AAS 10-155

# POST-MANEUVER ORBIT ACCURACY RECOVERY ANALYSIS

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This paper analyzes a series of maneuvers, measurement types, and maneuver uncertainties to determine how long it takes to recover a post-maneuver orbit solution using an optimal sequential filter. Filter convergence definitions are presented and evaluated to assess the key variables of interest for reducing the convergence time. The results are used to assess operator friendly useful "rules of thumb" for planning post-maneuver tracking schedules. The results are of particular interest to operators interested in evaluating maneuver performance and to those interested in rapid maneuver recovery for space situational awareness.

## INTRODUCTION

Maneuvers are a routine part of spacecraft operations. These consist of periodic station-keeping maneuvers, momentum dumps, or other thrusting events which perturb the orbit. Post-maneuver tracking data is then collected and used to solve for the new orbit. This can be particularly challenging for geostationary satellites due to the poor tracking geometry when using ground based measurements. This paper analyzes different maneuvers, measurement models, and maneuver uncertainties to determine how long it takes to recover the orbit solution to an acceptable level of accuracy using an optimal sequential filter orbit determination process.

Orbit maneuvers are typically performed with the majority of the thrust along a single axis (e.g., radial in-track, or cross-track); therefore this paper will focus on these simple maneuvers. More complicated maneuvers can be represented as combinations of single-axis maneuvers without loss of generality.

It's not uncommon for a real maneuver to be different than the theoretically desired or expected maneuver. Sources of error include the maneuver time, magnitude, and direction. Operators are interested in evaluating maneuver performance to determine if additional maneuvers are necessary and to incorporate observed performance into future maneuver plans. For the SSA community, the "error" of interest is whether the spacecraft has maneuvered at all – in this case the expected maneuver was no maneuver. Timeliness of the solution is important for both groups of people.

We'll first establish definitions for filter convergence, propose some rules for scheduling tracking data passes, simulate various maneuver scenarios and tracking schedules and evaluate the convergence time. All analysis was performed using Orbit Determination Toolkit (ODTK) v6.0.2.[1]

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## CONVERGENCE DEFINITION

Least squares orbit determination systems are commonly used for obtaining an orbit solution. A new fit span is typically begun after a maneuver is completed and extends over successive passes of tracking data. A common method of evaluating convergence is to perform ephemeris overlap tests as more measurements are obtained post-maneuver. When the difference between successive solutions falls below an operationally significant threshold (defined by the operator), the orbit solution is said to have converged. How long this takes depends on the type of measurements, measurement accuracy, number of passes, and pass schedule. Operators often establish a standard post-maneuver fit span and only perform one solution after the requisite time has passed. In the author's experience a fit span of 24-30 hours was typically used for geostationary satellites. A converged solution may have been possible sooner, but the effort to evaluate the intermediate solutions was prohibitive. Waiting 24-30 hours is not ideal for SSA purposes.

A sequential filter does not have a fit span and can continue to process right across the maneuver without having to restart at the end of the maneuver.[2] When the maneuver occurs, an additional force is applied to the spacecraft and additional process noise due to the uncertainty in the maneuver itself. This causes the position and velocity uncertainty in the covariance to spike. Therefore a different definition of convergence must be used.

Filter convergence can be defined in an absolute sense and in a relative sense. The filter has converged absolutely if the errors in the state estimates have reached a steady state behavior relative to the truth. Unfortunately the truth is not known other than in simulations, so operators modify this to a more practical definition based on the filter's covariance. The filter has converged absolutely when the uncertainty in the state parameter has fallen below desired operational limit *and stays below it*, absent another perturbing event. The uncertainties some are obtained from the diagonals of the covariance as represented in sigma-correlation form and scaled to 3-sigma values (99% confidence). We can define an absolute convergence criterion

$$3\sigma < X \tag{1}$$

where $X$ is a threshold value defined by the operator.

Convergence can be defined in a relative sense by examining the change in the state uncertainty during a measurement update. As each measurement is processed, the filter corrects the state estimate and its uncertainty. A decrease in the uncertainty indicates the parameter was observable. The most significant change occurs at the beginning of a new pass of measurements. Additional corrections are made on subsequent measurements, although they are not as large since very little time has elapsed between measurements within a pass as compared to the time gap between passes. We define a relative convergence criterion

$$\frac{|\sigma_M - \sigma_T|}{\sigma_T} < 10\% \tag{2}$$

Where $\sigma_T$ is the uncertainty after the filter time update of the first measurement in the pass and $\sigma_M$ is the uncertainty after the measurement update has been performed. A threshold value of 10%

was selected as the threshold in this case primarily because the uncertainties from the covariance are typically accurate only to two significant digits.[*]

The metric of interest is the time it takes the filter to converge after the maneuver. This is a function of the observability of the parameters being estimated and the limits induced by the process noise from the force models. Some key drivers are the type of measurement, the uncertainty in the measurements, and the distribution of measurements. Figure 1 shows the RIC position error uncertainty increasing at the time of a maneuver, then decreasing as tracking data passes are processed.

