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ANTI-SATELLITE ENGAGEMENT VULNERABILITY

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This work uses simple orbital dynamics to initially assess the vulnerability of a satellite to a missile. This vulnerability can be represented as an engagement volume for a specific missile relative to its launch platform. Alternately, the vulnerability can be represented as a geographical footprint relative to satellite position that encompasses all possible launcher locations for a specific missile. An interceptor missile's final two-body orbital energy is determined from its burnout altitude and velocity. Assuming a ballistic trajectory from launch, the burnout energy is used to find the interceptor's initial velocity for a given launcher's altitude. Three engagement solutions are then found that account for spherical earth rotation. One solution finds the maximum missile range for an ascent-only trajectory while another solution accommodates a descending trajectory. In addition, the ascent engagement for the descending trajectory is used to depict a rapid engagement scenario. These preliminary solutions are formulated to address ground-, sea-, or air-launched missiles. The approach presented is not limited to satellites and is equally valid for determining vulnerability to mortars, artillery, SCUDs, Surface-to-Air missiles, etc.

INTRODUCTION

There is a growing concern about low earth orbit vulnerability based upon recent successful intercepts of orbiting satellites. A hit-to-kill anti-satellite missile was launched from the Xichang Space Center by the People's Republic of China on January 11, 2007, using a direct-ascent profile to intercept their inoperative FengYun 1C polar-orbiting weather satellite[1]. In 2008 the United States modified an SM-3 missile, launched it from the USS Lake Erie, and used a descending trajectory to successfully intercept the malfunctioning USA 193 satellite while also facilitating the decay of the resulting orbital debris[2]. Previously, in a 1985 test, the U.S. launched an anti-satellite missile (ASM-135) from an F-15 Eagle and hit the Solwind P78-1 satellite[3]. The FengYun 1C interceptor was ground based, the SM-3 missile was sea launched, and the ASM-135 was air launched. Based on this history, it is beneficial to have a vulnerability assessment tool capable of addressing ground-, sea-, and air-launched missiles for both ascending and descending intercept trajectories.

An interceptor has three phases of flight: the boost phase (launch to burnout), the midcourse phase (coast or free-flight), and the terminal phase (exoatmospheric intercept guidance to precisely strike the target satellite)[4]. Several simplifying assumptions can be made to initially assess the threat. If one has an Earth-Centered, Earth-Fixed (ECEF) ephemeris for a given missile, then the final orbital energy in the Earth-Centered Inertial (ECI) frame is deduced from the state at burnout; staging, mass loss, and atmospheric drag effects can all be included to arrive at the proper burnout state. In the ECI frame, this final

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orbital energy is the total cumulative energy imparted to the missile during boost. For preliminary analysis this energy can be assumed attainable and constant regardless of launch direction, trajectory, dynamic losses, or guidance scheme. An initial determination of vulnerability can then be made by assuming the interceptor is given all its energy at launch and follows a ballistic trajectory to the target, thereby approximating all three phases as one simple phase. The resulting trajectory begins at the launcher altitude and follows a simple two-body dynamical path until reaching the target satellite. The mathematical equations for such free-flight representation are explained by Bate, et al[5], in describing ballistic missile trajectories for a spherical, rotating earth. This work modifies their approach because the target is in orbit and the launcher can be located above the earth's surface. The effect of earth rotation is addressed by transforming from the ECEF frame to an ECI frame as needed.

An engagement volume relative to the interceptor's launch platform is determined by creating a sufficient family of forward trajectories based on different initial launch azimuths and elevations; this volume is sometimes referred to as a "kill basket." Entry and exit times through this volume define the bounds of satellite vulnerability for a specific launcher and specific type of missile on an ascending or descending intercept profile. Alternately, a vulnerability region can be represented as a geographical footprint relative to satellite position that encompasses all possible launcher locations for a specific interceptor. This is accomplished by creating a sufficient family of backward trajectories from the target satellite using maximum range equations for ascending or descending intercepts based on the missile's burnout energy.

These volumes and footprints are created by simply knowing the interceptor's final ECEF or ECI energy and are only approximations due to the aforementioned assumptions. The intercept trajectory time of flight from launch to burnout will be underestimated because the actual powered flight segment takes longer than its ballistic representation. The intercept range from launch to burnout will be overestimated because the actual powered flight covers less ground distance than its ballistic representation. This causes the preliminary analysis to be conservative, creating an oversized volume or footprint. Because of this, the results are adequate for understanding and visualizing the threat, as well as determining if more detailed analysis is required. The actual missile fly-out profile for a specific engagement would be required to more accurately assess the threat.

FUNDAMENTAL COMPUTATIONS

This section shows how to find the necessary orbital parameters, planar angles, and time of flight that define a specific missile's intercept trajectory. We start with the missile's burnout altitude (alt_bo) and velocity (V_bo), convert to the burnout radius (r_bo) and determine the missile's total energy ξ

$$\xi = \frac{V\_bo^2}{2} - \frac{\mu}{r\_bo} \tag{1}$$

where the missile's trajectory is considered ballistic from launch with an orbital semi-major axis of

$$a = \frac{-\mu}{2 \cdot \xi} . \tag{2}$$

The energy considered for this type of scenario should be negative to ensure a closed orbit. If it is not, then the semi-major axis (a) is arbitrarily set to 10e8 km.

