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EFFICIENT NUMERICAL INTEGRATION OF COUPLED ORBIT AND ATTITUDE TRAJECTORIES USING AN ENCKE TYPE CORRECTION ALGORITHM

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A technique for integrating orbit trajectories influenced by attitude dependent forces is presented. The method employs an Encke type correction algorithm to account for the attitude dependence not modeled in the main orbit integration process. The formulation improves computational efficiency over the straight-forward integration of the fully coupled system by eliminating the need to compute the acceleration due to the geopotential at the time step required for the attitude dependent forces. Examples are shown for the integration of coupled orbital and attitude motion and for the integration of the orbit trajectory when the attitude profile is known a priori.

INTRODUCTION

The computation of coupled orbit and attitude trajectories is a challenging problem, which can be addressed at many levels. The first step is to define what it means to have coupled orbit and attitude states. In some circumstances, the integration of the orbit state may be dependent upon the attitude, while the attitude is considered to independent of the orbit. The attitude of the satellite typically affects the orbit via the interaction of the three dimensional configuration of the satellite with atmospheric drag or solar radiation pressure (SRP). It has even been proposed in recent studies to use the interaction of the spacecraft geometry with the SRP to provide orbit control for formation flying[1]. In other circumstances the integration of the attitude may be dependent upon the position and velocity of the satellite[2], while the orbit trajectory is not influenced significantly by the attitude. This type of situation can occur when the attitude of the satellite is maintained in accordance with a control law while the satellite is experiencing torques due to solar radiation pressure, atmospheric drag, gravity gradient or Earth's magnetic field. In a completely coupled case, the integration of the orbit and attitude parts of the state must be performed simultaneously.

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The integration of orbit trajectories which depend upon the satellite attitude and the simultaneous integration of the orbit and attitude introduce a problem in the selection of a step size for the integration process. In practice, it is typically assumed that the spacecraft attitude changes much faster than its position. This means that, if the two motions are not coupled, the numerical integration of attitude trajectory will require more (smaller) steps than the numerical integration of orbit trajectory for comparable duration and accuracy. The goal in computing coupled solutions is to avoid evaluating the computationally expensive accelerations on the satellite position at the frequency required to adequately sample the attitude. One approach to the problem is to use weakly coupled numerical integration where two separate integrators are employed: one for orbit propagation and one for attitude propagation. Their coupling is accomplished via periodic updates of their respective forcing terms with the state coming from the "companion" integrator. One possible implementation might involve using a two body approximation of the orbital motion inside the attitude integration. The attitude state could then be propagated for the duration of one orbit integration step allowing the subsequent step in the orbit integration process access to accurate attitude information. The initial state for the two body approximation is then updated based on the current orbit state and the process continues. Clearly, there will be errors when using this method in both the orbit and attitude trajectories.

The original procedure can be improved if additional characteristics of the trajectories and the exerted forces and torques are examined. Consider the effect of small orbit state errors on drag, SRP and gravity-gradient torques. Note that all of them are "differential" in nature, i.e. they are caused by differences in torques produced on individual parts of the spacecraft. Consequently, while not negligible, they are typically small. In addition, the "differential" nature of these torques reduces the effect of orbit state errors. The errors induce first order variations of the small torques, which makes them effectively second order with respect to the attitude state. The effect of attitude errors on the orbit propagation is different. Attitude dependent drag and SRP forces, while relatively small, are significant and are not "differential" in nature. More importantly, the attitude errors during the single (large) step taken by the orbit integrator cannot be treated as small. Indeed, while the orbit integrator may assume fixed attitude during the step, the satellite may undergo fast spin, going through several revolutions. For example, if the satellite was facing the Sun with its largest area at the beginning of the orbit integration step, but was maneuvered to face the Sun with its smallest area sometime during this step, the orbit propagation errors can be comparable with those induced by missing the eclipse boundary with cylindrical shadow model. Such errors are known to be potentially significant[3-6].

