# A Preliminary Analysis of State Vector Prediction Accuracy

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Modern precise navigation services are creating increased applications for numerically generated state vectors for satellite operations. Traditional radar and optical techniques can achieve modest accuracy in orbit determination, but on-board GPS satellite receivers are changing the routine accuracy available. System requirements usually involve future locations, rather than past locations derived from OD techniques. This paper compares propagation of various satellite initial state vectors to independently produced Precision Orbit Ephemerides (POE's). The initial state of each satellite is varied to reflect expected orbital accuracy achievable through existing orbit determination techniques. Satellite ephemerides are compared to known POE's, and to precise reference ephemerides generated by state-of-the-art orbit determination techniques.

## INTRODUCTION

The requirements for precise orbit determination and propagation are becoming commonplace as numerical operations become standard. A distinction between post-processing and prediction is made in that most historical studies examine the ability to post-fit observational data. With all the input data measured and known, post processing often achieves accuracies of 2-10 cm radial position, even for orbits strongly influenced by non-conservative forces. Operational processing of the entire satellite population (TLE data from the analytical SGP4 theory for instance), achieves only km-level accuracy, but can achieve about 400 m (Phillips 1996) and perhaps 50-100 m today in post processing when using numerical methods. The discrepancy is primarily due to the lack of sufficient quantity of observations, missing force models, calibration, quality, and other computational and technical limitations.

Despite the original source of initial state vectors, the important part for an operational planner is the ability to assess the accuracy of propagations from a known initial state. This is essential to make well-informed operational decisions and affects everything from simple mission planning and maneuver operations, to the broader concepts of Space Situational Awareness (SSA) and Single Integrated Space Picture (SISP). Consider the case where a close conjunction is predicted between two satellites. If one satellite is well tracked and has a sophisticated orbit determination resulting in a final estimated position of xx m while the other satellite has reasonable data, but lacks the ability to track a dynamically changing drag perturbation and achieves a 60 m initial estimated state, does this tell us anything about the conjunction in the next week? Actually, it's a very rough indicator, but not very important to the operational planner. What's most important is the ability to understand how the initial differences will translate into the fine-grained predicted positions at the time of the encounter.†

This paper seeks to clarify the ability to accurately propagate a state vector into the future. There are several areas that are examined.

1. How well can the propagator propagate the initial state - no errors assumed in an OD process. This section seeks to qualify how well the propagator can emulate the perturbing forces, as used / defined in the Precision Orbit Ephemerides (POEs).

2. How well can the propagator propagate the initial state assuming various errors resulting from the OD process, say 1m and 1 mm/s. These errors are introduced in the in-track direction as the largest source of uncertainty is generally in this direction, resulting from non-conservative forces.

3. How well can an optimal filter/smoother process the POE as an observation, and then take that state and propagate into the future. This section seeks to determine if the recent satellite states are more important than the final estimate in achieving accuracy for the prediction period. A side portion of this step examines the

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† An accurate covariance matrix that can propagate into the future and "predict" how the accuracy will degrade over time will also do this, but that is another issue and the subject of another paper.

ability to process observations and form a reference orbit from which to make comparisons from. While not as accurate as a POE, it provides a glimpse of what is achievable.

4. Establish a framework from which to evaluate additional orbits as observational data and post-processed precise orbits become available.

The third point could be expanded to "fuzz" the final state to be more reflective of what a skin-track orbit might look like.

## OBJECTIVE

This paper shows numerical results for propagating satellite state vectors against known POE's and against precise ephemerides generated by Analytical Graphics Inc.'s Orbit Determination Tool Kit (ODTK) optimal filter/smoother. All propagations are performed with AGI's Satellite Tool Kit/High Precision Orbit Propagator (STK/HPOP). The time span ranges from several days to a month and include tests for exact initial condition matches, to expected real-world difference in initial position. The POE's are considered "truth" for this paper as their uncertainty is generally much less than the accumulated error in the propagation. When reference orbits are generated, they are labeled as such to avoid confusion with truly independent reference orbits.

## STUDY PROCESS

To effectively examine the behavior of propagation using different initial condition accuracies for various satellites, a multi-step process was used, shown pictorially in Fig. 1 below.

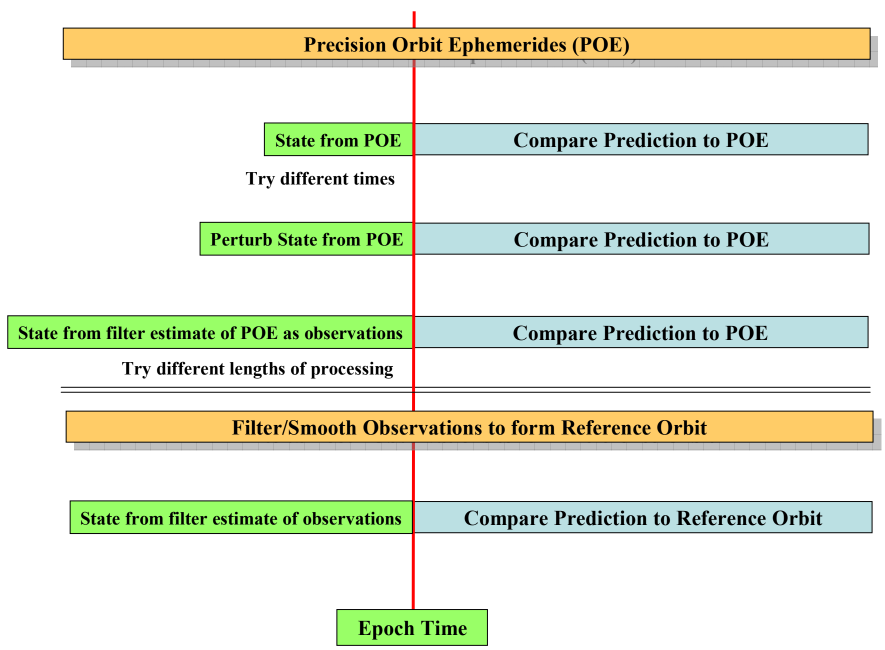



**Figure 1: General setup for Prediction Comparisons :** Several comparisons are found using the POE's directly. Other orbits require generation of reference orbits. The distinction is important as the independent nature and accuracy are not generally the same.

First, a vector from a POE was taken and used exactly in a propagation that then compared to the remaining POE. The external development and post-processed accuracy of these POE's is generally well established, and often is within 5-10 cm radial position for the entire span. Compared to the errors we would expect from propagation a satellite, these can be considered near-zero, and to be truth.

Next, recognize that orbit determination techniques are virtually never able to determine the exact satellite position to within 5-10 cm at epoch in real-time. In fact, many satellite with GPS receivers may have state vectors that are a meter or two from the "true" satellite position. To determine the impact of this uncertainty for real-time operations, we made different displacements to the initial state vectors, and repeated the comparison process of the first step.

The last step is only somewhat realistic as no orbit determination was available to "adjust" the orbital elements based on the available data. Thus, this step created reference orbits from existing GPS, SLR, or other data that was available. This let us examine an orbit determination solution that could vary all the individual input parameters, and then perform the propagation. Essentially, the last step was constrained to use the given $c_D$, $c_R$ , etc. which may have been in error, but this step allowed the OD technique to solve for, and find a time-varying value for each parameter. Because the underlying mathematical technique for ODTK is a real-time Kalman Filter (Vallado, 2007, Sec 10.6), the state vector used at any point can be either a smoothed result, or if real-world operations are a goal, the last state vector could be the any state vector, which will have the best accuracy at that point. There is no averaging and no fit spans necessary or required. Most studies that examine OD results need to go to extensive lengths to demonstrate the fit span is accurate, and one never really knows if the choice was correct and that all the dynamics were modeled when looking at the results in real-time. The filter does not suffer these limitations.

Several runs were made for satellites for which POE's were available to show the ease of processing, and the accuracy of the results.

The propagation span for each ephemeris was generally kept at about 7-14 days. Although the results at the end of this time showed some large differences, 4-7 days is generally about the event-horizon in which operational decisions are made. Differences are computed at many times during the ephemeris span to provide the user with a look at the time-varying trends.

## SATELLITES CONSIDERED

The initial task was to select a number of satellites with existing POE information. The satellites selected for this study form a spectrum from LEO to GEO satellites (Table 1). The epochs varied quite a bit due to the presence of available information. A month of data was selected for as many satellites as possible, and periods of higher solar activity were chosen to show the enhanced effects of atmospheric drag on the lower satellite orbits. The additional satellites in the LEO category were designed to better determine the results in the drag regime, and at the lower end of the solar radiation pressure regime.