The launcher's altitude (alt_launcher) and the target satellite's altitude (alt_sat) are converted to earth-centered r_launcher and r_sat. The true anomaly of the missile trajectory at the launcher is defined as

$$\nu\_launcher = \operatorname{acos}\left(\frac{\frac{1 - ecc^2}{r\_launcher}\cdot a - 1}{ecc}\right) \tag{3}$$

And the true anomaly of the ascending missile trajectory at the satellite becomes

$$\nu\_sat = \operatorname{acos}\left(\frac{\frac{1 - ecc^2}{r\_sat}\cdot a - 1}{ecc}\right) \tag{4}$$

The variable alltraj is set to 1 if all trajectories are permissible (alltraj=1). If so, the square of the missile's orbital eccentricity is computed by differentiating the range equation

$$\alpha = (\pi - \nu\_launcher) + (\pi - \nu\_sat) \tag{5}$$

with respect to eccentricity. The maximum value for α occurs when

$$ecc^2 = \frac{2\cdot r\_launcher\cdot r\_sat - 3\cdot a\cdot r\_sat + 4\cdot a^2 - 3\cdot a\cdot r\_launcher}{a\cdot(r\_sat - 4\cdot a + r\_launcher)} \tag{6}$$

and the interceptor reaches the satellite's altitude post-apogee. For an ascent-only intercept profile (alltraj=0), the intercept can occur anywhere prior to the missile's apogee where the range equation is defined as

$$\alpha = \nu\_sat - \nu\_launcher \tag{7}$$

and the maximum range occurs when the missile is at apogee ($\nu\_sat=\pi$) resulting in

$$ecc^2 = \left(\frac{r\_sat}{a} - 1\right)^2 \tag{8}$$

By using the eccentricity from Equation (6) in the range equation (7) one also gets the intercept solution for the ascending portion of the maximum range trajectory (alltraj=2). This result can be used to approximate a quick-ascent engagement trajectory. Such a trajectory could be used for a rapid engagement, thus reducing interceptor flight time and diminishing timely detection and response.

It is possible under certain conditions for Equation (6) to produce a negative result, meaning the missile has enough energy to intercept the target anywhere in its orbit at the prescribed altitude. For such a case the following parameters are set

$$a = \frac{r\_launcher + r\_sat}{2} \tag{9a}$$

$$ecc = \frac{r\_sat}{a} - 1 \tag{9b}$$

$$\alpha = \pi \tag{9c}$$

$$\beta = \frac{\pi}{2} - acos\left(\frac{r\_launcher}{r\_sat}\right) \tag{9d}$$

$$TOF = \frac{\pi \cdot a^{\frac{3}{2}}}{\sqrt{\mu}} \tag{9e}$$

where α is the missile's earth-centric free-flight range angle, β is the satellite's off-nadir angle to the launcher, and TOF is the time of flight from launch to intercept.

If the missile has insufficient energy to reach the target altitude, then the eccentricity will be greater than one and no engagement is possible. For such a case the parameters α, β, and TOF are set to some very small tolerance and the eccentricity is set to one.

The direct range from launcher to satellite is found from the law of cosines

$$range = \sqrt{r\_launcher^2 + r\_sat^2 - 2 \cdot r\_launcher \cdot r\_sat \cdot \cos(\alpha)} \tag{10}$$

to produce the satellite's off-nadir angle

$$\beta = asin\left(r\_launcher \cdot \frac{\sin(\alpha)}{range}\right) \tag{11}$$

The time of flight from launcher to satellite is found by differencing the time from perigee to launcher and perigee to satellite using eccentric anomaly.

$$sin\_E\_launcher = \frac{\sin(\nu\_launcher) \cdot \sqrt{1 - ecc^2}}{1 + ecc \cdot \cos(\nu\_launcher)} \tag{12a}$$

$$cos\_E\_launcher = \frac{ecc + \cos(\nu\_launcher)}{1 + ecc \cdot \cos(\nu\_launcher)} \tag{12b}$$

$$E\_launcher = atan2(cos\_E\_launcher, sin\_E\_launcher) \tag{12c}$$

$$TOF\_launcher = (E\_launcher - ecc\cdot sin\_E\_launcher)\cdot\sqrt{\frac{a^3}{\mu}} \tag{12d}$$

$$sin\_E\_sat = \frac{\sin(\nu\_sat)\cdot\sqrt{1 - ecc^2}}{1 + ecc\cdot\cos(\nu\_sat)} \tag{13a}$$

$$cos\_E\_sat = \frac{ecc + \cos(\nu\_sat)}{1 + ecc\cdot\cos(\nu\_sat)} \tag{13b}$$

$$E\_sat = atan2(cos\_E\_sat\ , sin\_E\_sat) \tag{13c}$$

$$TOF\_sat = (E\_sat - ecc\cdot sin\_E\_sat)\cdot\sqrt{\frac{a^3}{\mu}} \tag{13d}$$

If all trajectories are permissible (alltraj=1) then the time of flight from launcher to satellite becomes

$$TOF = 2\cdot\pi\cdot\sqrt{\frac{a^3}{\mu}} - TOF\_sat - TOF\_launcher \tag{14a}$$

else it is

$$TOF = TOF\_sat - TOF\_launcher \tag{14b}$$

In summary, given V_bo, alt_bo, alt_launcher, alt_sat, and alltraj, the above equations yield a, ecc, α, β, and TOF in an inertial frame.