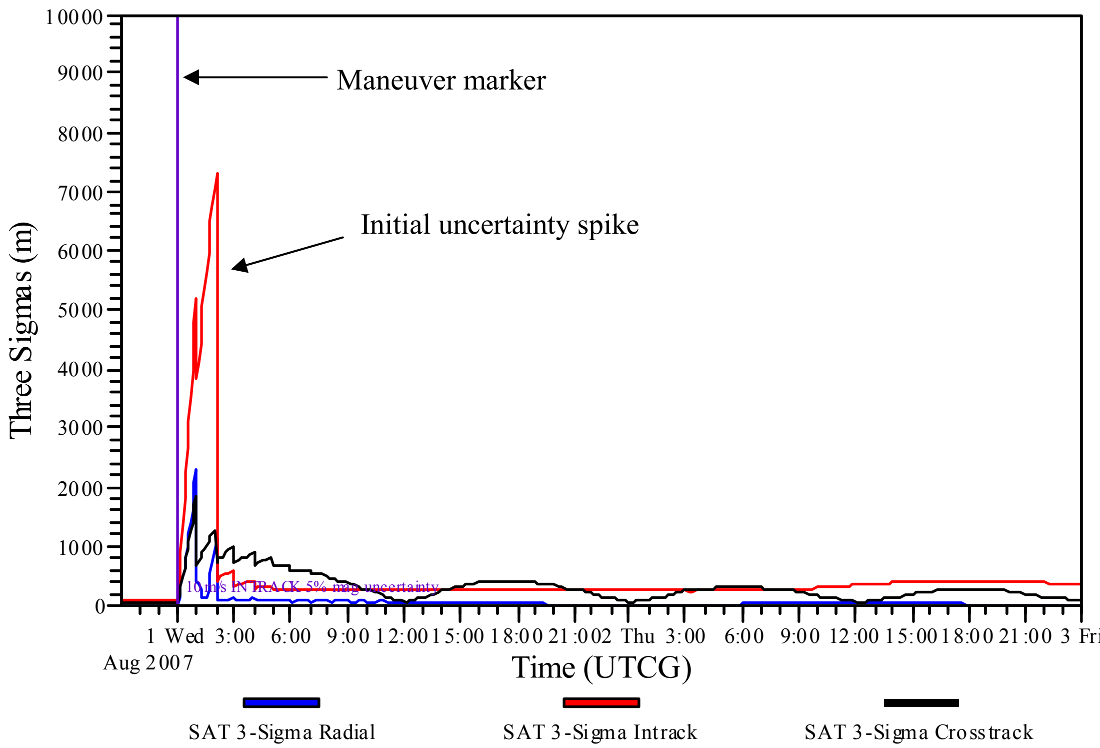


**Figure 1. RIC Position Uncertainty**

## OBSERVABILITY GUIDELINES

The observability of a maneuver error is dependent on the time since the maneuver and the type of measurements being used. Detailed simulations must be performed to evaluate the performance of a tracking system for a given situation. This is often time consuming and operators often don't have the proper tools for making such runs. Therefore various "rules of thumb" are often used in lieu of detailed simulations.

The basic premise is to understand when the position error is maximized and assume that this is a reasonable proxy for understanding when the error is most observable in the measurements. This leads to the following concepts where T is the orbit period of the new orbit:

- An intrack maneuver error maximizes the position error at ½T after the maneuver.

[*] Wright, James. Analytical Graphics, Inc., 2008. Personal communication.

- A crosstrack maneuver error maximizes the position error at ¼T and ¾T after the maneuver.
- A radial maneuver error maximizes the position error at ½T after the maneuver.