**Table 1: Satellite Orbital Parameters:** This table lists the general orbital parameters for each of the study satellites.

| Category | SSC # | Name | ***a*** **(km)** | ***e*** | ***i*** **(deg)** | **Period (min)** |
|---|---|---|---|---|---|---|
| LEO | 26405 | Champ | 6723 | 0.00030 | 87.23 | 91 |
| LEO | 27642 | ICESat | 6973 | 0.00024 | 94.00 | 97 |
| LEO | 21574 | Ers-1 | 7151 | 0.00333 | 98.24 | 100 |
| LEO | 23560 | Ers-2 | 7162 | 0.00012 | 98.54 | 100 |
| LEO | 26997 | Jason | 7715 | 0.00075 | 66.04 | 112 |
| LEO | 22076 | TOPEX | 7723 | 0.00051 | 66.04 | 112 |
| MEO | 25030 PRN-08 | GPS Block IIa | 26560 | 0.00375 | 56.01 | 718 |
| MEO | 28129 PRN-22 | GPS Block IIr | 26560 | 0.00375 | 54.51 | 718 |
| MEO | 29486-PRN-31 | GPS Block IIr-m | 26560 | 0.00375 | 55.11 | 718 |
| GEO | | | 35780 | 0.00007 | 0.03 | 1436 |

The study satellites permitted a quick look at various orbital classes, but the specific satellite parameters ($c_D$, $c_{SR}$, $m$, $A$, etc) were sometimes difficult to obtain. Table 2 lists some approximate values used in the analysis. Note that these values are somewhat arbitrary in most analyses and selection of one parameter to absorb the total error is often done. One could “claim” that $\rho$, $m$, $A$ are completely known and that all the remaining error is from a changing $c_D$ value (Bowman, 2007). However, without extensive processing and knowledge of each individual satellites, materials, atmospheric composition, etc, this is not realistic and ignores many previous studies on the physical nature of $c_D$ (Gaposchkin 1994 for instance).

**Table 2:** **Satellite Parameters:** Many of these parameters were assumed, or taken from information derived from the Internet. The mass should generally be assumed as the initial mass. These are not intended to be definitive values!

| Name | Apogee Alt (km) | Perigee Alt (km) | $c_D$ | Mass (kg) | Area ($m^2$) | $c_{SR}$ |
|---|---|---|---|---|---|---|
| Champ | 347 | 343 | 2.2 | 522 | 6.5 | 1.0 |
| ICESat | 596 | 594 | 2.2 | 970 | 2.0 | 1.0 |
| Ers-1 | 797 | 750 | 2.5 | 2377.13 | 11 | 1.0 |
| Ers-2 | 785 | 783 | 2.5 | 2377.13 | 11 | 1.0 |
| Jason | 1343 | 1332 | 2.2 | 489.1 | 9.536 | 1.2 |
| TOPEX | 1343 | 1332 | 2.2 | 2500 | 15.4 | 1.5 |
| GPS Block IIa | 20438 | 19927 | 2.2 | 1816 | 18.02 | 1.0 |
| GPS Block IIr | 20311 | 20052 | 2.2 | 2217 | 19.38 | 1.0 |
| GPS Block IIr-m | 20358 | 20005 | 2.2 | 2217 | 19.38 | 1.0 |

## DATA SOURCES

Most of the POEs came from the UT/Austin/ CSR website. These sources are extremely useful for studies as raw observational data is more difficult to come by, and introduces the aspect of determining the orbit as well. However, this is changing with some satellites having Satellite Laser Ranging (SLR) data, GPS measurements, and on-board accelerometer data.

For this paper, I commonly used the WGS-84/EGM-96 and EGM-96 gravity models shown below. The parameters are important to ensure compatibility with external organizations. STK/HPOP is designed to use an ASCII file for the gravity model, including the defining coefficients. This simplified matching other conventions used by other organizations.

For EGM-96

1. Gravitational Parameter $\mu$ =398600.4415 $km^3/s^2$
2. Radius of the Earth $r$ = 6378.1363 km
3. Flattening $f$ = 1/298.257
4. Rotation rate of the Earth $\omega$ = 7.292158553e-5 rad/s

For WGS-84/EGM-96

1. Gravitational Parameter $\mu$ = 398600.4418 $km^3/s^2$
2. Radius of the Earth $r$ = 6378.137 km
3. Flattening $f$ = 1/298.257223563
4. Rotation rate of the Earth $\omega$ =7.292158553e-5 rad/s

The sources of data for Earth Orientation Parameters (EOP) and space weather are somewhat standard and I used the data from CelesTrak which is a consolidated accumulation of the past, present, and future data from the defining data locations on the web (Vallado and Kelso, 2005).

Coordinate systems varied (ITRF, TOD, IAU-76/FK5, etc.) but this presented no difficulty as STK/HPOP accepts any of these systems and applies standard reduction techniques to accomplish the propagations (Vallado, 2007:228).

Finally, the study intervals were quite varied, depending on the available data. Some satellites contained maneuvers in the data. When the maneuvers were known, the analysis simply proceeded through each maneuver. When the maneuvers were unknown, different time periods were selected. Table 3 below shows the intervals for each satellite.

**Table 3:** **Study Intervals :** The time intervals for each satellite varied depending on availability and times to show certain perturbing effects.

| Name | Study Interval | Comments | |
|---|---|---|---|
| Champ | Oct 2000 | Solar flux increased | |
| ICESat | Feb 2003 | Modest solar flux, a few known maneuvers | |
| Ers-1 | Aug 1991 | Solar flux increased | |
| Ers-2 | Sep 1995 | Modest solar flux | |
| Jason | Apr 2002 | Modest solar flux | |
| GPS Block IIa | Dec 06 - Jan 07 | Maneuver during the first week | |
| GPS Block IIr | Dec 06 - Jan 07 | No maneuvers | |
| GPS Block IIm | Dec 06 - Jan 07 | No maneuvers | |
| GEO | Apr 2007 | Many known maneuvers | |

## PROPAGATING THE INITIAL STATE VECTOR

The first step was to propagate a state from the POEs, and compare the results to the POE. POEs are often regarded as "truth" because they are developed after all the data is collected, and extensive analysis, processing, smoothing, etc. can be applied to minimize errors throughout the ephemeris time span. This phase was intended to show the ability of the propagator to match the "truth". While the POEs are derived from independent processing, their accuracy is usually sub-meter, in which case we can consider them as truth for our purposes.

Note that most POE analyses perform multiple differential corrections on the POE to obtain a precise fit to the data with a certain set of force models (a data rich environment). Although this is the most accurate method (in the absence of detailed information between organizations), it lacks a real-world operations flavor because POEs are generally not available in near-real time. The approach I've taken here is to simply take the first state vector from the POE, and propagate it through time. This should conservatively bound the "expected" results one could see if the dynamics were known perfectly in the estimation process. The plots contain the results for an "exact" fit, and subsequent propagation.

However, as I showed in 2005, any number of propagation programs may be used to move the satellite state forward in time … as long as detailed information about the force models and parameters is known. Unfortunately, most sources provide extremely little additional information about the specific setup. The behavior of various satellites to individual perturbations (see Vallado 2007 for individual graphs) can eliminate the major sources of error between propagation runs, but generally atmospheric drag and solar radiation pressure contribute the largest uncertainty by at least an order of magnitude. Thus, difficulties in knowing what $c_D$, $c_{SR}$ were used created the largest uncertainty. Later, I introduce this through OD runs of the POE data.

### ICESat

Study interval: Feb 19, 2003 21:00:00.000 UTC – Mar 24, 2003 00:00:00.000 UTC

Information on the POD aspects of ICESat were obtained from the GLAS POD document (Rim and Schutz, 2002). For a starting estimate, I used a mass of 950 kg assuming the vehicle had used some maneuvering fuel. I'll show 4 graphs to illustrate the component (normal, tangential and cross-track, NTW) behavior of the predictions for varying time periods – 2 hours, 1 day, 1 week, and 1 month. This ensures the components remain valid for highly eccentric orbits should they be tested.

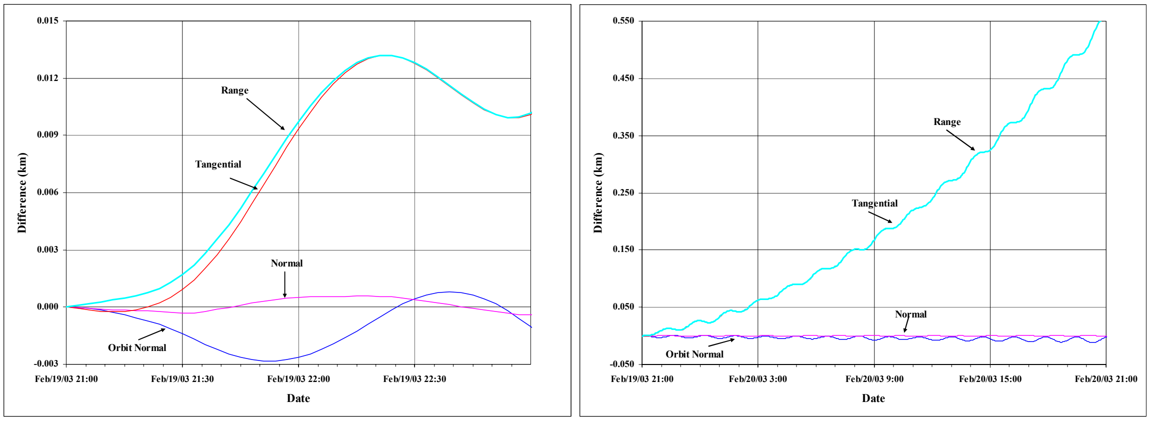



**Figure 2: ICESat POE Ephemeris Comparisons:** Results are given for ICESat at 2 hours (left) and at 1 day (right). Notice that the tangential component becomes dominant after only about 45 minutes.