BALLISTIC REPRESENTATION OF POWERED FLIGHT

The actual boost phase of an interceptor includes mass loss, atmospheric drag, and possibly staging. Given an Earth-Centered, Earth-Fixed (ECEF) ephemeris for this phase, the final orbital energy in this frame is computed from the burnout state. The interceptor's powered flight is represented as a ballistic trajectory by matching the burnout conditions and then propagating backwards to the launch altitude. A vulnerability determination begins by assuming the interceptor is given all its energy at launch and follows a ballistic trajectory to the target, thereby approximating all three phases (boost, coast, and terminal) as one simple exoatmospheric phase. The resulting trajectory begins at the launcher altitude and follows a simple two-body dynamical path until reaching the target satellite.

Figure 1 depicts two trajectories. The solid line represents a thrusting missile in a gravity turn and accounts for continuous mass loss as well as drag effects. The dotted line shows an exoatmospheric ballistic trajectory that precisely matches the interceptor's burnout altitude, velocity, and flight-path angle. As nominally shown in Figure 1, the interceptor range from launch to burnout will be overestimated because the actual powered flight covers less ground than its ballistic representation. This causes the preliminary analysis to be conservative, creating an oversized volume or footprint. If a satellite is outside the volume, or a launch platform is outside the footprint, then there is no threat. If a satellite is inside the volume, or a launch platform is inside the footprint, then further analysis should be considered to determine the feasibility of an intercept; the nearer to the boundary of the volume or footprint, the less likely it is that the interceptor can actually reach the target satellite.

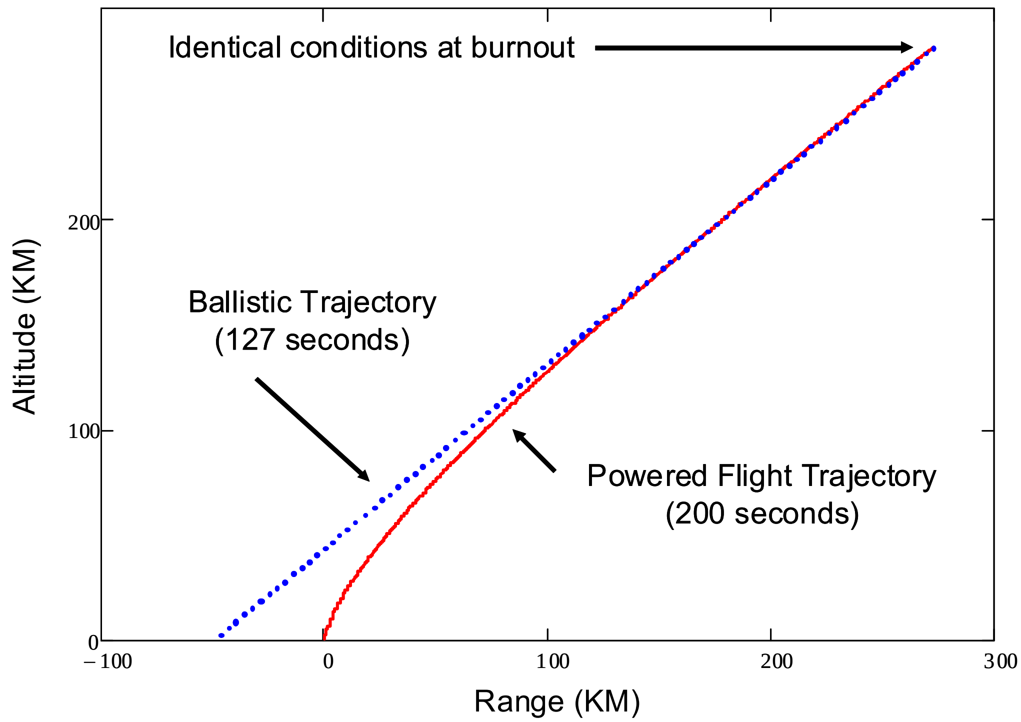

Figure 1. Ballistic Trajectory versus Powered Flight Trajectory.

DETERMINING INITIAL MISSILE VELOCITY

This section details how the earth-fixed missile velocity (Vm) can be determined from its state vector at or after burnout. Given such position (R_ECEF_bo) and velocity (V_ECEF_bo) vectors in the ECEF frame, the first step is conversion to the ECI frame. By assuming the position vector is at the reference longitude it is not necessary to know the associated time and, therefore, no rotation to the ECI frame is needed; R_ECI is simply set to R_ECEF_bo. The ECI velocity becomes

$$V\_ECI = V\_ECEF\_bo + \begin{pmatrix} 0 \\ 0 \\ \omega\_earth \end{pmatrix} \times R\_ECEF\_bo \tag{15}$$

where ω_earth is the earth's rotation rate.

The equations outlined in the Fundamental Computations section can then be used to represent a ballistic trajectory to the launcher's altitude. With the ECI launch vectors and

Time-of-Flight (TOF) now known, the missile's earth-fixed velocity vector can be found from the equations

$$\theta = \omega\_earth \cdot TOF \tag{16}$$

$$V\_ECEF\_launch = \begin{pmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \left[V\_ECI\_launch - \begin{pmatrix} 0 \\ 0 \\ \omega\_earth \end{pmatrix} \times R\_ECI\_launch \right] \tag{17}$$

Vm is simply the magnitude of V_ECEF_launch (muzzle velocity). Although this velocity was determined from a single, specific, launch trajectory, returning to an earth-fixed frame allows it to be used for multiple trajectories. Obviously, if the burnout vectors are already given in the ECI frame then the first step in this process is not needed.

