF. This alternative is rejected so as not to lose benefit of MF cross-correlations during the forward propagation of the MF copy.

There are no state-sized matrix inverse calculations required of the MF. This is particularly relevant with the incorporation of GPS carrier-phase measurement information.

SPAWNED FILTERS

The MF creates a spawned filter (SF) at each fixed-epoch. The SF state estimate (except for one parameter) and associated error covariance matrix are selected from the MF, the SF covariance matrix is checked for negative eigenvalues and corrected if necessary, and then the selected state estimate and covariance matrix are transferred to the SF. Each SF includes orbit, clock, and solar photon pressure parameters for a *particular* GPS NAVSTAR spacecraft, and coordinates and wet troposphere component for a *particular* GPS ground station. An additional parameter is added to the SF state estimate, a phase-as-range bias[2], initialized from the SF state estimate at the fixed epoch. All SF states will be estimated simultaneously from GPS carrier-phase and pseudo-range measurements. Let n denote the size of the SF state estimate. Then $15 < n < 20$ for each SF state estimate, with variations of n that depend on the number of parameters estimated for each clock and for solar pressure. The SF algorithm consists of the recursive application of two filter update algorithms, a time-update and a measurement-update. The SF time-update propagates the SF state estimate and covariance forward to the next measurement time-tag in the fixed-epoch time window. The SF measurement-update processes the new measurement at its time-tag. There are no state-sized matrix inverse calculations required of the SF. This is particularly relevant when processing GPS carrier-phase measurements.

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[2] Our phase-as-range bias is also known as *phase ambiguity*.

Our SF is similar to an EKF (extended Kalman filter) in that

- linearization about the last state estimate is employed
- the Kalman gain maps measurement residuals to state estimate corrections
- measurements are modeled with Gaussian white noise components

Our SF is unlike an EKF in that

- state estimate process noise covariance is derived from appropriate physics
- random acceleration errors are *not* modeled as white noise
- the SF calculates realistic state estimate error covariance functions
- the SF does not diverge[3]

SF orbits are always propagated with complete nonlinear numerical integrations.

FIXED-EPOCH SMOOTHERS

Each Frazer fixed-epoch smoother[4] (FES) has the same state estimate structure as the SF. An FES is initialized by the MF at each fixed-epoch. After each SF measurement-update, the FES linearly moves the new measurement information variation backwards to the fixed epoch, and combines the new information with the FES state estimate at the fixed epoch. The error magnitude of the FES state estimate is reduced with each operation of the FES at the fixed-epoch. An increasingly accurate smoothed state estimate with realistic error covariance is produced by an FES at each fixed epoch with variable time-lag behind the forward running SF. This enables the concentration of several hours of carrier-phase measurement information on the phase-as-range bias estimate at the fixed epoch, and on all other FES state estimate parameters.

There are no state-sized matrix inverse calculations required of the Frazer FES. This is particularly relevant when processing GPS carrier-phase measurements.

SUMMARY

MF is designed to map all GPS carrier-phase measurement information of each ground-station/NAVSTAR pass into the complete state estimate of all NAVSTAR parameters and all station parameters, with all serial correlations and all cross-correlations accounted for. The MF runs forward with time or pauses at fixed epochs. Each SF runs forward with time while the FES linearly maps measurement information backwards to fixed epochs. Thus the time-transfer algorithm is an autonomous forward running sequential estimator, with no state-sized matrix inverse calculations required.

NARRATIVE WITH FIGURES

The fully initialized MF state estimate and its covariance are assumed to exist at time t_{K-1}. The first pair of GPS carrier-phase and pseudo-range measurements for pass K are available at time-tag $t_K > t_{K-1}$ from the GPS Ground Station S1/NAVSTAR N1 contact. The MF state estimate and its covariance (or copy) are propagated from t_{K-1} to t_K.

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[3] See Jazwinski [5] Section 8 page 301 for divergence of the EKF when applied to orbit determination.

[4] Our fixed-epoch smoother is also referred to as a fixed-point smoother. See Meditch [15] Corollary 6.1 page 232 for Fraser's fixed-point smoother.