When the scope is expanded to one month, maneuvers become evident. The presence of unknown maneuvers in the data limited the length of immediate analysis that could be performed.

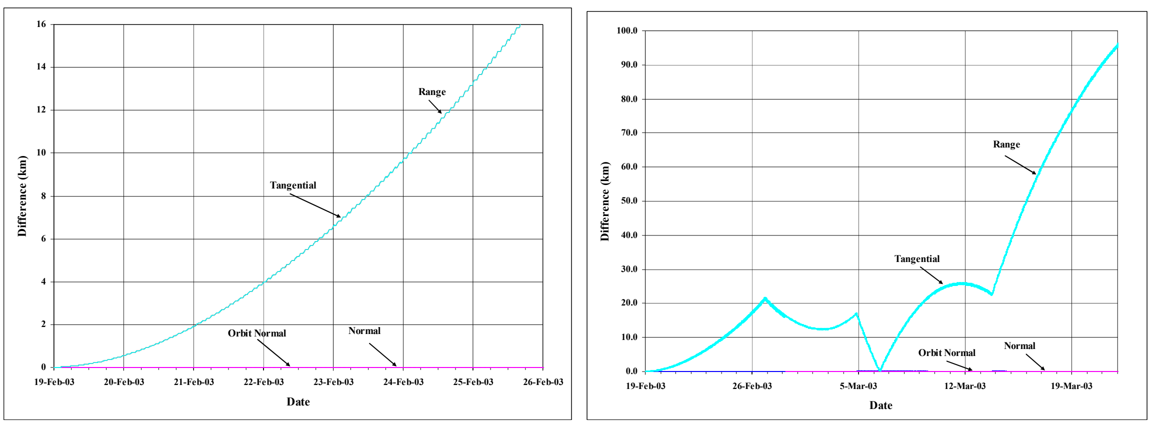



**Figure 3: ICESat POE Ephemeris Comparisons:** Results are given for ICESat at 1 week (left) and at 1 month (right). Notice that the pronounced appearance of maneuvers in the long range comparison.

The maneuvers were researched and inserted for February 27, March 5, and March 14, yielding the following results. Note that the ability to process through a maneuver is extremely useful for both orbit determination and propagation operations. I test only the propagation portion here, but will use the orbit determination feature in a filter in later operations.

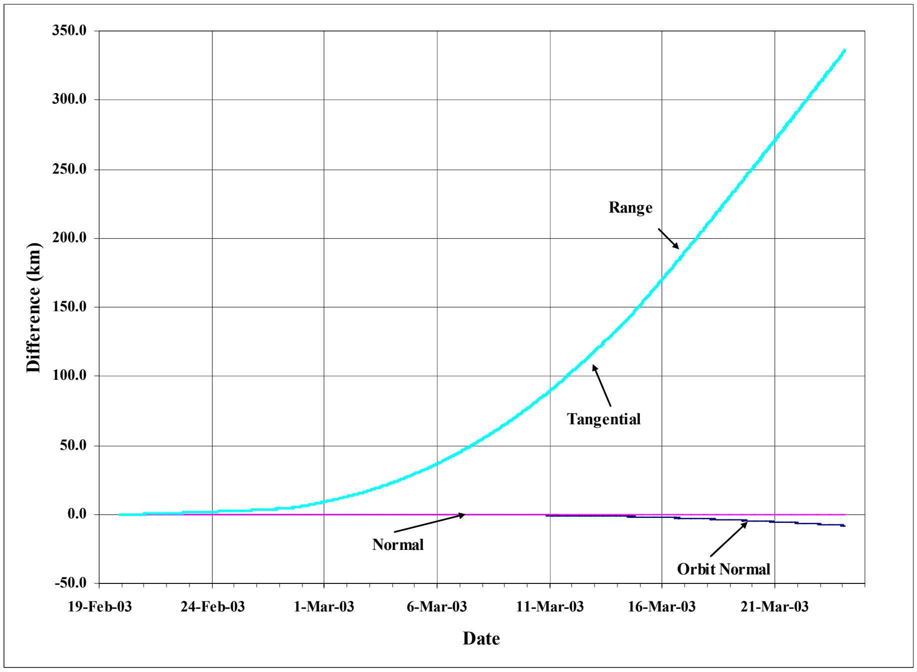
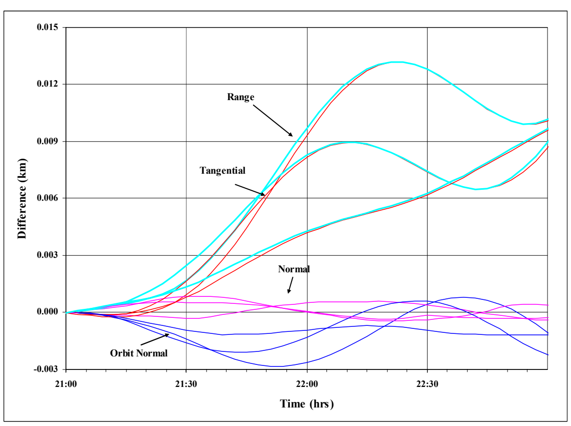



**Figure 4: ICESat POE Ephemeris Comparisons Including Maneuvers:** When the maneuvers are included, the performance is smoother, although not as good as it could be.

Several different start times were examined to determine if the initial state from the POE affected the results. The results were different for the short term, but quickly matched the preceding graphs. The following figure shows start times of 21:00, 21:20, and 21:40 UTC. Because the data starts at different times, the data is displaced, making it easier to see any trends. The general trend was the same, but especially in the 2 hour interval, the shape was quite different. Past 1 day, the results were essentially the same.



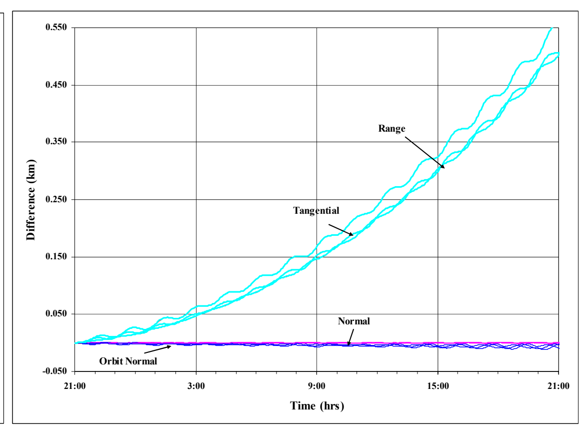




**Figure 5: ICESat POE Ephemeris Comparisons:** Results are given for ICESat at 2 hours (left) and at 1 day (right)using three different starting locations (each differs by 20 minutes). Notice the difference in the short term results.

This may be an artifact of the smoothing process used to create the original POEs, but time did not permit an adequate investigation.

**ERS-1**

Study interval: 1 Aug 1991 00:00:00.000 to 2 Sep 1991 00:00:00.000 UTC

There was not as much information available on the POE formation for the ERS-satellites, thus the estimates in Table 2 were used as a starting point. The mass was 2200 kg, and the area about 11 $m^2$. The results are very similar (in general trends) to the ICESat results including some apparent maneuvers in the long time period plot.