ROTATING EARTH COMPUTATIONS FOR MISSILE ENGAGEMENT VOLUME

This section shows how to incorporate a rotating earth by repeatedly computing the impact latitude (lat) and longitudinal offset (ΔN) for a given satellite altitude (alt_sat) from the missile's launch platform for a family of launch azimuths ($-\pi < AZ < \pi$). In addition to AZ, we must know the missile launcher latitude (lat_launcher) and altitude (alt_launcher), burnout velocity (V_bo), burnout altitude (alt_bo), and preferred trajectory (alltraj). To prevent mathematical difficulties associated with inverse trigonometric functions it is important to always examine numerators and denominators to ensure limits are not exceeded as a result of numerical imprecision. As an example, the latitude of the launcher can not be precisely at a pole; if so then a very small offset is introduced such that ($-\pi/2 < lat\_launcher < \pi/2$)

The first step is to perform the basic inertial computations of the previous section and determine preliminary values for a, ecc, α, β, and TOF. If α is equal to π then the missile has enough energy to intercept the target anywhere in its orbit at the prescribed altitude and

$$\Delta N = \pi \cdot \text{sign}(AZ) \tag{18}$$

$$lat = \frac{\pi}{2} - |AZ| \tag{19}$$

If α is less than π then iteration is required to assess the missile trajectory that results from a rotating earth. The earth-fixed missile velocity (Vm) is a combination of the interceptor's ballistic velocity deduced from the energy (Equation 1) and the topocentric velocity of the launch platform (V_boost) aligned to maximize Vm.

$$Vm = \sqrt{\frac{2 \cdot \mu}{r\_launcher} - \frac{\mu}{a}} + V\_boost \tag{20}$$

V_boost alignment is achieved by having the launching aircraft perform a pitch-up maneuver along the interceptor's desired azimuth. The rotating earth missile velocity (Vsez) is initialized to Vm. The eastward component of earth-induced velocity (V0) is

$$V0 = \cos(lat\_launcher) \cdot r\_launcher \cdot \omega\_earth \tag{21}$$

With Vm and V0 known, iteration can begin. The basic inertial computations of the previous section are accomplished with V_bo set to Vsez, and alt_bo set to alt_launcher to produce intermediate values for a, ecc, α, β, and TOF. To prevent mathematical difficulties, the absolute value of the earth central angle α must not exactly equal π or zero . If it does, then a slight adjustment is made to α. The angular momentum h is determined from

$$h = \sqrt{\left| a \cdot \left[1 - (ecc)^2 \right] \cdot \mu \right|} \tag{22}$$

The launch elevation angle ϕ is determined from

$$\phi = acos\left(\frac{h}{r\_launcher \cdot Vsez} \right) \tag{23}$$

To prevent mathematical difficulties h should be clipped if it is greater than the absolute value of the denominator in Equation (23). The intercept latitude (lat_sat) is

$$lat\_sat = \frac{\pi}{2} - acos\left(\cos(lat\_launcher) \cdot \cos\left(\frac{\pi}{2} - \alpha \right) \cdot \cos(AZ) + \sin(lat\_launcher) \cdot \sin\left(\frac{\pi}{2} - \alpha \right) \right) \tag{24}$$

Based on the new launch elevation angle, Vsez is recomputed for the next iteration

$$Vs = -Vm \cdot \cos(\phi) \cdot \cos(AZ) \tag{25a}$$

$$Ve = Vm \cdot \cos(\phi) \cdot \sin(AZ) + V0 \tag{25b}$$

$$Vz = Vm \cdot \sin(\phi) \tag{25c}$$

$$Vsez = \sqrt{Vs^2 + Ve^2 + Vz^2} \tag{25d}$$

Iteration ends when TOF converges to the user's tolerance. Longitudinal offset (ΔN) and impact latitude (lat), are then computed accounting for the earth rotation rate ω_earth

$$dN = acos\left(\frac{\cos(\alpha) - \sin(lat\_sat) \cdot \sin(lat\_launcher)}{\cos(lat\_sat) \cdot \cos(lat\_launcher)} \right) \tag{26}$$

$$\Delta N = dN \cdot sign(AZ) - \omega\_earth \cdot TOF \tag{27}$$

$$lat = lat\_sat \tag{28}$$

If the iterations exceed 50 steps it is assumed the missile has insufficient energy to reach the target altitude. For this insufficient energy case

$$\Delta N = 0 \tag{29}$$

$$lat = lat\_launcher \tag{30}$$

An engagement volume can be inferred that shows all possible satellite locations that are vulnerable to missile intercept. To accomplish this, a family of equally spaced azimuths is created as well as a family of satellite altitudes. The computations in this section are accomplished for each azimuth (AZ) at all specified target altitudes (alt_sat) coupled with the launcher latitude (lat_launcher) and altitude (alt_launcher), burnout velocity (V_bo), burnout altitude (alt_bo) and preferred trajectory (alltraj). The resulting family of points (ΔN, lat, alt_sat) are all located on the surface of the engagement volume and can be used to create a three-dimensional grid (convex hull) that represents the region of vulnerability.