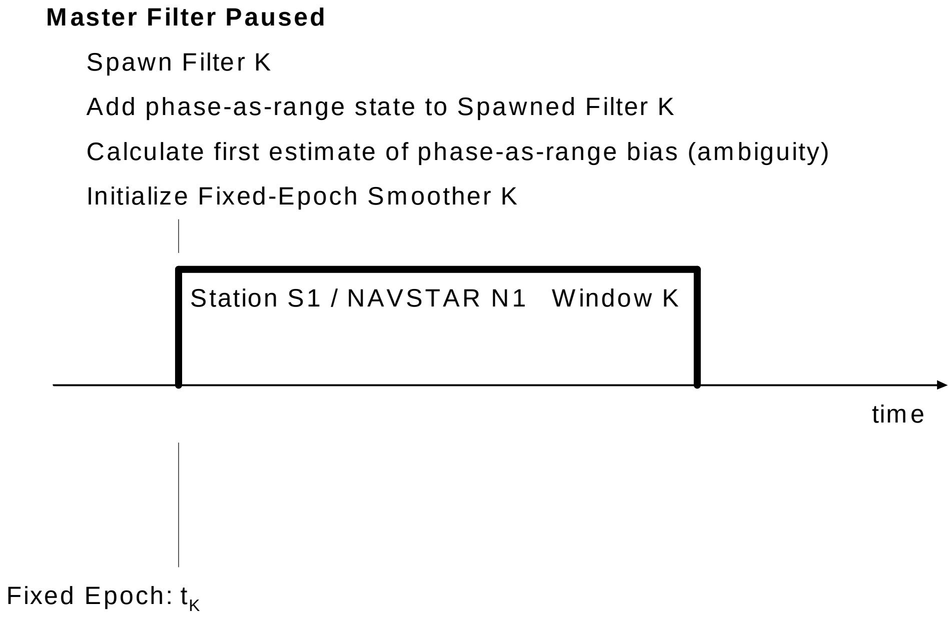

Figure 1. First carrier-phase measurement at time-tag and fixed-epoch t_K

As indicated by Figure 1, an SF K state estimate and covariance are spawned from the MF at t_K, and one state parameter is added, the phase-as-range bias. The initial phase-as-range bias estimate is calculated from the SF K state estimate, and its variance is calculated from the SF K covariance. FES K is initialized at t_K by the MF, and the time Window K is attached to t_K forward of t_K.

The second pair of GPS carrier-phase and pseudo-range measurements for pass K are presumed to be available at time-tag $t_{M1} > t_K$ as indicated in Figure 2. SF K performs a time-update from t_K to t_{M1}, and performs a measurement update at t_{M1}. FES K linearly maps the SF K measurement update variation backwards to t_K to improve the FES K estimate at the fixed epoch t_K. This filter-smoother estimation process by SF K and FES K is repeated on all pairs of GPS carrier-phase and pseudo-range measurements for pass K, while the MF remains paused at t_K.