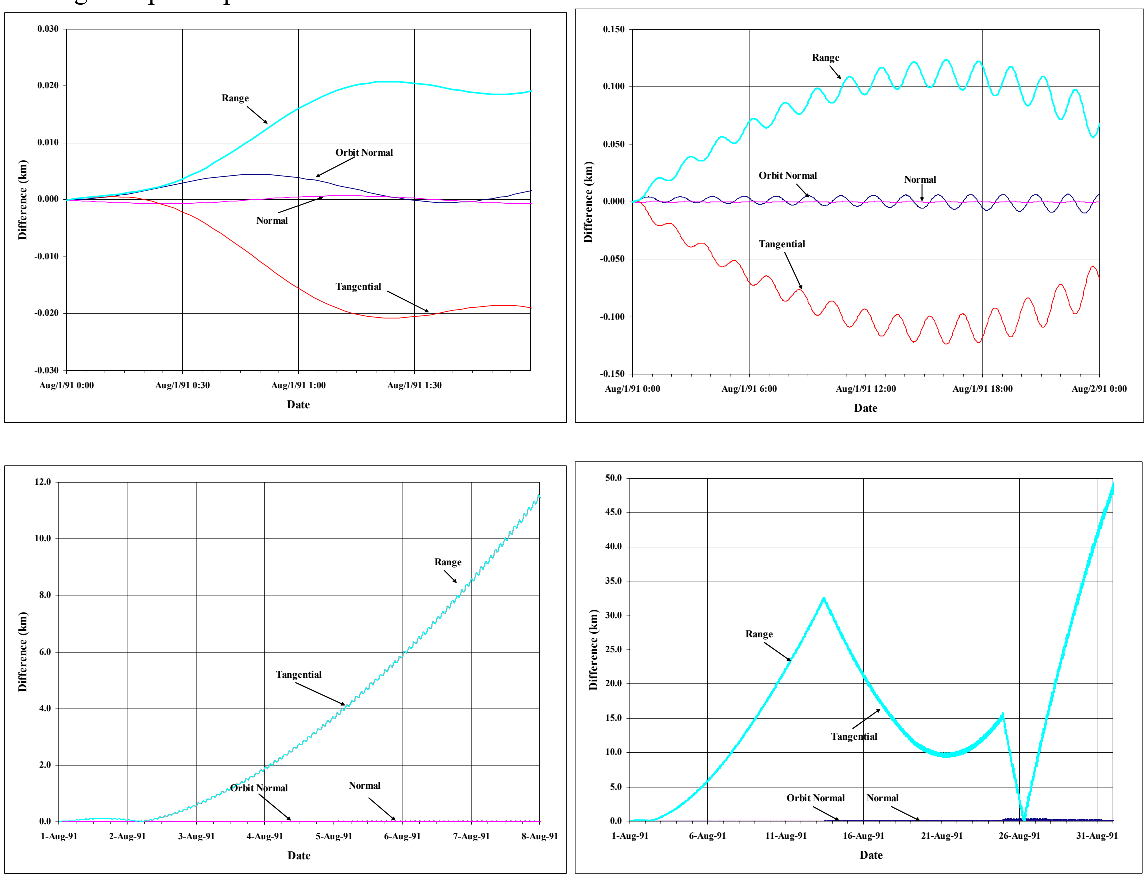



**Figure 6: ERS-1 POE Ephemeris Comparisons:** Results are given for ERS-1 at the four time intervals. Notice that the pronounced appearance of maneuvers in the long range comparison.

**ERS-2**

Study Interval: 8 Sep 1995 12:00:00.000 to 10 Oct 1995 00:00:00.000 UTC

The satellite parameters were kept the same as with ERS-1. The mass was 2200 kg, and the area about 11 $m^2$. The study interval was chosen as there were maneuvers on the $5^{th}$ and $6^{th}$ of September.

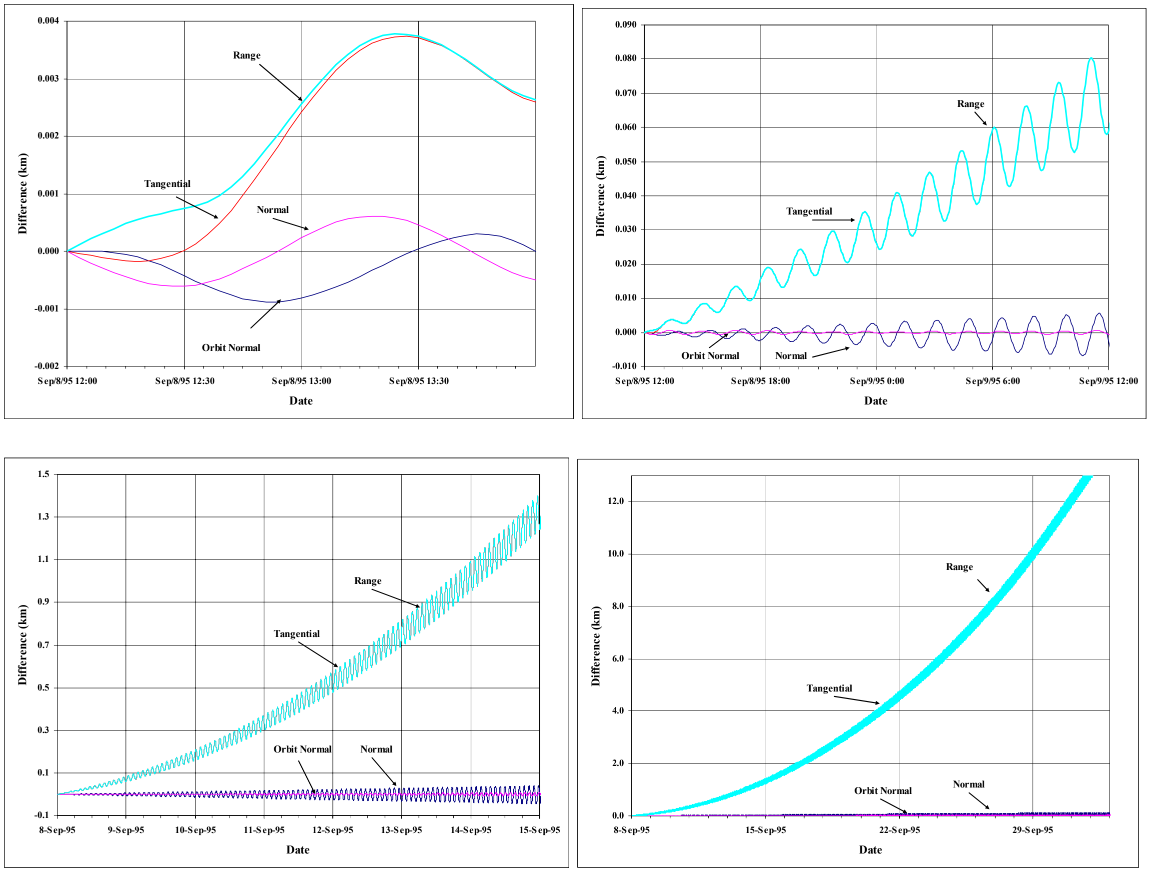



**Figure 7: ERS-2 POE Ephemeris Comparisons:** Results are given for ERS-2 at the four time intervals. There did not appear to be maneuvers in the long range comparison.

**JASON**

Study interval: 25 Mar 2002 15:01:00.000 – 14 May 2002 04:42:00.000 UTC

We would expect that the results for Jason would be better than the lower altitude satellites because atmospheric drag is significantly reduced at this altitude. There were not detailed information on the formation available, so I used the general values from Table 2. The mass was 475 kg and the area about 9.536 m$^2$. After about 24 hours, the following figure shows the prediction to be accurate to about 15 m. Notice the variation in the 1 day and 1 week plots. The scale is small, but the effect is simply a result of solving the second-order, nonlinear, equations of motion. The degree with which the oscillations occur is a function of the orbit, the relevant perturbations, the modeling of the satellite parameters, and mostly the scale. Note the largest variations occur about a much smaller difference in the orbits than the other figures for the same time interval.

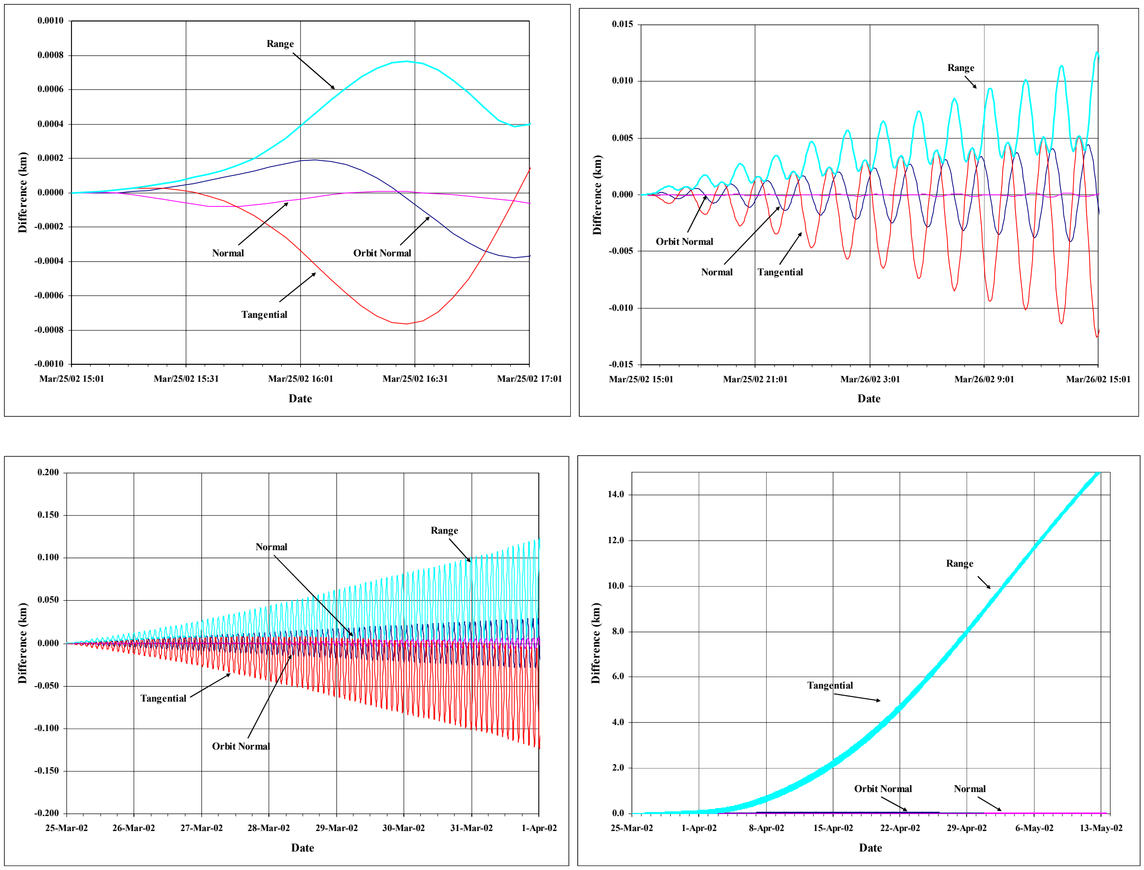



**Figure 8: Jason POE Ephemeris Comparisons:** Results are given for Jason at the four time intervals. The results are very similar to the ERS results.