ROTATING EARTH COMPUTATIONS FOR SATELLITE VULNERABILITY FOOTPRINT

A terrestrial footprint can be created that shows launcher geographical locations that could threaten the satellite in its current orbital position. To accomplish this, equally spaced, satellite-centric, intercept azimuths are created. Each azimuth (γ) is coupled with the satellite latitude (lat_sat) and altitude (alt_sat), burnout velocity (V_bo), burnout altitude (alt_bo), launcher altitude (alt_launcher), and preferred trajectory (alltraj) to determine latitudinal and longitudinal displacement from the satellite sub-point. As previously described, to prevent mathematical difficulties associated with inverse trigonometric functions it is important to always examine numerators and denominators to ensure limits are not exceeded as a result of numerical imprecision. As an example, the satellite can not be directly over a pole; if so, a very small offset is introduced such that ($-\pi/2 < lat\_sat < \pi/2$)

The first step is to perform the basic inertial computations previously described and determine preliminary values for a, ecc, α, β, and TOF. If α is equal to π then the missile has enough energy to intercept the target anywhere in its orbit at the prescribed altitude and

$$\Delta N = \pi \cdot \text{sign}(\gamma) \tag{31}$$

$$lat = \frac{\pi}{2} - |\gamma| \tag{32}$$

If α is less than π then iteration is required to assess the missile trajectory that results from a rotating earth. The earth-fixed missile velocity (Vm) is defined by Equation 17 and the rotating earth missile velocity (Vsez) is initialized to Vm.

To prevent mathematical difficulties, the absolute value of the intercept azimuth (γ) must not exactly equal π or zero . If it does, then a slight adjustment is made to γ. The basic inertial computations are accomplished with V_bo set to Vsez, and alt_bo set to zero to produce intermediate values for a, ecc, α, β, and TOF. The angular momentum h is determined from Equation 22 and the launch elevation angle ϕ is determined from Equation 23. The launcher latitude (lat_launcher) is determined from the spherical law of cosines as

$$\text{lat\_launcher} = \frac{\pi}{2} - \text{acos}(\cos(\alpha)\cdot\sin(\text{lat\_sat}) + \sin(\alpha)\cdot\cos(\text{lat\_sat})\cdot\cos(\gamma)) \tag{33}$$

Based on the new launcher latitude, the launch azimuth (AZ) is recomputed as

$$AZ = -\text{sign}(\gamma)\cdot\text{acos}\left(\frac{\sin(\text{lat\_sat}) - \sin(\text{lat\_launcher})\cdot\cos(\alpha)}{\cos(\text{lat\_launcher})\cdot\sin(\alpha)}\right) \tag{34}$$

The eastward component of earth-induced velocity (V0) becomes

$$V0 = \cos(\text{lat\_launcher})\cdot\text{r\_launcher}\cdot\omega\_\text{earth} \tag{35}$$

and the rotating earth missile velocity (Vsez) is

$$Vs = -Vm\cdot\cos(\phi)\cdot\cos(AZ) \tag{36a}$$

$$Ve = Vm\cdot\cos(\phi)\cdot\sin(AZ) + V0 \tag{36b}$$

$$Vz = Vm\cdot\sin(\phi) \tag{36c}$$

$$Vsez = \sqrt{Vs^2 + Ve^2 + Vz^2} \tag{36d}$$

Longitudinal offset (ΔN) and footprint latitude (lat), are then computed using the earth rotation rate ω_earth

$$dN = \text{acos}\left(\frac{\cos(\alpha) - \sin(\text{lat\_sat})\cdot\sin(\text{lat\_launcher})}{\cos(\text{lat\_sat})\cdot\cos(\text{lat\_launcher})}\right) \tag{37}$$

$$\Delta N = dN\cdot\text{sign}(\gamma) + \omega\_\text{earth}\cdot TOF \tag{38}$$

$$\text{lat} = \text{lat\_launcher} \tag{39}$$

If the iterations exceed 50 steps, then it is assumed the missile has insufficient energy to reach the target altitude. For this insufficient energy case

$$\Delta N = 0 \tag{40}$$

$$\text{lat} = \text{lat\_sat} \tag{41}$$

The resulting family of points on the earth's surface (ΔN, lat) can be plotted to show the footprint beneath the satellite. It is important to note that this footprint has accounted for the earth rotation as well as the missile's time of flight. The points represent the current location of possible launchers although the interceptor was launched before the satellite arrived at its present position.

EXAMPLE OF PRELIMINARY RESULTS

The first step is to determine the earth-fixed missile (muzzle) velocity Vm from a launch ephemeris or profile. The entire ephemeris is not needed, only the state vectors and times at launch and burnout in the ECEF (or ECI) frame: For the following conditions

$$\text{R\_launch\_ECEF} = \begin{pmatrix} 3.358348733 \times 10^{3} \\ 4.002324168 \times 10^{3} \\ 3.658348517 \times 10^{3} \end{pmatrix} \text{ km} \tag{42a}$$

$$\text{R\_burnout\_ECEF} = \begin{pmatrix} 3.368166448 \times 10^{3} \\ 4.074471725 \times 10^{3} \\ 3.718730716 \times 10^{3} \end{pmatrix} \text{ km} \tag{42b}$$

$$\text{V\_burnout\_ECEF} = \begin{pmatrix} 0.054871826 \\ 2.092102911 \\ 1.725722584 \end{pmatrix} \text{ km/sec} \tag{42c}$$

and a powered-flight time of 100 seconds, the computed muzzle velocity Vm is 2.99572978 km/sec. This velocity now characterizes the capability of this particular missile and can be applied to any terrestrial launch location. Applying the fundamental equations produces the following figures:

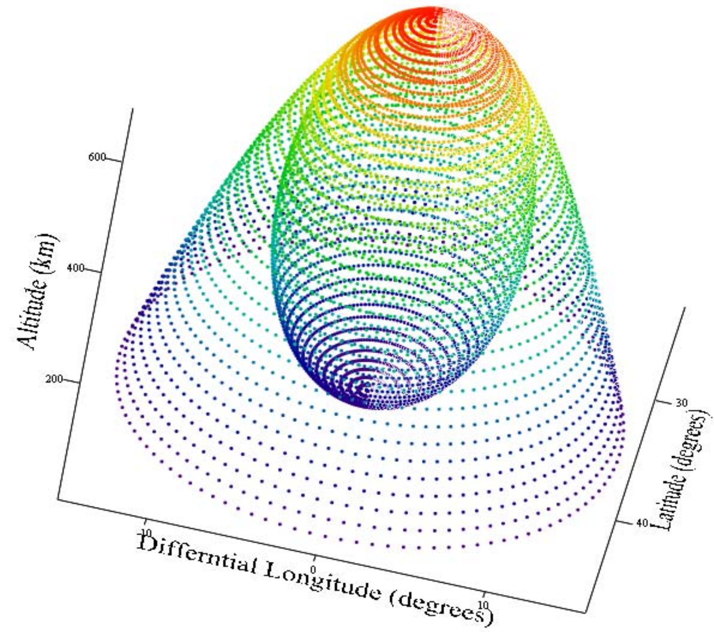

Figure 2. Ascending and Descending Intercept Volumes.

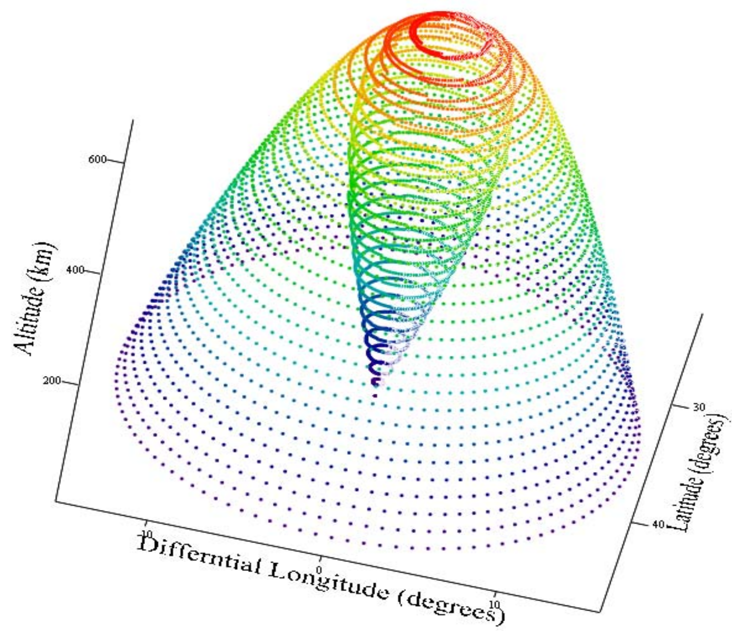

Figure 3. Descending and Rapid Engagement Intercept Volumes.

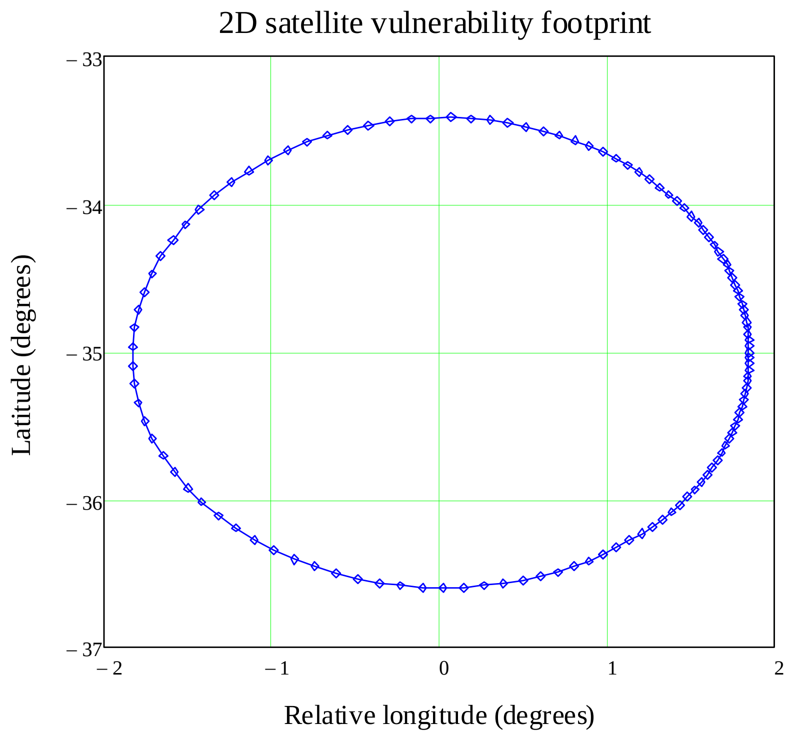

Figure 4. Geographical footprint of launcher locations relative to satellite position

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In Figure 2 the outer shell defines the engagement volume for a descending intercept and the inner shell shows the vulnerability to an ascent-only intercept. In Figure 3 the outer shell defines the engagement volume for a descending intercept and the inner shell shows the vulnerability to the ascending portion of the same intercept. This inner volume represents a quick-ascent engagement trajectory, one in which the flight time is short and timely threat detection diminished. Figure 4 shows the geographical footprint relative to satellite position that encompasses all possible launcher locations for a specific interceptor. Figures 5-8 are vulnerability depictions as implemented in the Analytical Graphics, Inc., SOLARA tool.