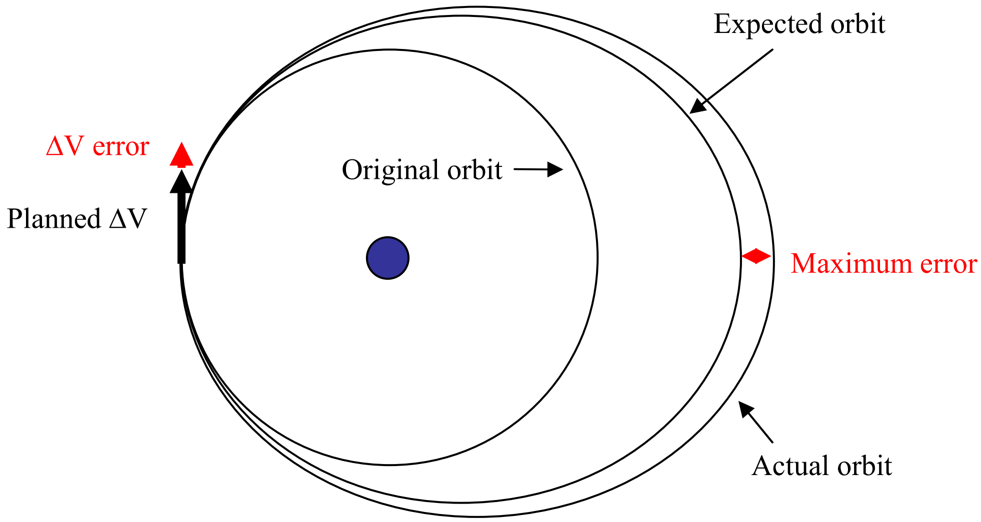


**Figure 2. Intrack Maneuver Position Error**

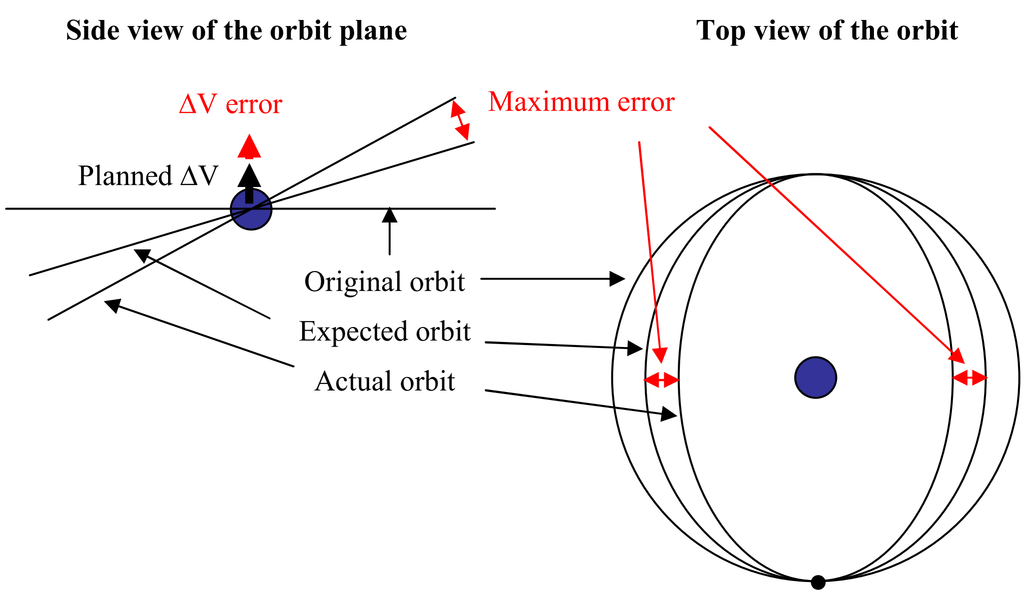


**Figure 3. Crosstrack Maneuver Position Error**

The first two are illustrated in Figure 2 and Figure 3. The radial error is similar to the intrack error in that the maneuver error and new velocity vector are in the orbit plane. It's important to note several limitations:

- They do not address the direction of the position error, only the magnitude.

- Errors may be observable at other points in the orbit as well.

How well a particular measurement can observe the error is effected by both of these. As a general guideline post-maneuver passes should ensure coverage at 1/4T, 1/2T, and 3/4T. Additional passes may be necessary as well. The next step is to identify how well the filter performs using tracking data throughout the orbit and establish how fast it can converge and what variables can affect it.

## SIMULATION CONFIGURATION

The simulation has two tracking facilities located in New York City and Rio de Janeiro and a geostationary satellite at roughly 4 deg West longitude. Tables 1 and 2 describe the location and measurement statistics of each facility. Tables 3 and 4 describe the satellite orbit and force model configuration.

**Table 1. Facility Properties**

| | Rio de Janeiro | New York |
|---|---|---|
| Latitude | 22.7215 deg S | 40.7143 deg N |
| Longitude | 43.4552 deg W | 74.006 deg W |
| Altitude | 0 m | 0 m |

**Table 2. Measurement Statistics**

| **Measurement Type** | **White Noise (1σ)** |
|---|---|
| Range | 5 m |
| Right Ascension | 5 arc sec |
| Declination | 5 arc sec |

**Table 3. Satellite Initial State**

| | |
|---|---|
| Epoch | 1 Jul 2007 00:00:00 |
| Coordinate Frame | True of Date |
| Semi-major Axis | 42166.34653 km |
| Eccentricity | 0.001 |
| Inclination | 0.002475 deg |
| RAAN | 287.9122 deg |
| Arg. of Perigee | 358.4398 deg |
| Arg. of Latitude | 346.7428 deg |

**Table 4. Force Models**

| | |
|---|---|
| Mass | 1000 kg |
| Gravity | 8 x 8 EGM-96 |
| $3^{rd}$ Body Gravity | Sun<br>Moon |
| Solar Radiation Pressure Model | Spherical |
| Area | 20 $m^2$ |
| Cr | 1.0 |

The map in Figure 4 illustrates the tracking geometry. These particular tracking station locations were chosen to obtain a reasonable good tracking geometry for a geostationary satellite – one in each hemisphere and a significant longitude separation relative to the satellite.

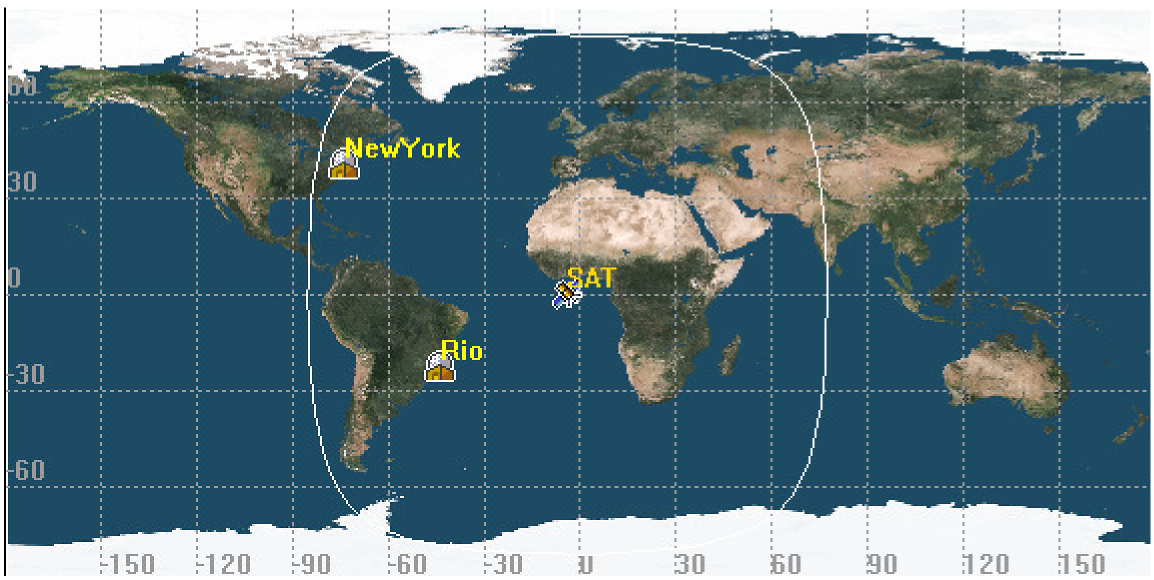


**Figure 4. Tracking Geometry**

Simulated tracking data measurements from each facility are created over a period of 41 days (1 July 2007 00:00 to 11 August 2007 00:00). Tracking passes are generated every hour on the hour (e.g. 00:00, 01:00, 02:00, etc.). Each pass consists of five measurements at one second intervals. The 31 days in July are used to establish a stable, baseline set of outputs from the filter. An impulsive maneuver is performed at 1 August 2007 00:00 and another 10 days of measurements are processed.