## GPS

To adequately examine the GPS satellites, specific information is required on the solar radiation pressure modeling. In general, there are different models for each block of satellites, and these make a large difference when trying to compare results. All the GPS models use 2 solar radiation pressure scaling coefficients (Scale, and Y-Bias). Scale multiplies the model in body-specific equations in the X and Z directions, while Y-Bias estimates the acceleration in the body Y-direction. Both values are usually near unity, but will differ. Table 4 lists some of the models available in STK/HPOP, and used in the GPS ephemeris generation.

**Table 4: GPS Solar Radiation Pressure Models :** The various GPS satellite blocks use different solar radiation pressure models, although they do share many similarities. Note that NGA uses the Bar-Sever models in the formulation of the GPS POE data.

| Block | Name | Reference | Update | Update |
|---|---|---|---|---|
| Block I | ROCK 4 | Rockwell, Fliegel, Gallini, and Swift | T10 | |
| Block II, II-A | ROCK 42 | Rockwell, Fliegel, Gallini, and Swift | T20 | GPSM.IIA.04, Bar Sever at JPL |
| Block II-R | Table look-up | Lockheed | | GPS.IIR.04, Bar Sever at JPL |
| Block IIR-M | Table look-up | Lockheed | | GPS.IIR.04, Bar Sever at JPL |

### GPS SVN-22

Study interval: 26 Nov 2006 00:00:00.000 to 31 Jan 2007 23:45:00.000 UTC

Using a/m .01569, therefore mass was about 1200 kg. The K1 and K2 parameters were both left at 1.0. We'll see later that this is not a very good approximation, but it serves to bound the problem.

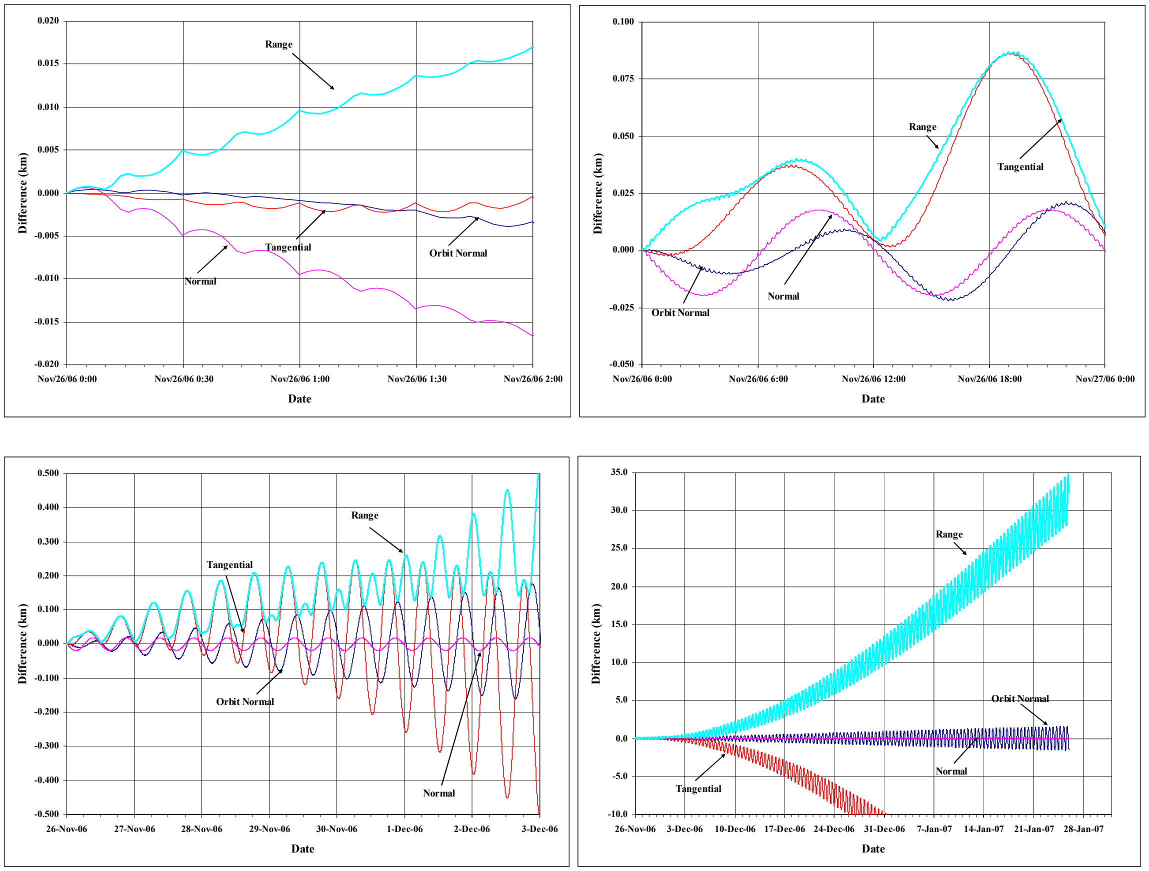
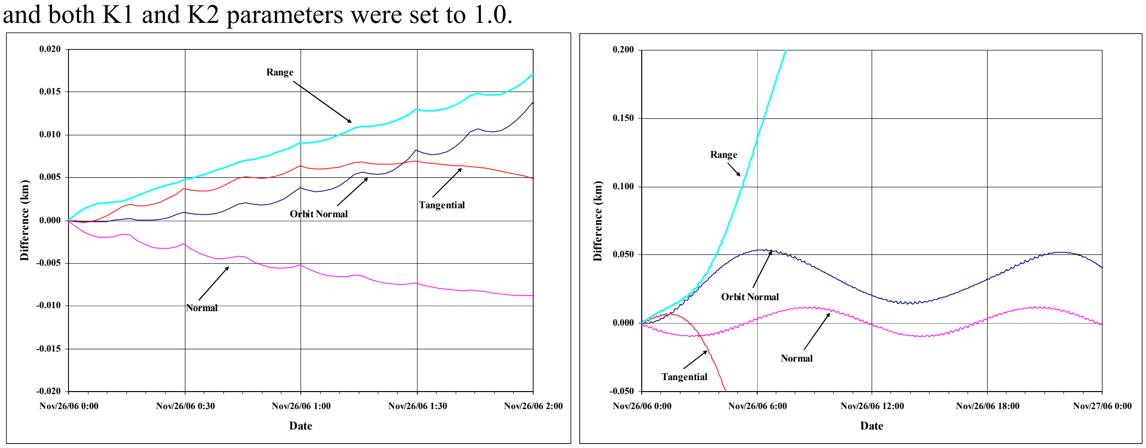



**Figure 9: GPS SVN-22 POE Ephemeris Comparisons:** Results are given for GPS SVN-22 at the four time intervals. Notice that the change in behavior at about the 2-3 week point.

**GPS SVN-31**

Study interval: 26 Nov 2006 00:00:00.000 to 31 Jan 2007 23:45:00.000 UTC

Using a/m .01569, therefore mass was about 1200 kg. The satellite enters eclipse in January 2007 and both K1 and K2 parameters were set to 1.0.



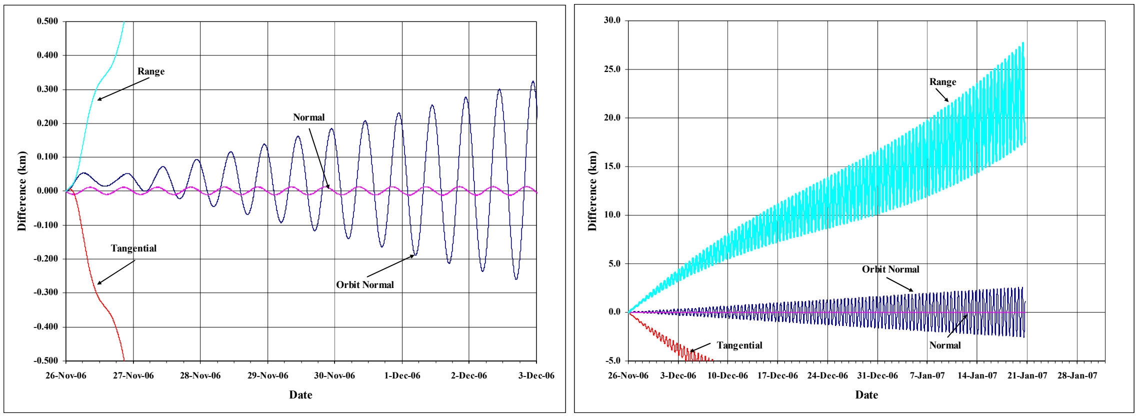



**Figure 10: GPS SVN-31 POE Ephemeris Comparisons:** Results are given for GPS SVN-31 at the four time intervals. The rapid departure suggests some of the modeling is not properly set.