An alternative approach is to integrate a correction to the orbit trajectory, taking as many steps as there are attitude updates, using an Encke type algorithm. The Encke algorithm uses a reduced force model that only computes changes in attitude dependent forces unaccounted for during the orbit integration step and the differential two body acceleration[7]. This algorithm has been used successfully to correct for missed eclipse boundaries and has the advantage over fully coupled integration that a much simpler force model is used during the integration of the high frequency component[8]. The resulting accuracy is comparable to that of the fully coupled orbit and attitude trajectory integration, but requires a fraction of the computation time. Note that the algorithm can be implemented to only account for attitude dependent translational accelerations or to provide complete coupling with the integration of the attitude part of the state. In either case, the only assumption being made is that the differences in higher order perturbations between the main orbit integrator and the Encke correction algorithm are negligible.

FORMULATION

Two formulations for orbit integration with attitude dependent forces and fully coupled orbit and attitude integration will be developed in the sections to follow. In each case, the problem will be separated into two parts. In part one, the main orbit integration step performs the integration of the equations of motion for the orbit. In part two, the correction step, a correction to the orbit state computed by part one, is determined along with the optional integration of the attitude state. This process is illustrated in Figure 1.

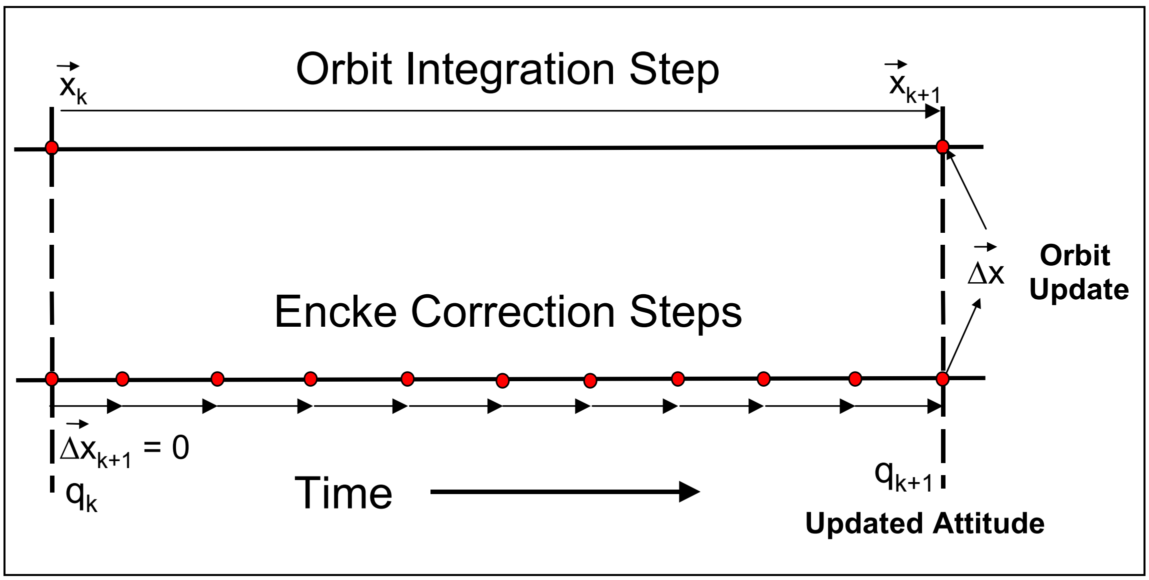

Figure 1. Encke correction timeline

The difference between the pair-wise formulations is the distribution of translational accelerations between the main orbit integration step and the correction step. The first formulation includes nominal accelerations due to atmospheric drag and solar radiation pressure as part of the main orbit integration model. This formulation will be referred to as the attitude correction formulation. In the second formulation, the computation of drag and SRP are only done in the correction step. This formulation will be referred to as the extended correction formulation.