Range measurements are used as they are the most common measurement used by owner-operators of geostationary satellites and the hourly tracking schedule is typical of a tracking system with dedicated antennas per satellite. The hourly schedule is also useful for ensuring sufficient sampling throughout the orbit to determine when convergence occurs. Angle measurements are used because they are the most common measurement used by the space surveillance community for tracking geostationary satellites. The tracking schedule in this case is overly aggressive and does not reflect the practical considerations of limited visibility due to the sun-satellite-observer phase angle and obscuration due to weather.

A parametric analysis was performed for a range of maneuver scenarios to determine which variables had an impact on the convergence of the orbital elements. Table 5 illustrates the variables and the range of values that were simulated. Note that right ascension and declination angle pairs were treated as one measurement type. The relative convergence criterion is 10% from Eq (2) and the absolute convergence thresholds are listed in Table 6. The absolute thresholds are conservative and were selected assuming weaker measurements (angles) were being used. The RIC and Keplerian thresholds were selected independently from each other, meaning one was not transformed to obtain the other.

**Table 5. Input Variables**

| | |
|---|---|
| Measurement | Range<br>Right Ascension<br>Declination |
| Maneuver Direction | Radial<br>Intrack<br>Crosstrack |
| Maneuver Magnitude | 0.01 m/s<br>0.1 m/s<br>1 m/s<br>10 m/s |
| Maneuver Magnitude Uncertainty (1σ) | 1 %<br>5 %<br>10 % |
| Maneuver Direction Uncertainty (1σ) | 0.001 deg<br>0.1 deg<br>1 deg<br>5 deg |

**Table 6. Absolute Convergence Criteria (3σ)**

| Parameter | Threshold |
|---|---|
| Radial Position | 1000 m |
| Intrack Position | 5000 m |
| Crosstrack Position | 2000 m |
| Radial Velocity | 0.30 m/s |
| Intrack Velocity | 0.10 m/s |
| Crosstrack Velocity | 0.20 m/s |
| Semi-major Axis | 1000 m |
| Eccentricity | 0.0005 |
| Inclination | 0.005 deg |
| RAAN | 2 deg |
| Arg. of Perigee | 2 deg |
| Arg. of Latitude | 2 deg |

## RESULTS

A parametric analysis was performed varying the maneuver magnitude, direction, magnitude uncertainty, and direction uncertainty, and the type of measurements resulting in 288 test cases. The results are examined along different lines of inquiry to establish which parameters were significant and draw conclusions regarding the validity of the proposed rules. The convergence times have a resolution of one hour due to the tracking schedule so it's not uncommon to see a convergence threshold just made or missed by one pass so the results sometimes vary by an hour or two between test cases as different variables are changed.

### Maneuver Magnitude Uncertainty

The first test examines the effect of the maneuver magnitude uncertainty. Range measurements are used and the maneuver magnitude and direction uncertainty fixed at 0.01 m/s and 0.001 deg with results in Table 7. The results are very optimistic as compared to the proposed rules with many of the parameters converging within the first hour. Only the results from the 10% magnitude uncertainty cases are in line with the expected values. The results from the full range of maneuver magnitudes were checked with the final case of 10 m/s shown in Table 8. The full set of results show that once the maneuver magnitude is 0.1 m/s or larger there is a sufficient increase in the sigmas that then takes the expected time to resolve. The inclination, RAAN, Arg. of Perigee, and Arg. of Latitude elements still show very short convergence times. Inspection of the filter output graphs show that these results are correct. The range measurements are strong enough that the filter can easily resolve these parameters.

**Table 7. Relative Convergence Time for Range, ΔV = 0.01 m/s, Dir. σ = 0.001 deg**

| | | Relative Convergence Time (hrs) | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | Magnitude Sigma | | | | Magnitude Sigma | | |
| ΔV Direction | | 1% | 5% | 10% | | 1% | 5% | 10% |
| RADIAL | Radial Pos | 11 | 11 | 11 | Semimajor Axis | 11 | 11 | 11 |
| | Intrack Pos | 4 | 4 | 4 | Eccentricity | 11 | 11 | 11 |
| | Crosstrack Pos | 1 | 1 | 1 | Inclination | 1 | 1 | 1 |
| | Radial Vel | 8 | 8 | 8 | RAAN | 1 | 1 | 1 |
| | Intrack Vel | 11 | 11 | 11 | Arg of Perigee | 2 | 2 | 2 |
| | Crosstrack Vel | 1 | 1 | 1 | Arg of Latitude | 1 | 1 | 1 |
| INTRACK | Radial Pos | 1 | 1 | 10 | Semimajor Axis | 1 | 1 | 10 |
| | Intrack Pos | 1 | 1 | 1 | Eccentricity | 1 | 1 | 10 |
| | Crosstrack Pos | 1 | 1 | 1 | Inclination | 1 | 1 | 1 |
| | Radial Vel | 1 | 1 | 1 | RAAN | 1 | 1 | 1 |
| | Intrack Vel | 1 | 1 | 10 | Arg of Perigee | 1 | 1 | 1 |
| | Crosstrack Vel | 1 | 1 | 1 | Arg of Latitude | 1 | 1 | 1 |
| CROSSTRACK | Radial Pos | 11 | 11 | 11 | Semimajor Axis | 11 | 11 | 11 |
| | Intrack Pos | 4 | 4 | 4 | Eccentricity | 11 | 11 | 11 |
| | Crosstrack Pos | 1 | 1 | 1 | Inclination | 1 | 1 | 2 |
| | Radial Vel | 8 | 8 | 8 | RAAN | 1 | 1 | 1 |
| | Intrack Vel | 11 | 11 | 11 | Arg of Perigee | 2 | 2 | 2 |
| | Crosstrack Vel | 1 | 1 | 2 | Arg of Latitude | 1 | 1 | 2 |