## PERTURBING THE INITIAL STATE VECTOR

For real operations, the state is not known with the same level of accuracy as is available in post-processing. Thus, even rapid ephemerides from a GPS receiver will differ by perhaps 1 m, and / or 1 mm/s at any epoch. To simulate this more realistic scenario, the initial state vector from the POE is changed to include this offset (in the tangential direction), and the propagate through time is performed. This should conservatively bound the “expected” results one could see if the last update (say from a Kalman filter of on-board GPS measurements) was used for prediction calculations.

The plots contain the results for an initial state that was perturbed in the tangential position axes by 1m, and each velocity axis by 1 mm/s. These errors were intended to simulate performance under actual conditions when the initial state would not have cm-level accuracy (as in the POE), but something larger.

Because these analyses are being done against “truth” (the POE), one would expect the errors to grow due to two factors. First, the inadequacy of the force models, solar data, etc. (as seen in the first tests where the exact initial state was used). Next, the initial displacement from the POE also introduces an error. Although the argument can be made that in practice, you would have better knowledge of the satellite characteristics, the departure from “truth” will still result from both sources. For satellites experiencing significant atmospheric drag, this effect can overwhelm any benefit of a good initial state vector, or satellite characteristic, in just a few revolutions, depending on the orbit. From previous analyses conducted by the author, the effect of velocity uncertainty is sometimes more influential than that of position. The primary goal was to examine what impact the additional uncertainty in the initial state vectors had on the transient behavior. Thus, only the first few hours were examined as after that time, the non-conservative forces would mask any initial uncertainty.

The first runs were conducted on ICESat and ERS-2. As expected, the results look very similar, with the ERS-2 performance being slightly better because its in a slightly higher orbit.

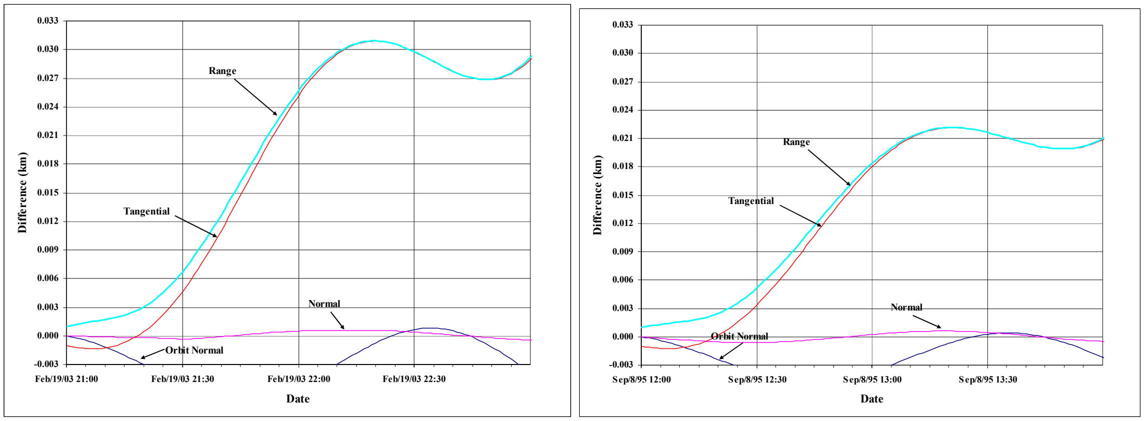



**Figure 11: POE Ephemeris Comparisons:** Results are given for ICESat (left) and ERS-2 (right). The similarity of the orbital altitudes produces very similar results. Notice the difference in the orbit normal component. In both cases, the dominant feature is the initial tangential displacement. The scales are the same between the two graphs.

Offsets were applied in different axes to determine if the transient effects would differ greatly. For example consider ERS-2. If we perturb the initial normal component, the tangential difference still quickly masks any transient changes (note the scale change from Fig. 7). ICESat is very similar as in Figs. 2 and 3.

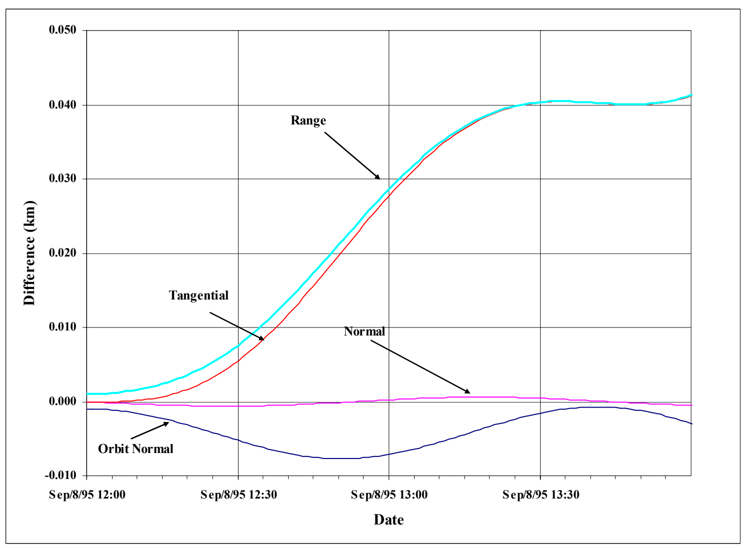



**Figure 12: ERS-2 POE Ephemeris Comparisons:** Results are given for ERS-2 with an initial normal displacement. The initial normal difference is quickly masked by the tangential component.

## OD USING THE POE'S

The most realistic approach to investigate prediction accuracy is to use actual observational data that would be available in near-real time, perform a differential correction, and then propagate the result into the future. Comparing to the POE (at a later time), would then reveal the uncertainty involved with this operation. This process has several important benefits, including comparison with a truly external and independent source for producing the POE. In addition, most POE's are developed by fusing data from several sources, thus providing an additional level of confidence in the final result.

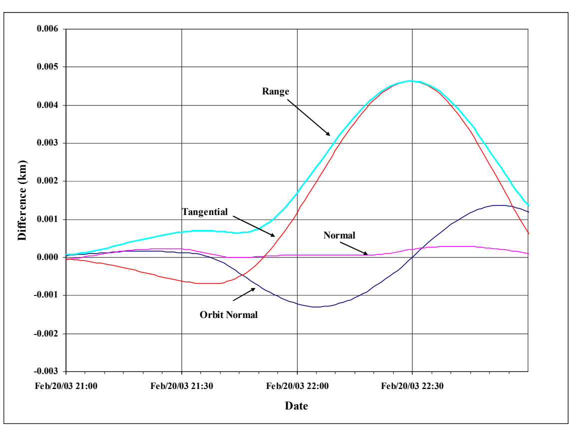
The difficult step is obtaining the data. Satellite Laser Ranging Data (SLR) exists for many satellites, and GPS and accelerometer data are available from some sources. However, obtaining these data at the same time a POE is developed and available becomes difficult. In the absence of raw observational data, one can also use the POE itself as a data source – the case used here. This requires that the POE be long enough that you could get a "converged" solution and still have enough data to perform a comparison with. The absence of maneuvers is also desirable. Several satellites met these criteria.

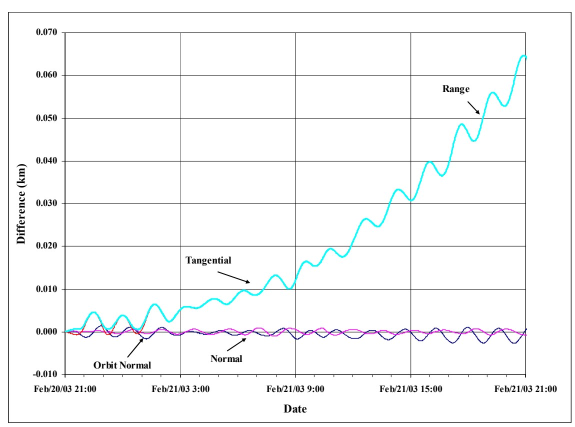
**ICESat**

Study period: Feb 19, 2003 21:00:00.000 – Mar 1, 2003 21:00:00.000 UTC

The initial satellite parameters were used as identified previously. The GLAS document (Rim and Schutz, 2002) provided additional details about the formation of the POE. From this report, several key points were obtained to assist the setup of the prediction runs. The *"position of the GLAS instrument should be known with an accuracy of 5 and 20 cm in radial and horizontal components, respectively."* To obtain this accuracy, "*the adopted approach for ICESat/GLAS POD is the dynamic approach with gravity tuning and the reduced-dynamic solutions will be used for validation of the dynamic solutions.* In addition, to "*account for the deviations in the computed values of density from the true density, the computed values of density,* $\rho_c$*, can be modified by using empirical parameters which are adjusted in the orbit solution. Once-per revolution density correction parameters [Elyasberg et al., 1972; Shum et al., 1986] have been shown to be especially effective for these purposes*. As is often done, to "*account for the unmodeled forces, which act on the satellite or for incorrect force models, some empirical parameters are customarily introduced in the orbit solution. These include the empirical tangential perturbation and the one-cycle per-orbital-revolution (1 cpr) force in the radial, transverse, and normal directions [Colombo, 1986; Colombo, 1989]. Especially for satellites like ICESat/GLAS which are tracked continuously with high precision data, introduction of these parameters can significantly reduce orbit errors occurring at the 1 cpr frequency and in the along track direction [Rim et al., 1996]."*

The number of "additional" parameters (1 cpr, unmodeled accelerations, reduced dynamics, etc) in the POE formation suggested separate OD runs using the POE data as an input. In Analytical Graphics Inc. ODTK, the process is relatively straightforward to estimate these parameters from the POEs. The .sp3 vectors are formatted as NAV solutions (.navsol) and the filter then processes the state vectors as position and velocity vectors. The bias values were set to zero, and the bias sigmas were set to 0.5m. With full force models (70x70 EGM-96 gravity, NRLMSIS-00 atmospheric drag, Sun and Moon third body, solar radiation pressure, solid and ocean tides, albedo) we perform a 1-day OD on the filter POE information, and then predict for about a week and a half. The results show a marked improvement over the state propagation in Figs. 2 and 3. Note the maneuver at the end of the week prediction, and thus, no month long prediction plot.