Orbit integration model

The equations of motion for the main orbit integrator for the attitude correction formulation are given as

$$\ddot{\vec{\rho}}_{AC} = \vec{\nabla} U + \vec{a}_{Sun} + \vec{a}_{Moon} + \vec{a}_{drag_0} + \vec{a}_{SRP_0}, \tag{1}$$

while the orbital equations of motion for extended correction formulation are,

$$\ddot{\vec{\rho}}_{EC} = \vec{\nabla} U + \vec{a}_{Sun} + \vec{a}_{Moon}, \quad (2)$$

where $\vec{\nabla} U$ is the gradient of the gravitational potential function, $\vec{a}_{Sun}$ is the third body gravitational acceleration due to the Sun, $\vec{a}_{Moon}$ is the third body gravitational acceleration due to the moon, $\vec{a}_{drag_0}$ is the acceleration due to atmospheric drag and $\vec{a}_{SRP_0}$ is the acceleration due to solar radiation pressure. The zero subscripts on the accelerations due to atmospheric drag and SRP in Eq. (1) indicate that a constant reference area is used instead of the instantaneous area in the main orbit integrator. The form of the atmospheric drag and solar radiation pressure accelerations are given in Eq.(3) and (4) respectively,

$$\vec{a}_{drag} = -\frac{C_D A_D}{2M} \rho_{Atm} V_F \vec{V}_F, \quad (3)$$

$$\vec{a}_{SRP} = -\kappa \frac{C_R A_R}{2M} \frac{F_{mean}}{c\, R_{Sun}^2} \hat{R}_{Sun}, \quad (4)$$

where C_D is the coefficient of drag, A_D is the projected area of the spacecraft in the planet fixed velocity direction, $\vec{V}_F$, ρ_{Atm} is the atmospheric density, M is the mass of the satellite, C_R is the coefficient of radiation, A_R is the projected area of the spacecraft in the apparent Sun direction, $\vec{R}_{Sun}$, F_{mean} is the mean solar flux at 1 AU, c is the speed of light and $\kappa \in [0,1]$ is the fraction of visible solar disk.

Encke correction model

The dependence of the orbit trajectory on the satellite attitude is realized through the variation of the areas in Eqs. (3-4). During the correction phase of the trajectory integration, the areas are allowed to vary continuously. The differential equation for the trajectory update is therefore given by

$$\Delta\ddot{\vec{r}}_{AC} = \frac{\mu}{\rho^3}\left[\left(1 - \frac{\rho^3}{r^3}\right)\vec{r} - \Delta\vec{r}_{AC}\right] + \frac{A_D - A_{D0}}{A_{D0}}\vec{a}_{drag} + \frac{A_R - A_{R0}}{A_{R0}}\vec{a}_{SRP}, \quad (5)$$

for the attitude correction formulation and by

$$\Delta\ddot{\vec{r}}_{EC} = \frac{\mu}{\rho^3}\left[\left(1 - \frac{\rho^3}{r^3}\right)\vec{r} - \Delta\vec{r}_{EC}\right] + \vec{a}_{drag} + \vec{a}_{SRP}, \quad (6)$$

for the extended correction formulation. In Eqs. (5-6) A_D and A_R are the instantaneous attitude dependent areas exposed in the Earth-Fixed velocity and Sun directions respectively, $\vec{r}$ is the corrected state, $\vec{\rho}$ is the uncorrected state, μ is the gravitational parameter and the state correction $\Delta\vec{r}$ is defined as

$$\Delta\vec{r} = \vec{r} - \vec{\rho}. \tag{7}$$

The first term in Eq.(5-6) is the difference in the two body accelerations between the corrected and uncorrected trajectories. The second and third terms account for the improper modeling of the acceleration due to drag and SRP during the main integration step in the attitude correction formulation and complete accelerations due to drag and SRP in the extended correction formulation. The advantage of computing complete accelerations for drag and SRP in the extended correction formulation is that less expensive force model evaluations are required in the main orbit integration process. The disadvantage is that the step by step corrections to the main integration state will be larger and have a greater potential to violate the assumption that the effect of the corrections on higher order perturbations is negligible.