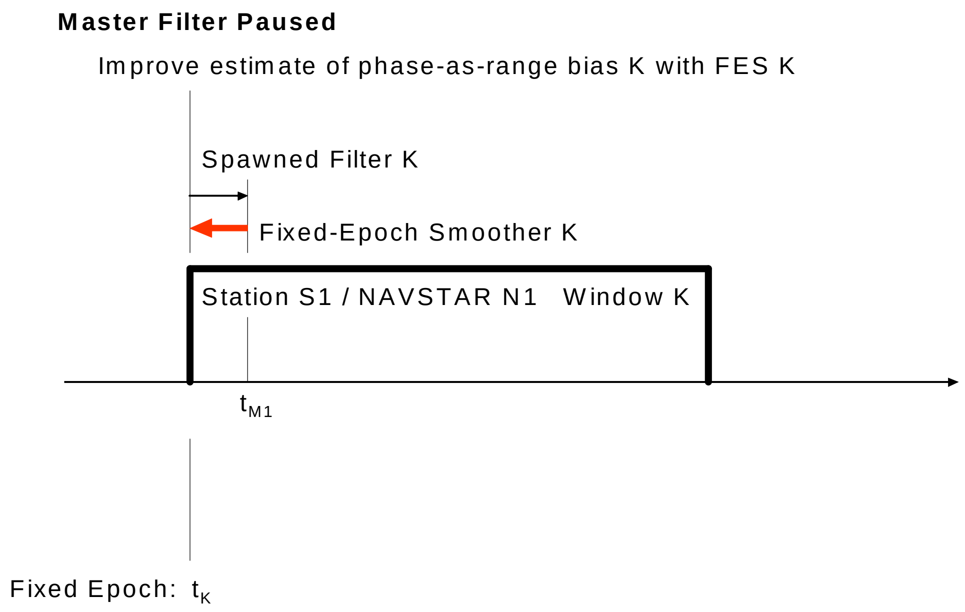

Figure 2. Run SF & FES across Window K

While filter-smoother SF-FES K is processing measurements in Window K, the first pair of GPS carrier-phase and pseudo-range measurements for pass K+1 with Station S2 and NAVSTAR N2 become available at time-tag t_{K+1} as indicated in Figure 3. A copy of the MF is propagated to t_{K+1}, and SF K+1 is spawned from the MF copy.

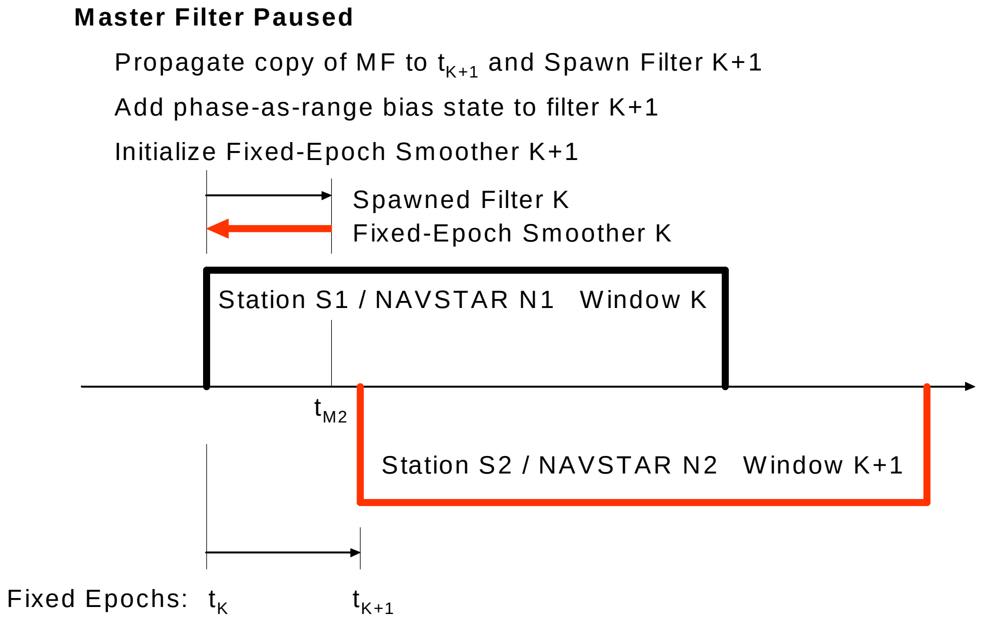

Figure 3. New Pass Window K+1

FES K+1 is initialized at t_{K+1} to complete the initialization of SF-FES K+1. As indicated with Figure 4, both filter-smoothers SF-FES K and SF-FES K+1 operate simultaneously and asynchronously sending carrier-phase measurement information to fixed epochs t_K and t_{K+1} respectively, with the MF paused at t_K.