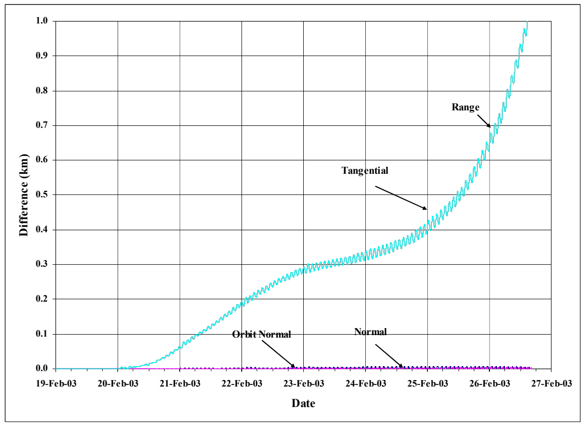



**Figure 13: ICESAT OD and POE Ephemeris Comparisons:** Results are given for ICESAT after a 1 day OD of the POE data. The tangential difference still dominates very quickly, but the overall results are much better than the original prediction because the satellite parameters are better known from the OD portion of the run.

Several things stand out.

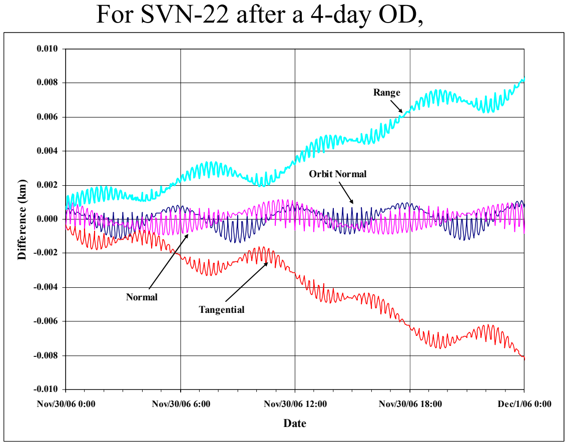
1. The expected result of using information from an estimation process to predict into the future is demonstrated. The OD provides the detailed information on the satellite parameters and coefficients that may not be available from the associated literature of the POE ($c_D$, $c_{SR}$, $A$, $m$, etc). This is also confirmed in Vallado (2005).

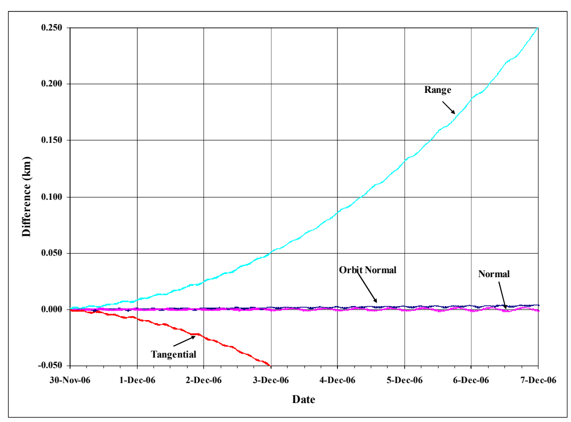
2. The prediction can be quite accurate, even in the presence of non-conservative forces if the proper input data is used. 1 km over about a week for a LEO satellite should not be considered as too bad in the presence of atmospheric drag. More importantly, the error is only about 300 m at 3-4 days in the future. This should be quite adequate for many near-term planning operations.

## GPS

The GPS satellite orbits have longer spans of data from which reasonable analysis may be obtained. Because the original results differed for the two satellites, both were examined.

For SVN-22 after a 4-day OD,



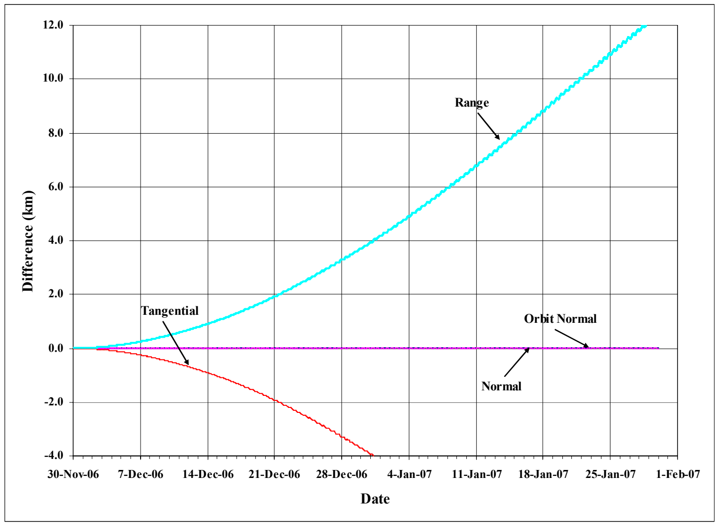
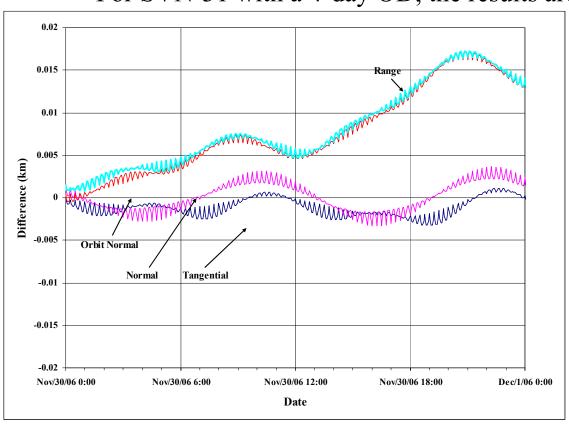
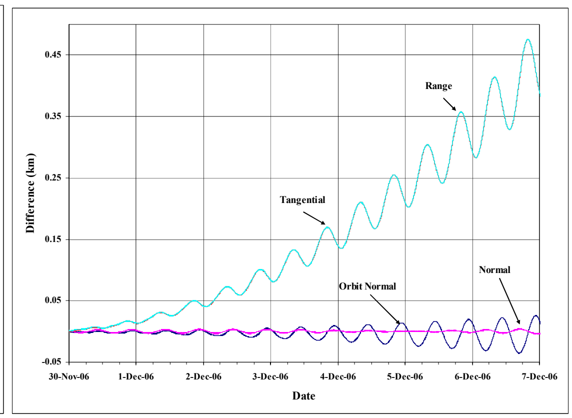



**Figure 14: GPS SVN-22 OD and POE Ephemeris Comparisons:** Results are given for GPS SVN-22 after a 4 day OD of the POE data. Notice the smoother overall performance of the differences.

For SVN-31 with a 4-day OD, the results are as follows.



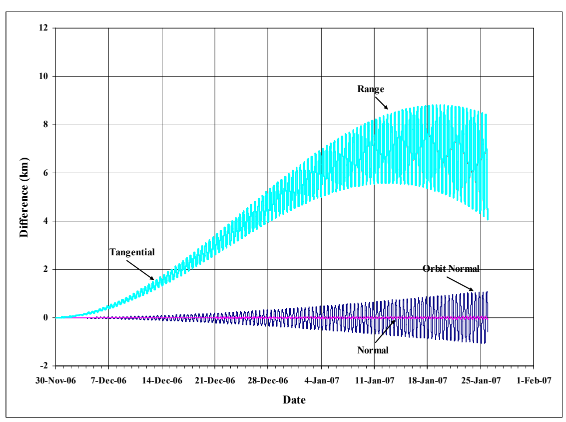




**Figure 15: GPS SVN-31 OD and POE Ephemeris Comparisons:** Results are given for GPS SVN-31 after a 4 day OD of the POE data. Notice the smoother overall performance of the differences and the significant improvement from Fig. 10.

The results in Fig. 15 show the improvement when more accurate satellite parameters are known.

## SELF GENERATED REFERENCE ORBITS

The previous section examined using the POE as observations, and then comparing to the POE. Unfortunately, POEs aren't available for all satellite classes, and there are cases where the satellite observational data is available without any POE. We examine that case here.

### LEO

LEO orbits are probably best analyzed using accelerometer and GPS data that exist for some satellites. Time did not allow for these evaluations for this paper.

