DETERMINATION OF CLOSE APPROACHES FOR EARTH-FIXED LAUNCH TRAJECTORIES

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The deployment of large constellations of satellites in Low Earth Orbit increases the probability of close approaches with newly launched satellites. A new algorithm for determining launch window blackout intervals based on the avoidance of close approaches for launch trajectories which are fixed relative to the Earth Fixed coordinate system is presented. This algorithm is more robust than the technique of performing close approach analyses by sampling the launch window and significantly more efficient. The example case presented shows an improvement in efficiency by a factor of 200 while detecting blackout intervals as small as 0.3 seconds in duration.

INTRODUCTION

As the number of objects in orbit about the Earth increases, the determination of close approaches between objects is becoming an increasingly important aspect of satellite operations. A recent study by Jenkins and Schumacher[1] indicates the growing importance of close approach prediction for the Shuttle and the Mir space station. The basic problem is to determine when two objects will have a conjunction where the risk of collision is unacceptably large. There are many ways of defining what constitutes risk. These definitions range in complexity from the specification of a minimum allowable separation distance between the two objects to using complex probability density functions to determine the statistical probability of collision during a conjunction.

Prior studies of the close approach problem have focused on the case where all objects are assumed to be in orbit about the Earth and the nominal ephemeris of the objects is known. It is also assumed that no propulsive forces are applied. The method of locating close approaches to the primary object typically entails using a set of filters in order of increasing computational burden to eliminate objects which are candidates for close approaches from consideration. The source of data which defines the orbital elements of the tracked objects in orbit about the Earth is the space catalogue maintained by the United States Space Command. Hoots et al.[2] designed a series of three filters through which candidate objects have to pass before a final determination of the close approach distance is made. One of the filters is purely geometrical and two utilize known properties of the orbital motion of the two objects. These filters serve to “weed out” the majority of the objects in the catalogue and greatly reduce the number of computations needed. After the

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application of the filters, the trajectories of the remaining candidate objects are sampled to determine the actual close approach periods. The exclusion zone is modeled as a sphere centered at the primary satellite. A special adaptation of the algorithms presented in Hoots et al.[2] has been developed to allow for efficient predictions of close approaches for entire constellations of satellites[3].

Alfano and Negron[4] developed a technique for modeling the distance between two objects using localized cubic polynomials. In this approach, the geometrical filter and first orbital motion-dependent filter developed by Hoots et al.[2] are still applied, but the final filter, referred to as the time filter, is removed. The trajectories of the vehicles are then sampled at large time steps (up to 10 minutes) to create waveforms describing either the relative distance[4] or range rate[5] between the satellites. This waveform provides a model from which estimates of the time of closest approach and the entrance and exit times for crossing an exclusion zone boundary are made. The work of Alfano and Negron models the exclusion zone boundary as an ellipsoid centered at the primary satellite to account for uncertainties in the along-track position of the objects being greater than the uncertainty in the cross-track and radial directions. Other authors have approached restricted versions of the problem considering only the distance between the orbital paths[6] or only circular orbits[7].

The detection of close approaches to satellites during the launch and early post-deployment phase of their lifetimes is an important subset of the overall problem. Potential collisions during this period can usually be avoided by adjusting the time of launch. Standard close approach detection methods cannot be applied since most of the assumptions made in satellite to satellite close approach algorithms are violated when launch trajectories are considered. First, the trajectory of the launch vehicle is heavily influenced by thrusting. As a result, any filters that depend on an assumption of orbital motion can not be used. Second, the launch time is typically restricted to be within a launch window, but the exact time of launch is not known prior to liftoff. This complication is even more troublesome than the first, since in this case the position of the primary object is not uniquely defined at a given point in time.

One approach to solving the close approach problem for a launch vehicle is to generate the trajectory of the launch vehicle based on possible launch times throughout the launch window. Each of these trajectories can then be analyzed for close approaches and the results accumulated. This process can be very time consuming since the time steps through the launch window must be extremely small to account for the short duration of the conjunctions. This paper addresses an alternative algorithm which has been developed for the case where the launch trajectory can be assumed to be known in the Earth-Centered Earth-Fixed (ECEF) reference frame. This new process can be used to determine close approaches based on a launch time anywhere within the launch window and the corresponding blackout times during the launch window in a small number of runs without concern for the sampling frequency.

COMPUTATION OF CLOSE APPROACHES

The launch window close approach prediction algorithm begins with the generation of an ephemeris for the launch vehicle. This ephemeris is maintained in the ECEF reference frame with the time stored as seconds after launch. This time reference will be referred to as Mission Elapsed Time (MET). The ephemeris of the launch vehicle may, therefore, be held fixed regardless of the launch time. The generated ephemeris is assumed to start from the surface of the Earth and will reach some maximum radius during the span of the generated ephemeris. Since this maximum radius is independent of the time of launch, an apogee/perigee filter can be applied. The application of this filter removes candidate objects from consideration which do not come within the maximum close approach distance of having an altitude overlap with the launch vehicle, see Figure 1.