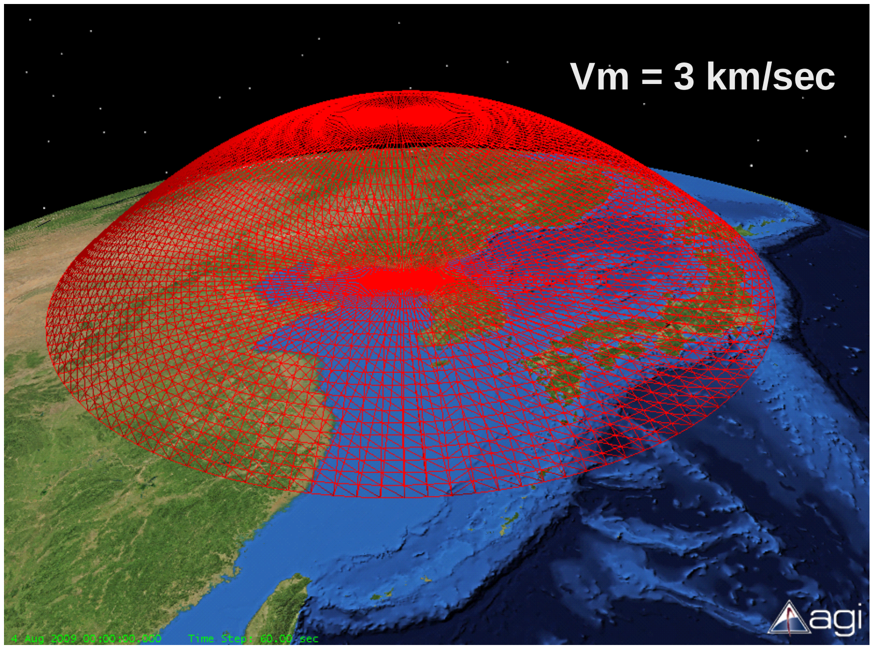

Figure 5. SOLARA maximum range descending

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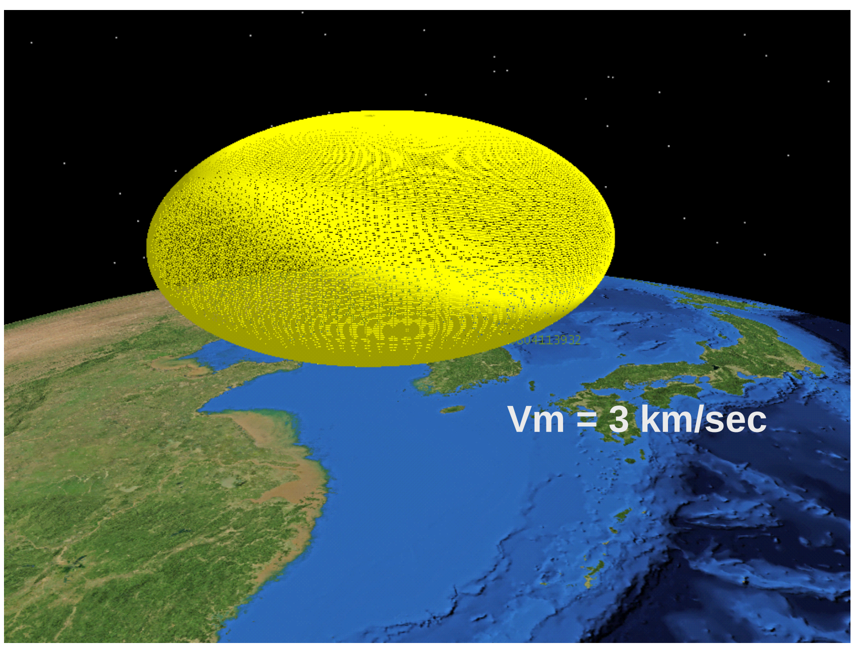

Figure 6. SOLARA maximum range ascending

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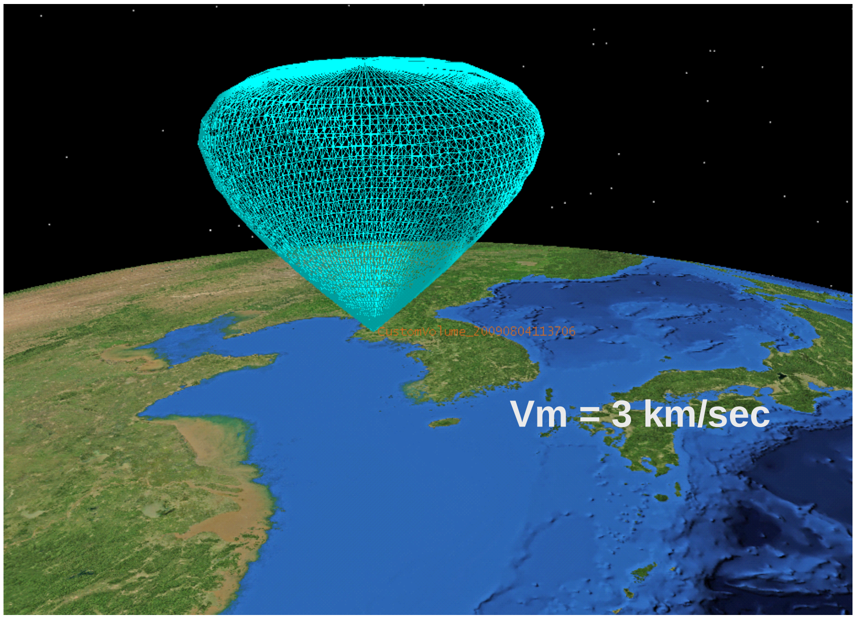