It is important to note that the reference trajectory for the Encke correction is a numerically integrated trajectory, not a two body trajectory as in the classical Encke formulation. To compute the position and velocity of the spacecraft at various times along the reference solution as required during the integration of the Encke correction, 7^{th} order Lagrangian interpolation is used in conjunction with a history of the computed orbit states.

Attitude Integration Model

The equations of motion for the attitude state, assuming rigid body motion, are

$$\dot{q} = \frac{1}{2}\Omega(\vec{\omega})\, q, \tag{8}$$

$$\dot{\vec{\omega}} = I^{-1}\left(\vec{T} + (I\vec{\omega}) \otimes \vec{\omega}\right), \tag{9}$$

where q is the attitude quaternion, $\vec{\omega}$ is the angular velocity in the body frame, I is the moment of inertia matrix and $\vec{T}$ is the sum of the torques acting on the satellite. The skew-symmetric matrix Ω is given as,

$$\Omega = \begin{bmatrix} 0 & \omega_z & -\omega_y & \omega_x \\ -\omega_z & 0 & \omega_x & \omega_y \\ \omega_y & -\omega_x & 0 & \omega_z \\ -\omega_x & -\omega_y & -\omega_z & 0 \end{bmatrix}. \tag{10}$$

To facilitate the verification of the attitude trajectory, no torques were added into the implementation of Eq.(9). A torque due to SRP was computed during the integration of the attitude to provide a more realistic value for the computation time, but it was set to zero. This step made the equations of motion for the attitude state independent of the equations of motion of the orbit state. It is important to note that the nullification of the torque is done purely to aid in the verification of the process and does not affect the formulation of the equations of motion or the matter in which they are implemented.

Implementation

The correction to the orbit has been implemented using the Runge-Kutta-Fehlberg 7-8 single step integration procedure[9]. This integration procedure has the benefit of automatic step size based error control and is therefore a good choice for cases where the rates of the attitude dynamics may vary significantly. The error control senses the attitude motion directly when the orbit correction and attitude are integrated simultaneously and indirectly through atmospheric drag and solar radiation pressure when the attitude is known a priori.

The main orbit integrator has also been implemented using the Runge-Kutta-Fehlberg 7-8 single step integration procedure. The Encke correction algorithm is simply applied at the end of integration steps over which a change in the attitude of the satellite has occurred. The updated state at the leading node of the integration process is then used as the starting state for the next integration step. It is important to note that computation of the Encke correction may require multiple steps of the numerical integrator to cover a single time step of the main orbit integrator. Implementation of the main orbit integrator using a multi-step integration procedure was not investigated, but would be complicated due the state updates at every integration step.

TEST CASES

A single initial condition set for a tumbling cylinder in LEO was run using varying truncations of the geopotential to evaluate the accuracy and computational efficiency of the correction algorithms. The initial conditions for the orbit state are given in Table 1 and the initial conditions for the attitude state are given in Table 2.

TABLE 1. ORBIT INITIAL CONDITIONS

Epoch	Apo Alt (km)	Peri Alt (km)	I (deg)	Ω (deg)	ω (deg)	ν (deg)	C_p	C_D
1 Jan 2001 00:00:00	430.0	360.0	98.5	0.0	0.0	0.0	1.6	2.0

TABLE 2. ATTITUDE INITIAL CONDITIONS

Epoch	(313) Euler Angles (deg)	$\vec{\omega}$ (deg/s)	I_{xx} (kg m^2)	I_{yy} (kg m^2)	I_{zz} (kg m^2)
1 Jan 2001 00:00:00	20.0 40.0 118.0	3.0 1.0 2.0	1000.0	1000.0	600.0