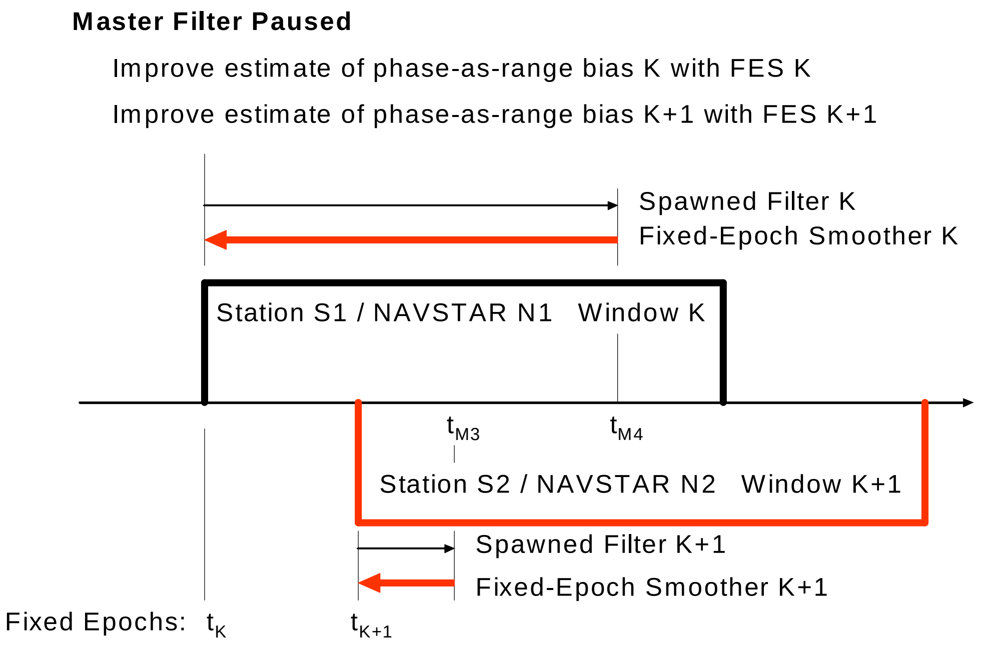

Figure 4. Asynchronous Processing in Windows K & K+1

Filter-smoother SF-FES K eventually completes the processing of GPS carrier-phase measurements in Window K, as illustrated in Figure 5. The MF converts the state estimate for FES K to a multidimensional pseudo-measurement, and converts the covariance matrix for FES K to a multidimensional pseudo-measurement covariance matrix. The MF processes the pseudo-measurement using the pseudo-measurement covariance matrix. The MF then destroys filter-smoother SF-FES K. Last, the MF propagates the MF state estimate and MF covariance matrix from the fixed epoch t_K to the fixed epoch t_{K+1}. Filter-smoother SF-FES K+1 continues processing GPS carrier-phase measurements in Window K+1, as indicated in Figure 6.