**Table 8. Relative Convergence Time for Range, ΔV = 10 m/s, Dir. σ = 0.001 deg**

| | | Relative Convergence Time (hrs) | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | Magnitude Sigma | | | | Magnitude Sigma | | |
| ΔV Direction | | 1% | 5% | 10% | | 1% | 5% | 10% |
| RADIAL | Radial Pos | 9 | 9 | 9 | Semimajor Axis | 9 | 9 | 9 |
| | Intrack Pos | 2 | 2 | 2 | Eccentricity | 9 | 9 | 9 |
| | Crosstrack Pos | 1 | 1 | 1 | Inclination | 1 | 1 | 1 |
| | Radial Vel | 7 | 7 | 7 | RAAN | 1 | 1 | 1 |
| | Intrack Vel | 9 | 9 | 9 | Arg of Perigee | 1 | 2 | 2 |
| | Crosstrack Vel | 1 | 1 | 1 | Arg of Latitude | 1 | 1 | 1 |
| INTRACK | Radial Pos | 11 | 11 | 11 | Semimajor Axis | 11 | 11 | 11 |
| | Intrack Pos | 4 | 4 | 4 | Eccentricity | 11 | 11 | 11 |
| | Crosstrack Pos | 1 | 1 | 1 | Inclination | 1 | 1 | 1 |
| | Radial Vel | 8 | 8 | 8 | RAAN | 1 | 1 | 1 |
| | Intrack Vel | 11 | 11 | 11 | Arg of Perigee | 2 | 2 | 2 |
| | Crosstrack Vel | 1 | 1 | 1 | Arg of Latitude | 1 | 1 | 1 |
| CROSSTRACK | Radial Pos | 11 | 11 | 11 | Semimajor Axis | 11 | 11 | 11 |
| | Intrack Pos | 4 | 4 | 4 | Eccentricity | 11 | 11 | 11 |
| | Crosstrack Pos | 6 | 6 | 6 | Inclination | 5 | 5 | 5 |
| | Radial Vel | 8 | 8 | 8 | RAAN | 5 | 5 | 5 |
| | Intrack Vel | 11 | 11 | 11 | Arg of Perigee | 5 | 5 | 5 |
| | Crosstrack Vel | 5 | 5 | 5 | Arg of Latitude | 5 | 5 | 5 |

The results show that when using range measurements the maneuver magnitude uncertainty does not impact the relative convergence time. The radial and intrack orbital elements have relative convergence within ½ T and the crosstrack components within ¼ T. The same is true of the Keplerian elements with semimajor axis and eccentricity converging within ½ T and Inclination, RAAN, Arg. of Perigee, and Arg. of Latitude converging within ¼ T.

Table 9 shows the absolute convergence results using the same case as Table 8. The results are very good – a combination of a strong measurement (range) and absolute convergence thresholds that are conservative.  Figure 5 shows the RIC position sigmas – the filter has clearly met the operational thresholds. This highlights the need to evaluate both convergence definitions.

**Table 9. Absolute Convergence Time for Range, ΔV = 10 m/s, Dir. σ = 0.001 deg**

| | | Absolute Convergence Time (hrs) | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | Magnitude Sigma | | | | Magnitude Sigma | | |
| ΔV Direction | | 1% | 5% | 10% | | 1% | 5% | 10% |
| RADIAL | Radial Pos | 1 | 1 | 1 | Semimajor Axis | 1 | 1 | 1 |
| | Intrack Pos | 1 | 1 | 1 | Eccentricity | 1 | 1 | 1 |
| | Crosstrack Pos | 1 | 1 | 1 | Inclination | 1 | 1 | 1 |
| | Radial Vel | 1 | 1 | 1 | RAAN | 1 | 1 | 1 |
| | Intrack Vel | 1 | 1 | 1 | Arg of Perigee | 1 | 1 | 1 |
| | Crosstrack Vel | 1 | 1 | 1 | Arg of Latitude | 1 | 1 | 1 |
| INTRACK | Radial Pos | 1 | 1 | 1 | Semimajor Axis | 2 | 2 | 2 |
| | Intrack Pos | 1 | 1 | 1 | Eccentricity | 1 | 1 | 1 |
| | Crosstrack Pos | 1 | 1 | 1 | Inclination | 1 | 1 | 1 |
| | Radial Vel | 1 | 1 | 1 | RAAN | 1 | 1 | 1 |
| | Intrack Vel | 1 | 1 | 1 | Arg of Perigee | 1 | 1 | 1 |
| | Crosstrack Vel | 1 | 1 | 1 | Arg of Latitude | 1 | 1 | 1 |
| CROSSTRACK | Radial Pos | 1 | 1 | 1 | Semimajor Axis | 2 | 2 | 2 |
| | Intrack Pos | 1 | 1 | 1 | Eccentricity | 1 | 1 | 1 |
| | Crosstrack Pos | 1 | 1 | 1 | Inclination | 1 | 1 | 1 |
| | Radial Vel | 1 | 1 | 1 | RAAN | 1 | 1 | 1 |
| | Intrack Vel | 1 | 1 | 1 | Arg of Perigee | 1 | 1 | 1 |
| | Crosstrack Vel | 1 | 1 | 1 | Arg of Latitude | 1 | 1 | 1 |