The Jacchia-Roberts atmospheric density model was used in the computation of the acceleration due to atmospheric drag. Environmental inputs to the atmospheric density model were held constant at the following values: F10.7 cm flux 180, average F10.7 cm flux 165 and a K_p geomagnetic index of 4.3. The geopotential model was EGM-96. The positions of the Sun and Moon were computed from the JPL DE405 ephemeris and the apparent position of the Sun was used in the computation of SRP. Errors can be introduced when the trajectory crosses a lighting condition boundary during the numerical integration process. To avoid these trajectory errors and the additional complexity in comparing trajectories, the test cases for this paper were computed without a shadow model for the Earth. This is equivalent to setting $\kappa = 1$ in Eq.(4). The spacecraft body was modeled as a cylinder with a radius of 1.5 m and a height of 3 m. The duration of the analysis span was one day. Orbit and attitude trajectories were computed simultaneously using a fully coupled formulation for use as a reference for the accuracy and efficiency of the trajectory computations performed using the correction formulations.

The frequency of the attitude dynamics modeled in this test case is much higher than the frequency of the orbit dynamics. To illustrate this point, the time history of the area to mass ratios for drag and solar radiation pressure are plotted over a one hour time span in Figures 2 and 3. The area to mass ratio for drag was computed based on the area of the cylinder facing the direction of the Earth-Fixed velocity vector. The area to mass ratio for SRP was computed based on the computed based on the area of the cylinder facing the direction of the apparent Sun. The dynamic values of the area to mass ratios were computed based on a specification of an area to mass ratio of 0.02 m^2/kg along the axis of the cylinder.