### GEO

POEs are not readily available for Geosynchronous satellites, but I obtained some data which permitted me to form a reference orbit (using the filter-smoother combination in ODTK). I then took a subset of that data, performed an orbit determination, and predicted the remainder of the time. While it would not be valid to perform an OD over the entire interval of data and then take a point and predict through times that were already processed, the filter can rapidly establish the orbit, enabling a slightly longer prediction time analysis.

For the case in which the OD used about 2 days of data, the following results are shown.

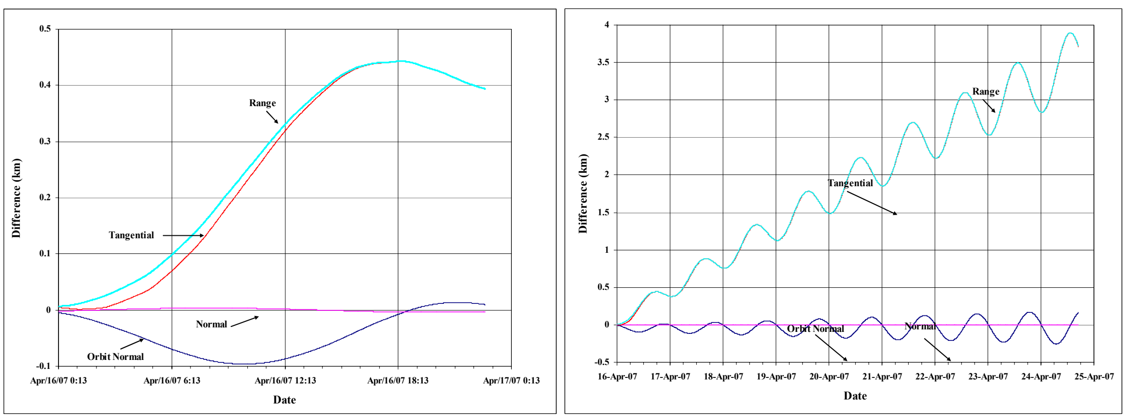



**Figure 16: GEO OD Ephemeris Comparisons:** Results are given for a GEO satellite after a 2 day OD of observational data. The difference after 1 day is about 400m.

Notice the immediate departure in the data, although reasonable performance considering the orbit is geosynchronous. The behavior is also remarkably similar (though not in scale) to the LEO cases. If we had used 3 days to establish the orbit, we get

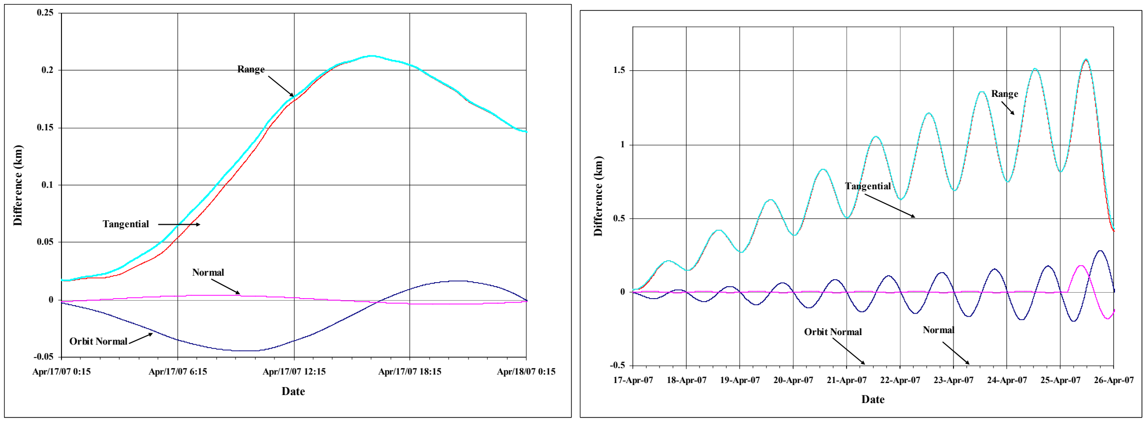



**Figure 17: GEO OD Ephemeris Comparisons:** Results are given for a GEO satellite after a 3 day OD of observational data. The difference after 1 day is about 200m.

The graphs (Fig. 14 and 15) do not start on the same date due to the difference in the length of the initial OD. However, notice that at 4 days into the future, the 3-day initial OD performs nearly 3-times better than the 2-day OD case. Obviously with more time (data) in the original OD, the fit will get better – up until a point. For this case, the filter continually improves with the new data, although after about 3 days, the improvements become much smaller. The key is that with just a few days observational data, fairly accurate predictions can be made for a "long" period into the future. The dominance of the tangential error growth is shown again.

## SUMMARY AND CONCLUSIONS

The basic nature of prediction accuracy has been introduced. Four types of prediction examples were examined. The first took the exact state from the POE and processed the data. While not realistic in an operational sense, it showed what is reasonably possible with some modern tracking techniques. Unfortunately, sparse information about the satellite parameters limited the performance of each case. The second section examined a few cases where the initial POE state vector was perturbed by an amount approximately equal to what one could expect from an on-board GPS receiver (about 1 m and about 1 cm/s). The results departed very quickly from the POE as expected. The third phase examined Orbit Determination on a section of the POE, and a subsequent prediction with comparison to the POE. Again, while not entirely independent, it showed that more precise modeling of the satellite parameters could dramatically improve the performance of the prediction accuracy. Part of this is also due to the fact that the force models are aligned exactly between the OD and prediction runs. This is possible with differing flight dynamics programs (Vallado 2005), but requires strict attention to detail on several fronts. The final case examined creating a reference orbit with observations, and then running the filter on a portion of the reference orbit data, and comparing to the longer ephemeris. While this is perhaps the most realistic scenario from an operational perspective, the precision of the reference orbit is clearly not as good as the independently developed POEs used in the previous tests. This phase did permit a look at what is possible for these orbital classes. Table 5 summarizes the results from the various runs conducted.

A similar comparison using Two-Line Element set (TLE) data was conducted (Kelso, 2007) with GPS orbits. While the magnitudes of the differences were much larger, some of the same behavior was noted.

**Table 5: Summary Prediction Values :** This table summarizes the maximum error, per time interval, for each of the study satellites. All values are in km.

| Satellite | Condition | 2 hours | 1 day | 1 week | 1 month | 2 months | Period (min) |
|---|---|---|---|---|---|---|---|
| ICESat | | 0.014 | 0.550 | 16.000 | manv | X | 91 |
| ICESat | Pert T | 0.032 | X | X | X | X | 91 |
| ICESat | OD 1 day | 0.005 | 0.065 | 1.000 | manv | X | 91 |
| ERS-1 | | 0.020 | 0.125 | 12.000 | manv | X | 97 |
| ERS-2 | | 0.004 | 0.080 | 1.400 | 13.000 | X | 100 |
| ERS-2 | Pert T | 0.022 | X | X | X | X | 100 |
| ERS-2 | Pert N | 0.040 | X | X | X | X | 91 |
| Jason | | 0.001 | 0.012 | 0.120 | 15.000 | X | 100 |
| GPS SVN-22 | | 0.017 | 0.080 | 0.400 | X | 34.000 | 112 |
| GPS SVN-22 | OD 4 day | X | 0.008 | 0.250 | X | 12.000 | 112 |
| GPS SVN-31 | | 0.016 | 1.000 | 5.000 | X | 26.000 | 718 |
| GPS SVN-31 | OD 4 day | X | 0.017 | 0.480 | X | 8.200 | 718 |
| GEO | | X | X | X | X | X | 1436 |
| GEO | OD 2 day | X | 0.400 | 3.000 | X | X | 1436 |
| GEO | OD 4 day | X | 0.210 | 1.400 | X | X | 1436 |

Some general observations:

- Even using cm-level accurate initial state vectors, the best predictions are immediately in the tens to hundreds of meters or more.
- After about half a period, the tangential error begins to dominate the error characteristics, no matter what orbital regime.
- Performing an OD significantly improves the results due to the consistent force modeling, and ability to solve for potentially unknown satellite parameters.
- Almost all results were less than 1 km at a week, which should suffice for many operations.
- A properly formulated filter/smoother can easily process through maneuvers and provide valuable information about a variety of operations.

It appears there are three "phases" with respect to the orbital prediction accuracy. During the first phase of a few hours, the current models do rather well in accounting for each perturbation. In the second phase, larger systemic problems emerge – primarily in the non-conservative forces. This manifests itself in the tangential direction, along the velocity vector and quickly masks any transient effects in the other axes. The third phase appears in the long term behavior of the satellite motion. This last area is interesting because it encompasses the area in which almost all modern POE formulations use un-modeled accelerations, track weighting, segmented ballistic coefficient, white noise sigmas and bias half-lifes, etc. These parameters, while useful to match data in a post processing situation, appear to be of diminished use for long term prediction. While the imprecision of future indices (solar output and the associated atmospheric drag indices, EOP parameters, etc.) is well known, and likely accounts for a good deal of the imprecision, it is the authors opinion that additional fundamental research is needed to describe the exact physical effects that produce these discontinuities, requiring additional parameters.

## ACKNOWLEDGMENTS

I appreciate Bob Schutz from the Center For Space Research for providing data on some of the maneuvers that were contained in the POE's, plus additional information on the satellite parameters.

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