Figure 7. SOLARA rapid ascent

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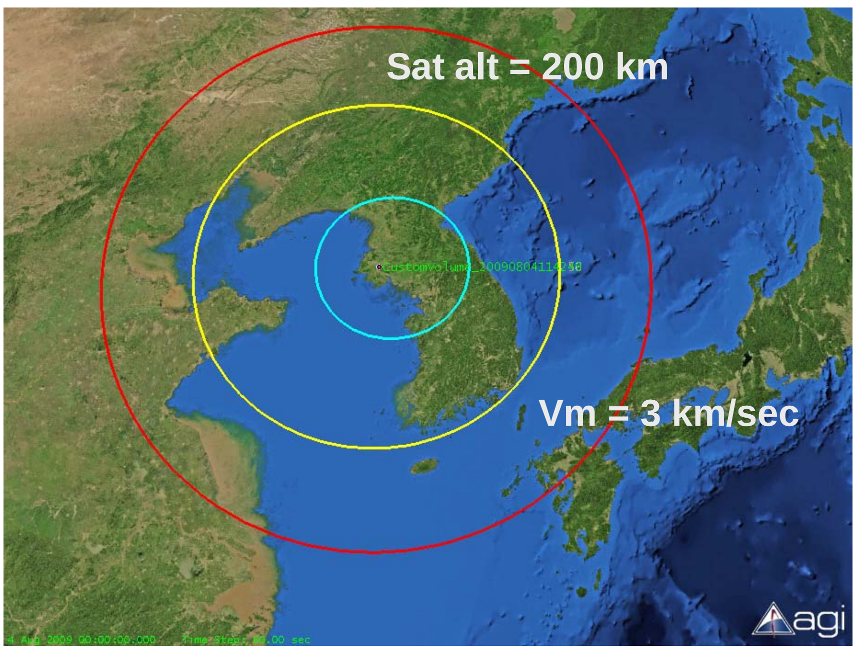

Figure 8. SOLARA geographical footprints of locations relative to satellite position

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CONCLUSION

Simple orbital dynamics were used to initially assess the vulnerability of a satellite to a missile. For a specific interceptor this vulnerability was represented as an engagement volume relative to its launch platform and also as a geographical footprint relative to satellite position that encompassed all possible launcher locations. This approach assumed a ballistic trajectory from launch by working backwards from the interceptor's burnout state. Three engagement solutions were found that account for spherical earth rotation. The first solution found the maximum interceptor range for an ascent-only trajectory while the second accommodated a descending trajectory. The third solution used the ascending potion of the descending trajectory to depict a rapid engagement scenario. All three solutions were formulated to address ground-, sea-, or air-launched missiles. The results are adequate for understanding and visualizing the threat, as well as determining if more detailed analysis is required. The volume or footprint will be oversized because the actual powered flight covers less ground distance than its ballistic representation. A more accurate threat assessment would require a precise missile fly-out profile tailored to a specific engagement. The approach shown here is not limited to satellites and is equally valid for determining vulnerability to mortars, artillery, SCUDs, Surface-to-Air missiles, etc.

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NOMENCLATURE

a	=	semi-major axis
alltraj	=	variable to toggle intercept trajectory (ascending or descending)
alt_bo	=	interceptor burnout altitude
alt_launcher	=	launcher altitude
alt_sat	=	target satellite altitude
AZ	=	launch azimuth
cos_E_launcher	=	cosine of eccentric anomaly (at launcher)
cos_E_sat	=	cosine of eccentric anomaly (at target)
dN	=	longitudinal offset
ecc	=	interceptor orbital eccentricity
E_launcher	=	eccentric anomaly (at launcher)
E_sat	=	eccentric anomaly (at target)
h	=	interceptor angular momentum
lat	=	target latitude at impact
range	=	direct range from launcher to target
r_bo	=	interceptor burnout radius
r_launcher	=	launcher radius
sin_E_launcher	=	sine of eccentric anomaly (at launcher)
sin_E_sat	=	sine of eccentric anomaly (at target)
r_sat	=	target satellite radius
TOF	=	time of flight from launcher to target
TOF_launcher	=	time of flight from perigee to launcher
TOF_sat	=	time of flight from perigee to target
Vm	=	earth-fixed interceptor velocity (i.e. muzzle velocity)
Ve	=	east component of rotating earth interceptor velocity at launch
Vs	=	south component of rotating earth interceptor velocity at launch
Vsez	=	magnitude of rotating earth interceptor velocity at launch
Vz	=	up component of rotating earth interceptor velocity at launch
V0	=	eastward component of earth-induced velocity
V_bo	=	interceptor burnout velocity
V_boost	=	topocentric velocity of the launch platform
α	=	earth-centric free-flight range angle from launch to intercept
β	=	target's off-nadir angle to the launcher
ΔN	=	longitudinal offset including earth rotation
ξ	=	total orbital energy of interceptor
γ	=	satellite-centric intercept azimuth
ϕ	=	launch elevation angle
μ	=	earth gravitational parameter
ν_launcher	=	true anomaly of interceptor trajectory at the launcher
ν_sat	=	true anomaly of interceptor trajectory at the target
ω_earth	=	earth rotation rate