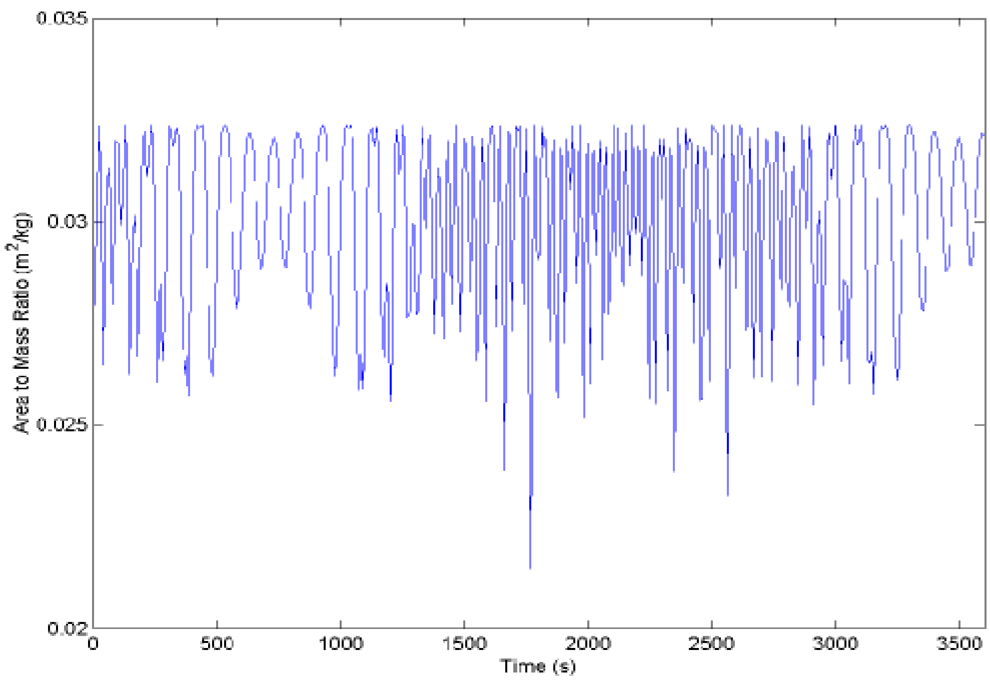

Figure 2. Area/Mass in drag direction

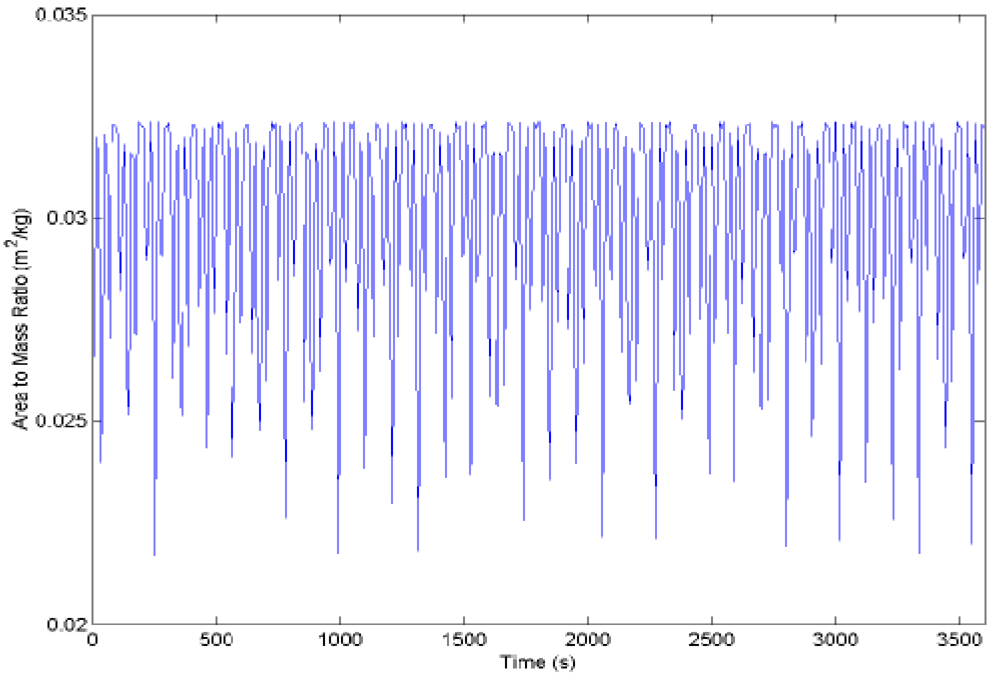

Figure 3. Area/Mass in Sun direction

Trajectories were computed using a variety of truncations of the geopotential. The results of these tests are given in Table 3. The listed computation times have been normalized by dividing by the amount of time required to compute the fully coupled solution. The maximum positional differences between the trajectory computed as part of the fully coupled system and those computed using the correction formulations are also given. The attitude solution was in near perfect agreement in all cases with a solution generated separately from the orbit model.

The information in Table 3 shows that the improvement in computational efficiency increases rapidly as the size of the geopotential increases. The data also indicates that the accuracy of trajectories generated using the correction formulations relative to the fully coupled solution is independent of the size of the gravity. It also appears that the extended correction formulation provides a slight computational advantage over the attitude correction formulation, as was expected, but does not significantly increase the errors in the trajectory. To verify that the assumption that size of the orbit state correction is small in reference to higher order perturbations, the maximum magnitude of the corrections to the position were computed for the case where a 4x4 geopotential was used. The maximum correction to the position was 2.2 cm for the attitude correction formulation and 6.2 cm for the extended correction formulation, thus justifying the assumption.

TABLE 3. EFFICIENCY AND ACCURACY FOR COUPLED INTEGRATION

Geopotential Truncation	Attitude Correction		Extended Correction	
	Computation Time	Maximum Diff (cm)	Computation Time	Maximum Diff (cm)
4x4	1.12	35	1.05	35
8x8	0.98	30	0.94	21
12x12	0.86	45	0.80	40
21x21	0.66	35	0.63	35
36x36	0.35	37	0.34	31
50x50	0.27	44	0.26	42