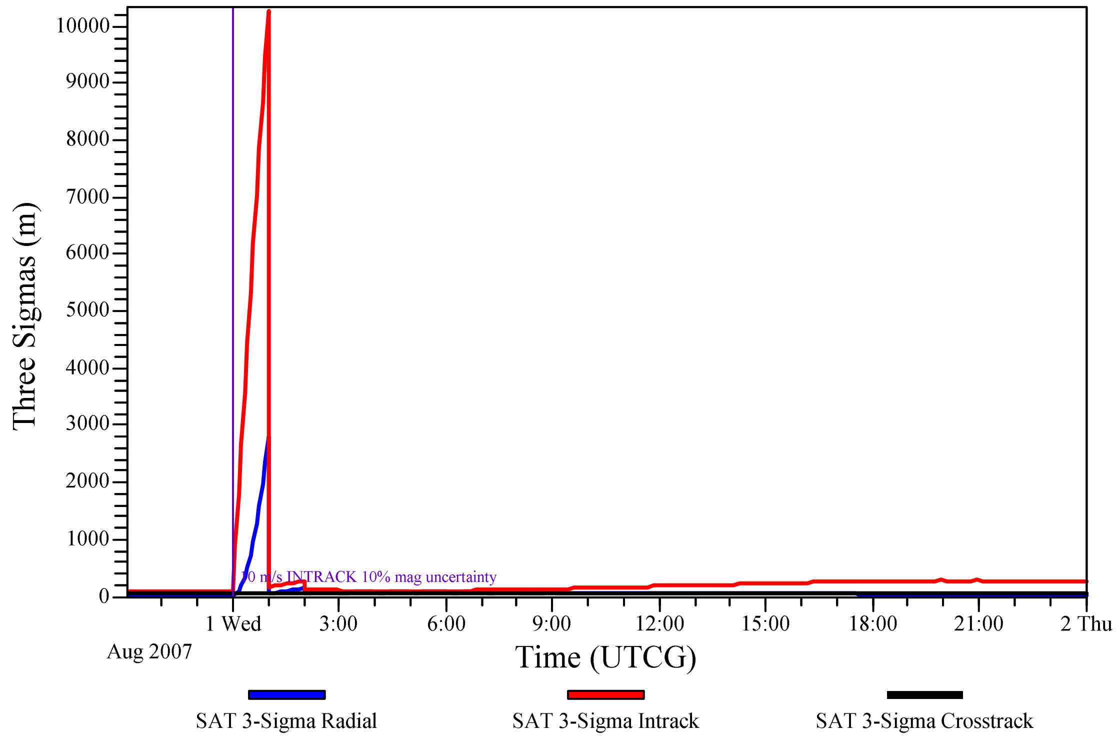


**Figure 5. RIC Position Uncertainty for Range, Intrack ΔV = 10 m/s, Dir. σ = 0.001 deg**

## Maneuver Direction Uncertainty

The next test examines the effect of the maneuver direction uncertainty. Range measurements were used and the maneuver magnitude and magnitude uncertainty fixed at 10 m/s and 1% with results in Table 10. Results from smaller maneuvers were consistent with those seen in Table 7 and indicated rapid relative convergence. Once the maneuver magnitude was 0.1 m/s or larger they were consistent with the results in Table 10 so they are not included in the paper. It's worth noting that the direction sigma has some influence on the relative convergence time with some parameters taking as long as 5/8 T (15 hrs). The absolute convergence results are practically identical to those seen in Table 9 and therefore are not included.

**Table 10. Relative Convergence Time for Range, ΔV = 10 m/s, Mag. σ = 1%**

| | | Relative Convergence Time (hrs) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | Direction Sigma (deg) | | | | | Direction Sigma (deg) | | | |
| ΔV Direction | | 0.001 | 0.1 | 1 | 5 | | 0.001 | 0.1 | 1 | 5 |
| RADIAL | Radial Pos | 9 | 14 | 14 | 14 | Semimajor Axis | 9 | 13 | 13 | 13 |
| | Intrack Pos | 2 | 7 | 7 | 7 | Eccentricity | 9 | 10 | 10 | 10 |
| | Crosstrack Pos | 1 | 6 | 7 | 7 | Inclination | 1 | 6 | 7 | 7 |
| | Radial Vel | 7 | 4 | 4 | 4 | RAAN | 1 | 6 | 7 | 7 |
| | Intrack Vel | 9 | 15 | 15 | 15 | Arg of Perigee | 1 | 6 | 7 | 7 |
| | Crosstrack Vel | 1 | 5 | 6 | 6 | Arg of Latitude | 1 | 6 | 7 | 7 |
| INTRACK | Radial Pos | 11 | 14 | 14 | 14 | Semimajor Axis | 11 | 13 | 13 | 13 |
| | Intrack Pos | 4 | 7 | 7 | 7 | Eccentricity | 11 | 13 | 13 | 13 |
| | Crosstrack Pos | 1 | 1 | 7 | 7 | Inclination | 1 | 1 | 7 | 7 |
| | Radial Vel | 8 | 4 | 4 | 4 | RAAN | 1 | 1 | 7 | 7 |
| | Intrack Vel | 11 | 14 | 14 | 14 | Arg of Perigee | 2 | 5 | 7 | 7 |
| | Crosstrack Vel | 1 | 1 | 6 | 6 | Arg of Latitude | 1 | 1 | 7 | 7 |
| CROSSTRACK | Radial Pos | 11 | 14 | 14 | 14 | Semimajor Axis | 11 | 13 | 13 | 13 |
| | Intrack Pos | 4 | 7 | 7 | 7 | Eccentricity | 11 | 13 | 13 | 13 |
| | Crosstrack Pos | 6 | 6 | 7 | 7 | Inclination | 5 | 6 | 7 | 7 |
| | Radial Vel | 8 | 4 | 4 | 4 | RAAN | 5 | 6 | 7 | 7 |
| | Intrack Vel | 11 | 14 | 14 | 14 | Arg of Perigee | 5 | 6 | 7 | 7 |
| | Crosstrack Vel | 5 | 6 | 6 | 6 | Arg of Latitude | 5 | 6 | 7 | 7 |

## Measurement Choice

The test cases so far all used range measurements and easily achieved the operational thresholds. The next step is to investigate the results when using angle measurements. The cases in Table 7 and Table 8 were repeated using angle measurements and the relative convergence results presented in Table 11 and Table 12. The convergence times are still invariant with respect to the maneuver sigma, however, there's an overall degradation in the convergence times relative to the equivalent range cases. Instead of converging near the smaller side of ½ T (11 hours) we now see times on the larger side of ½ T (14 hours).