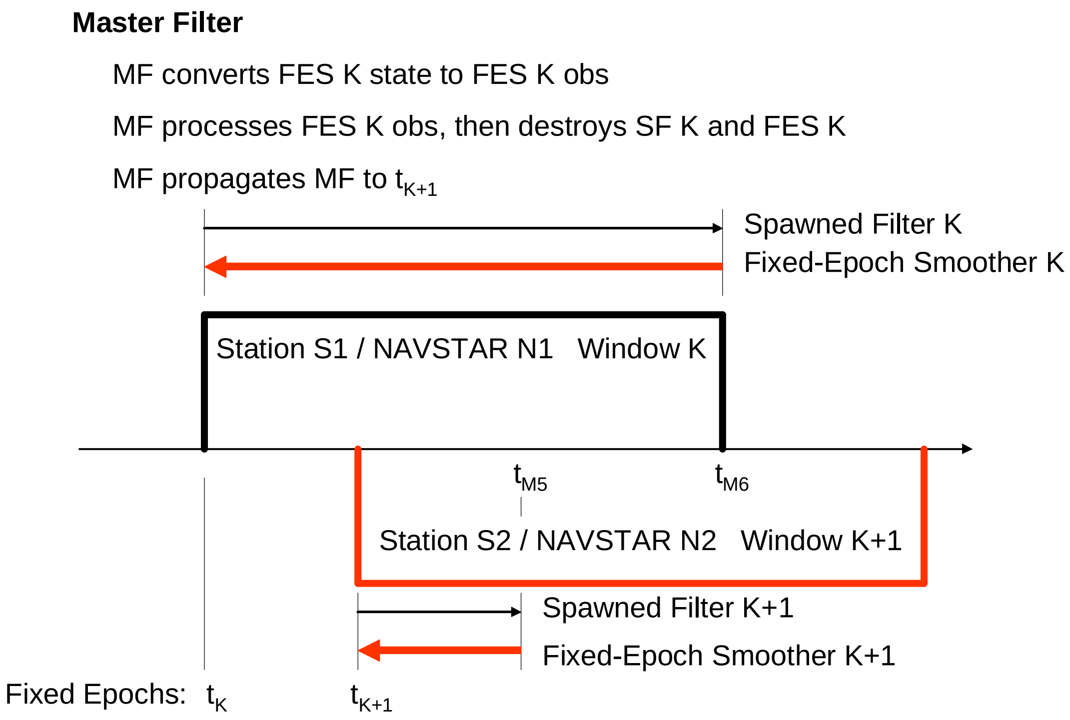

Figure 5. Finish Window K

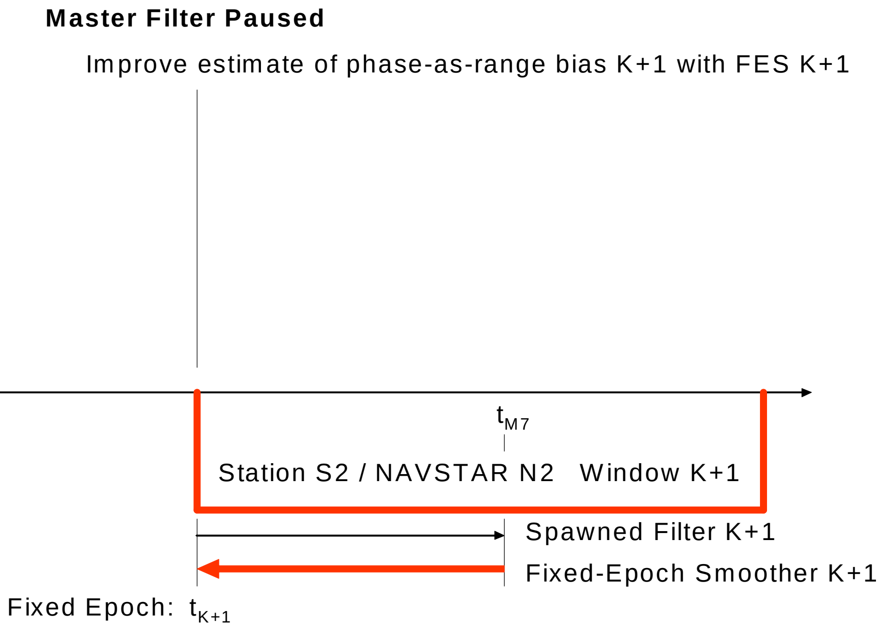

Figure 6. Continue with Window K+1

Figure 4 illustrates both filter-smoothers SF-FES K and SF-FES K+1 operating simultaneously and asynchronously sending carrier-phase measurement information to fixed epochs t_K and t_{K+1} respectively, with the MF paused at t_K. In fact the MF can process *many* SF-FES filter-smoothers simultaneously and asynchronously sending carrier-phase measurement information to their respective fixed epochs, while the MF is appropriately paused at the fixed epoch for the earliest window being processed.

The MF state estimate and error covariance can be saved at each fixed epoch to create a useful history of MF state estimates and error covariance matrices. At any time the MF state estimate and error covariance may be propagated from its current fixed epoch to any time of interest in the future. The current and propagated MF state estimates will provide the most accurate and realizable real-time and predicted state estimates, together with realistic MF covariance matrices for uncertainty assessment.