To understand the importance of modeling the attitude dependence of the accelerations due to drag and SRP, a reference is needed to determine what the effect would be of not modeling the variation in area. Matlab™ was used to compute the time average of the area to mass ratios, Figures 2-3, for drag, 0.030036662 m^2/kg, and solar radiation pressure, 0.0299 m^2/kg. The area to mass ratios were set to the average values and held constant for the duration of integration time span without use of a correction algorithm. The maximum difference in the resulting orbit trajectory from a trajectory computed simultaneously with the attitude using fully coupled equations of motion was 9.5 meters using a 50x50 degree and order truncation of the geopotential. If the area to mass ratio is changed by only 0.1%, the trajectory difference becomes 160 meters. This indicates the extreme sensitivity of this test case to the area projected in the direction of the Earth-Fixed velocity direction. It should also be noted that it was possible to compute the average areas in this case due to the fact that the attitude could be integrated independently. In the case where environmental torques depending on the position and velocity of the satellite are included, coupled trajectories would have to be generated to compute the average areas. The computation time for independent integration of the orbit and attitude motion was also computed. The results in normalized time units compared with the computation time for the extended correction formulation are shown in Figure 4. It is seen that time required for coupled integration using the correction algorithm approaches the lower limit set by independent integration as the size of the geopotential increases.