The absolute convergence results in Table 13 are much more interesting. It's the same case as Table 11 – a very small maneuver and direction uncertainty. The RIC elements are taking significantly longer to converge along with the some of the Keplerian elements. The weaker angle measurements are not as effective at observing the parameters. The relative convergence performance indicates that the filter was effective at removing the initial uncertainty spike, but the limited observability means it will take longer to drive the uncertainty within the operational threshold as seen in Figure 6. The intrack maneuver has excellent recovery times as the uncertainty is still along the intrack axis. The radial and crosstrack maneuvers are not as observable and take much longer to recover. Larger maneuvers have correspondingly worse recovery, even the intrack ones.

**Table 11. Relative Convergence Time for Angles, ΔV = 0.01 m/s, Dir. σ = 0.001 deg**

| | | Relative Convergence Time (hrs) | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | Magnitude Sigma | | | | Magnitude Sigma | | |
| ΔV Direction | | 1% | 5% | 10% | | 1% | 5% | 10% |
| RADIAL | Radial Pos | 14 | 14 | 14 | Semimajor Axis | 14 | 14 | 14 |
| | Intrack Pos | 14 | 14 | 14 | Eccentricity | 14 | 14 | 14 |
| | Crosstrack Pos | 1 | 1 | 1 | Inclination | 1 | 1 | 1 |
| | Radial Vel | 14 | 14 | 14 | RAAN | 1 | 1 | 1 |
| | Intrack Vel | 14 | 14 | 14 | Arg of Perigee | 1 | 2 | 2 |
| | Crosstrack Vel | 1 | 1 | 1 | Arg of Latitude | 1 | 1 | 1 |
| INTRACK | Radial Pos | 1 | 1 | 1 | Semimajor Axis | 1 | 1 | 1 |
| | Intrack Pos | 1 | 1 | 1 | Eccentricity | 1 | 1 | 1 |
| | Crosstrack Pos | 1 | 1 | 1 | Inclination | 1 | 1 | 1 |
| | Radial Vel | 1 | 1 | 1 | RAAN | 1 | 1 | 1 |
| | Intrack Vel | 1 | 1 | 1 | Arg of Perigee | 1 | 1 | 1 |
| | Crosstrack Vel | 1 | 1 | 1 | Arg of Latitude | 1 | 1 | 1 |
| CROSSTRACK | Radial Pos | 14 | 14 | 14 | Semimajor Axis | 14 | 14 | 14 |
| | Intrack Pos | 14 | 14 | 14 | Eccentricity | 14 | 14 | 14 |
| | Crosstrack Pos | 1 | 1 | 1 | Inclination | 1 | 1 | 1 |
| | Radial Vel | 14 | 14 | 14 | RAAN | 1 | 1 | 1 |
| | Intrack Vel | 14 | 14 | 14 | Arg of Perigee | 1 | 2 | 2 |
| | Crosstrack Vel | 1 | 1 | 1 | Arg of Latitude | 1 | 1 | 1 |

**Table 12. Relative Convergence Time for Angles, ΔV = 10 m/s, Dir. σ = 0.001 deg**

| | | Relative Convergence Time (hrs) | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | Magnitude Sigma | | | | Magnitude Sigma | | |
| ΔV Direction | | 1% | 5% | 10% | | 1% | 5% | 10% |
| RADIAL | Radial Pos | 12 | 13 | 13 | Semimajor Axis | 12 | 13 | 13 |
| | Intrack Pos | 12 | 12 | 12 | Eccentricity | 12 | 12 | 12 |
| | Crosstrack Pos | 1 | 1 | 1 | Inclination | 1 | 1 | 1 |
| | Radial Vel | 12 | 12 | 12 | RAAN | 1 | 1 | 1 |
| | Intrack Vel | 12 | 13 | 13 | Arg of Perigee | 1 | 1 | 1 |
| | Crosstrack Vel | 1 | 1 | 1 | Arg of Latitude | 1 | 1 | 1 |
| INTRACK | Radial Pos | 14 | 14 | 14 | Semimajor Axis | 14 | 14 | 14 |
| | Intrack Pos | 14 | 14 | 14 | Eccentricity | 14 | 14 | 14 |
| | Crosstrack Pos | 1 | 1 | 1 | Inclination | 1 | 1 | 1 |
| | Radial Vel | 14 | 14 | 14 | RAAN | 1 | 1 | 1 |
| | Intrack Vel | 14 | 14 | 14 | Arg of Perigee | 1 | 2 | 2 |
| | Crosstrack Vel | 1 | 1 | 1 | Arg of Latitude | 1 | 1 | 1 |
| CROSSTRACK | Radial Pos | 14 | 14 | 14 | Semimajor Axis | 14 | 14 | 14 |
| | Intrack Pos | 14 | 14 | 14 | Eccentricity | 14 | 14 | 14 |
| | Crosstrack Pos | 10 | 10 | 10 | Inclination | 11 | 11 | 11 |
| | Radial Vel | 14 | 14 | 14 | RAAN | 11 | 11 | 11 |
| | Intrack Vel | 14 | 14 | 14 | Arg of Perigee | 11 | 11 | 11 |
| | Crosstrack Vel | 11 | 12 | 12 | Arg of Latitude | 11 | 11 | 11 |