An optimal estimate of the difference between the phase of the stable real-time UTC ground clock and the phase of each NAVSTAR spacecraft clock will thus be available at all past MF fixed epochs, at the current fixed-epoch, and at any propagated time of interest in the future. The error variance will provide the uncertainty on the time-transfer estimate.

Given two distinct ground stations with UTC real-time clocks and capabilities to process GPS carrier-phase measurements from the same GPS NAVSTAR across a common time interval, then time-transfer between the ground clocks is enabled by differencing the time-transfer difference estimates referred to the same NAVSTAR clock.

REAL-DATA VALIDATION

McReynolds' rigorous filter-smoother (FS) consistency test has been in successful operational use for twenty-seven years. The FS test is sensitive to modeling errors. We propose to apply it to the time-transfer problem using the SF (spawned filter) and FES at each fixed epoch. The FS test statistic for clock phase-difference is defined by a ratio. The numerator is the difference between filtered and smoothed estimates (smoothed - filtered) of the clock phase-difference, and the denominator is the square-root of the difference (filtered - smoothed) between the variances of the filtered and smoothed estimates of the clock phase-difference. When this unitless ratio is between -3 and +3 at least 99% of the time, then the FS test is said to be satisfied. Otherwise the FS test is failed. The FS test is appropriately applied to every component of the state estimate common to the SF and FES. The FS test thereby points to sources of modeling problems on FS failure, and provides powerful validation on FS success. Proof[5] for this theorem (McReynolds) derives from hypotheses of Gaussian distributions on all state estimation errors, and from use of the two-filter representation theorem (Donald Fraser [4]) for the smoother.

We will enable the operator to initialize global fixed-epoch smoothers (GFES), at operator specified global fixed epochs, where each GFES sends measurement information back to all global fixed epochs after each MF pseudo-measurement update. This will enable the application of FS testing to the complete state space simultaneously.

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[5] A derivation of the FS theorem is currently available from AGI.

STOCHASTIC ESTIMATION

Physically-connected process noise covariance will be applied additively to all SF and MF covariance propagations. A three-dimensional stochastic sequence for solar photon pressure acceleration error will be estimated and applied for each NAVSTAR orbit. An associated three-dimensional stochastic estimation error sequence will be used to drive the solar pressure acceleration error process noise covariance. Options for the stochastic sequence will include three-dimensional Wiener, Ornstein-Uhlenbeck, and six-dimensional Vasicek sequences. The three-dimensional stochastic acceleration will be defined on the same body-fixed coordinate system used in Bar-Sever's deterministic model [2]. Appropriate clock parameter estimation and covariance propagation will be applied for each GPS clock; see Wright [23], [24]. White noise sequences and their variances will be used to model thermal noise in GPS carrier-phase and pseudo-range measurements. Realistic state estimate error covariance functions will be produced by the master filter, all spawned filters, and all fixed-epoch smoothers. By *realistic*, we refer to satisfaction of McReynolds' rigorous filter-smoother consistency test described above.

ADOPTED CONVENTIONS

- Use geopotential EGM 2008 or GRACE 2C for Earth gravity field
- Ionospheric effects will be removed with appropriate linear combinations of L1 and L2 carrier-phase and pseudo-range measurement data
- GPS carrier-phase and pseudo-range measurements will be edited at elevations below particular thresholds for multi-path effects (Larson, et. al. [9])
- GPS carrier-phase measurements from receivers calibrated to the GPS simulator for electrical delays will be preferred or required (Plumb, et. al. [20])
- The deterministic Bar-Sever model [2] for NAVSTAR solar pressure will be employed
- The deterministic Bar-Sever model [3] for GPS yaw attitude will be employed

BENEFITS

- Optimal time-transfer estimates and realistic error variances in *near-real-time*
- All cycle slipped carrier-phase measurements autonomously identified and edited
- All carrier-phase-as-range biases (ambiguities) optimally and autonomously estimated
- One-day predicted IGS orbits replaced by near-real-time simultaneously estimated orbits
- IGS orbit one-day overlap discontinuities eliminated
- IGS radial position errors will not alias into clock phase estimates for time-transfer
- Complete MF state estimate and realistic error covariance functions always available

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