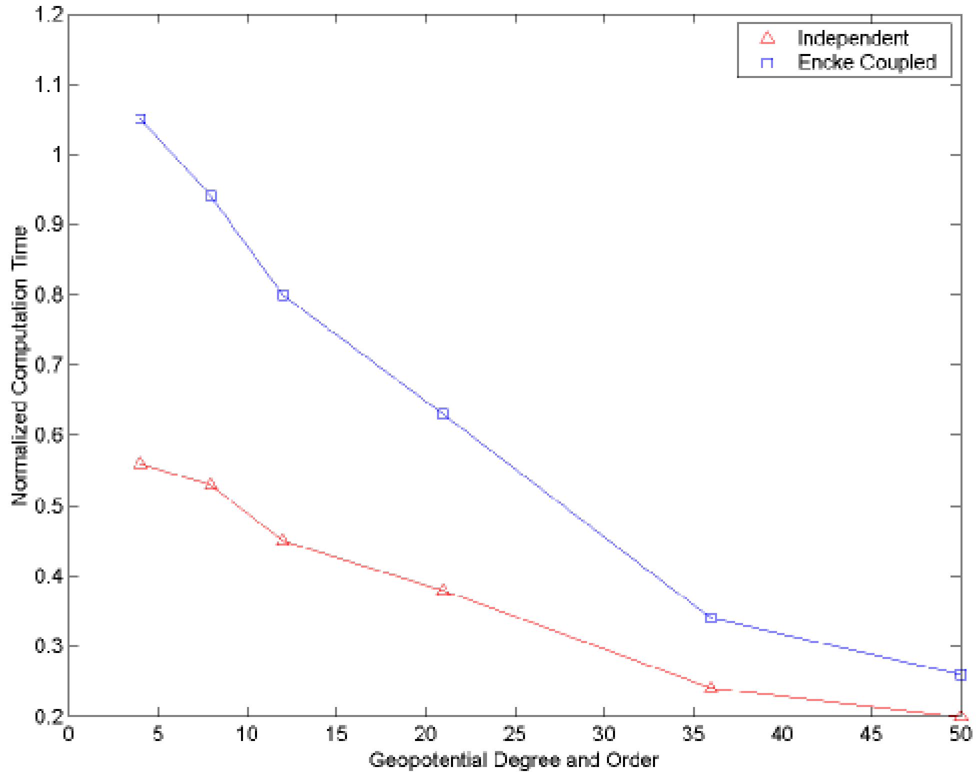

Figure 4. Computation time comparison

A separate test was performed for the case where the attitude motion is known and available as input to the orbit integration. In this case, the attitude is no longer part of the integrated state and therefore does not have a direct effect on the step size selection of the integrator. The effect of the attitude is now sensed indirectly by the integrator via the attitude effects on the accelerations due to drag and SRP. The ability of the integrator to sense changes in the accelerations due to drag and SRP will be dependent on the size of those changes relative to the other computed accelerations. In addition to the two correction based formulations, trajectories were also generated using only the main orbit integrator, but allowing the projected areas to change based on the attitude. This formulation will be referred to as the variable attitude formulation. Each case was run twice for each formulation. In the first run, the integration step size selection was performed purely based on the relative error sensed in the integrated state. In the second run, the step size was restricted to be at most 10 seconds for integration of the attitude dependent effects. In the case where the attitude dependent accelerations were computed as part of the main orbit integration, the variable attitude formulation, this means that entire integration process has a capped step size. In the cases where an Encke correction is used, only the correction process has a capped step size. The results of this test are given in Table 4. The listed computation times have been normalized by dividing by the amount of timed required to compute a solution with varying area in the main orbit integration loop with the step size capped at 10 seconds. The maximum positional differences are measured based on the trajectory computed as part of the fully coupled system.

TABLE 4. EFFICIENCY AND ACCURACY FOR VARYING AREA

Geopotential Truncation	Step Size Selection	Attitude Correction		Extended Correction		Variable Attitude	
		Comp. Time	Maximum Diff (m)	Comp. Time	Maximum Diff (m)	Comp. Time	Maximum Diff (m)
4x4	Error Control 10 sec max.	0.77 1.11	26 0.7	0.61 1.03	26 0.7	0.29 1.0	35 0.45
8x8	Error Control 10 sec max.	0.74 1.05	25 0.8	0.66 0.97	25 0.8	0.29 1.0	66 0.5
12x12	Error Control 10 sec max.	0.63 0.91	25 0.7	0.59 0.85	22 0.8	0.30 1.0	47 0.5
21x21	Error Control 10 sec max.	0.47 0.61	22 0.6	0.43 0.58	25 0.6	0.30 1.0	40 0.5
36x36	Error Control 10 sec max.	0.31 0.39	25 1.6	0.30 0.38	23 1.6	0.29 1.0	53 0.5
50x50	Error Control 10 sec max.	0.26 0.31	22 0.7	0.24 0.29	25 0.8	0.29 1.0	74 0.5

The data in Table 4 indicates that the use of the Encke correction procedure increases the sensitivity to attitude dependent accelerations. The differences from the fully coupled trajectory in this example are about half the size of the differences seen when the attitude is allowed to vary in the main integration step. The significant differences found for the trajectories computed using only the standard error control are indicative of the sensitivity of the trajectories to the proper computation of the drag area. In these cases, it is suspected that the sampling of the area to mass ratio during the integration procedure was not fine enough to ensure that an accurate average area could be realized. The results of the test cases where the step size was capped at 10 seconds show similar levels of accuracy for all cases, indicating adequate sampling of the attitude motion, but the use of the correction procedures allows the computation to be done in significantly less time for geopotential sizes above 12^{th} degree and order. As was the case for the coupled integration test cases, the difference in accuracy between the attitude correction and attitude extended correction is not significant.

CONCLUSIONS

Two algorithms for computing coupled orbit and attitude trajectories using a main orbit integration step paired with an Encke type correction step have been developed. Both algorithms, which differ in the distribution of accelerations between the main orbit integrator and the correction step, have been shown to be computationally efficient for both simultaneous integration of orbit and attitude trajectories and integration of orbit trajectories with attitude dependence when the attitude is known. The new algorithms are seen to be more computationally efficient than the integration of fully coupled equations when the truncation of the geopotential is beyond degree and order 12 for the test case analyzed. The exact point of tradeoff will vary with the relative weight of other forces being modeled, but the trend of the reduction in computation time as the time required for computation of the gravitational acceleration begins to dominate the computation time for the sum of the accelerations. A similar improvement in computational efficiency is seen for the case when the attitude motion is known and only the orbit trajectory has to be computed.

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