**Table 13. Absolute Convergence Time for Angles, ΔV = 0.01 m/s, Dir. σ = 0.001 deg**

| | | Absolute Convergence Time (hrs) | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | Magnitude Sigma | | | | Magnitude Sigma | | |
| ΔV Direction | | 1% | 5% | 10% | | 1% | 5% | 10% |
| RADIAL | Radial Pos | 38 | 38 | 38 | Semimajor Axis | 17 | 17 | 17 |
| | Intrack Pos | 117 | 117 | 117 | Eccentricity | 1 | 4 | 5 |
| | Crosstrack Pos | 1 | 1 | 1 | Inclination | 1 | 1 | 1 |
| | Radial Vel | 171 | 171 | 171 | RAAN | 1 | 1 | 1 |
| | Intrack Vel | 15 | 15 | 15 | Arg of Perigee | 1 | 1 | 1 |
| | Crosstrack Vel | 1 | 1 | 1 | Arg of Latitude | 1 | 1 | 1 |
| INTRACK | Radial Pos | 1 | 1 | 1 | Semimajor Axis | 1 | 1 | 1 |
| | Intrack Pos | 1 | 1 | 1 | Eccentricity | 1 | 1 | 1 |
| | Crosstrack Pos | 1 | 1 | 1 | Inclination | 1 | 1 | 1 |
| | Radial Vel | 1 | 1 | 1 | RAAN | 1 | 1 | 1 |
| | Intrack Vel | 1 | 1 | 1 | Arg of Perigee | 1 | 1 | 1 |
| | Crosstrack Vel | 1 | 1 | 1 | Arg of Latitude | 1 | 1 | 1 |
| CROSSTRACK | Radial Pos | 38 | 38 | 38 | Semimajor Axis | 17 | 17 | 17 |
| | Intrack Pos | 117 | 117 | 117 | Eccentricity | 1 | 4 | 5 |
| | Crosstrack Pos | 1 | 1 | 1 | Inclination | 1 | 1 | 1 |
| | Radial Vel | 171 | 171 | 171 | RAAN | 1 | 1 | 1 |
| | Intrack Vel | 15 | 15 | 15 | Arg of Perigee | 1 | 1 | 1 |
| | Crosstrack Vel | 1 | 1 | 1 | Arg of Latitude | 1 | 1 | 1 |

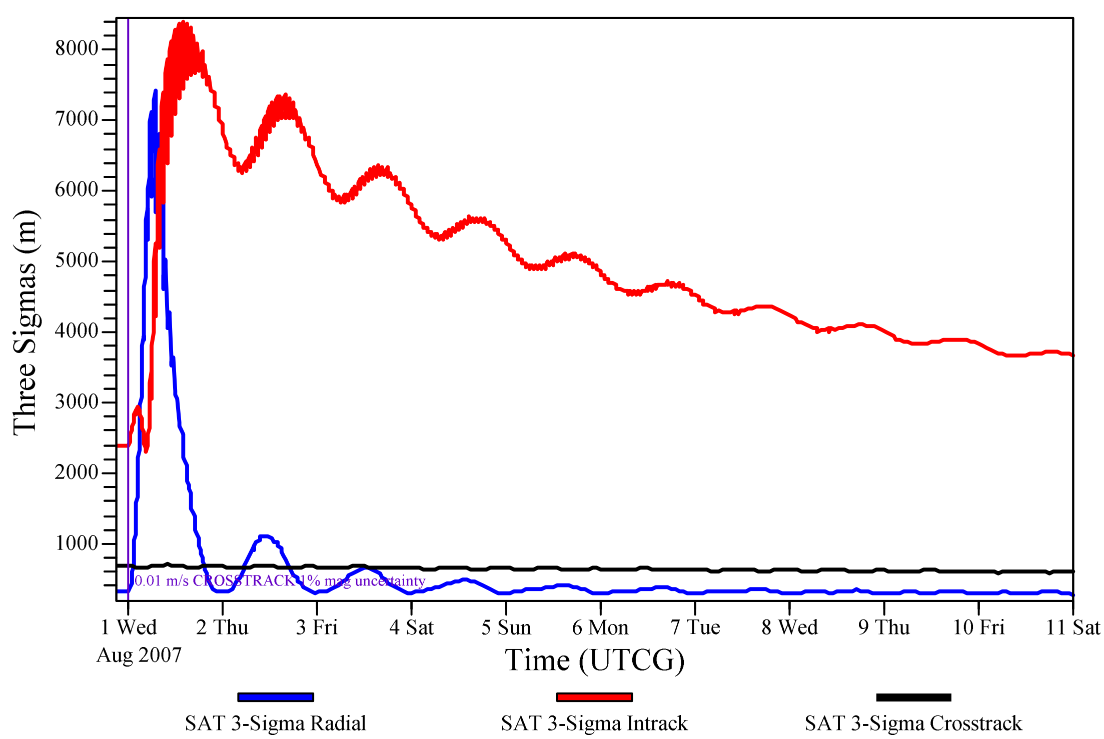


**Figure 6. RIC Position Uncertainty for Angles, Crosstrack ΔV = 0.01 m/s, Mag σ = 1%, Dir. σ = 0.001 deg**

## CONCLUSIONS

The relative convergence performance confirms that the first ¾ T hours following a maneuver is significant for obtaining a converged orbit solution and only ½ T hours is necessary in many cases. This is consistent with the proposed rules of thumb. However, the operational thresholds may be met much sooner or later than ½ T, depending on the type and quality of the available tracking data. Meeting the relative convergence threshold is not a necessary condition for meeting the operational thresholds, as demonstrated with the angle measurements. A simulation must be performed to analyze the absolute performance.

The maneuver magnitude and direction sigmas are not primary drivers for affecting the convergence times. This assumes that any errors in the maneuver fall within the specified sigmas. If not, the filter is likely to diverge.

It would be useful to extend this analysis to future work examining the impact of the following additional variables: measurement accuracy, tracking schedules, additional orbits regimes such as LEO or GTO, measurement types such as Doppler and GPS, and using a variable lag smoother

## REFERENCES

[1] Hujsak, Richard S., Woodburn, James W., Seago, John H., "The Orbit Determination Tool Kit (ODTK) – Version 5," Paper AAS 07-125, presented at the AAS/AIAA Space Flight Mechanics Meeting, Sedona, AZ, January 2007.

[2] Hujsak, Richard S, "Orbit Determination during High Thrust and Low Thrust Maneuvers", Paper AAS 05-136, Presented at the 15th AAS/AIAA Space Flight Mechanics Conference, San Diego, CA, January 2005.