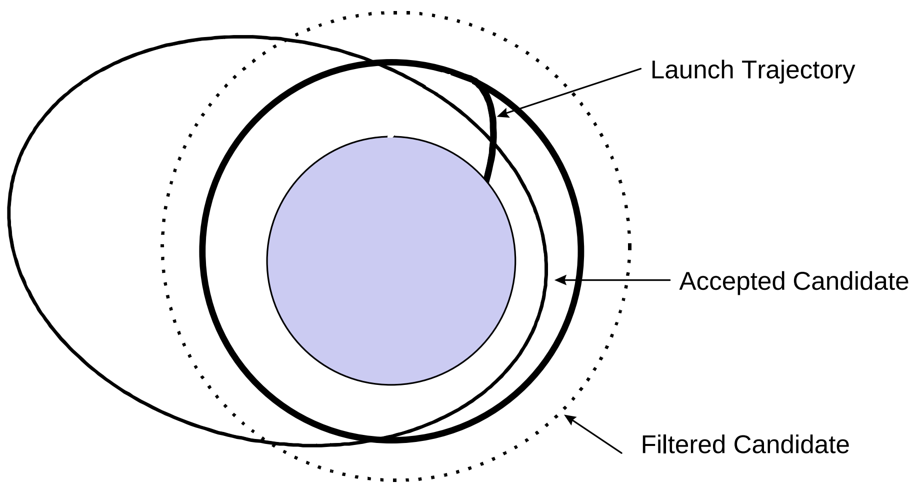

Figure 1. Application of the apogee/perigee filter

The second step in the process is to find the "minimum possible range" between the launch vehicle and the remaining candidate objects at sampled points along the nominal launch vehicle trajectory. Each sample point along the launch vehicle trajectory corresponds to a unique MET. The actual time of launch could be any time within the launch window. The minimum possible range is computed by sampling the range between the launch vehicle and the candidate object, where the position of the candidate object is computed in the ECEF coordinate system based on a small set of launch times throughout the launch window, see Figure 2. These sample launch times must include the end points of the launch window and several points in between. A simple extremum solution algorithm is then used to determine the time within the launch window that corresponds to the smallest distance between the target and candidate objects at each sample location as shown in Figure 3. This method of sampling imposes a constraint on the use of this algorithm that the launch window under consideration cannot be longer in duration than the shortest of the orbital periods of the candidate objects. If the launch window were longer than the orbital period of a candidate object, then it is possible that two minima could occur during the launch window.

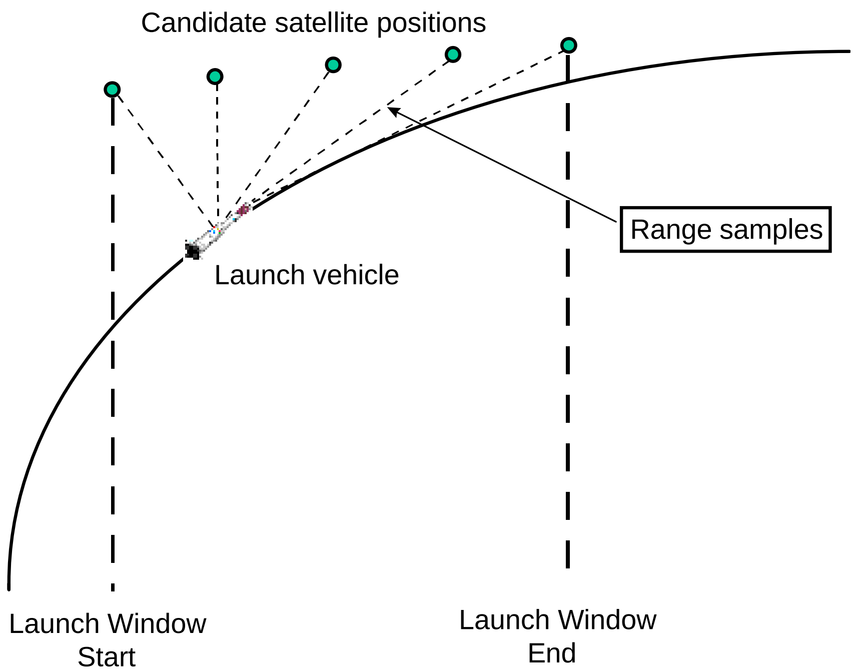

Figure 2. Locus of possible Earth-Centered Earth-Fixed positions for a candidate object considering the launch window

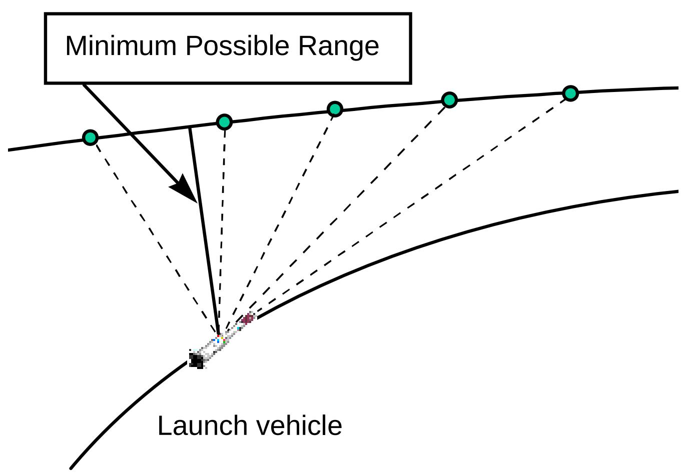

Figure 3. Minimum possible range at a specific time past time of launch

The sampled values of the minimum possible range are subjected to a threshold-crossing detection algorithm to determine when the minimum possible range violates a user defined boundary. The solutions of this process represent intervals in MET when a close approach could occur dependent upon the time of launch. It is very important that the threshold crossing algorithm be able to detect the possibility of crossing pairs between sampled points. This is necessary since the close approach periods may be much smaller than the sampling rate. A simple way to detect this type of crossing is to nest the extremum solution within the threshold crossing algorithm as depicted in Figure 4. Extrema are computed and tested against the threshold value when the slope of the minimum possible range between the two object with respect to the MET changes from negative to positive.

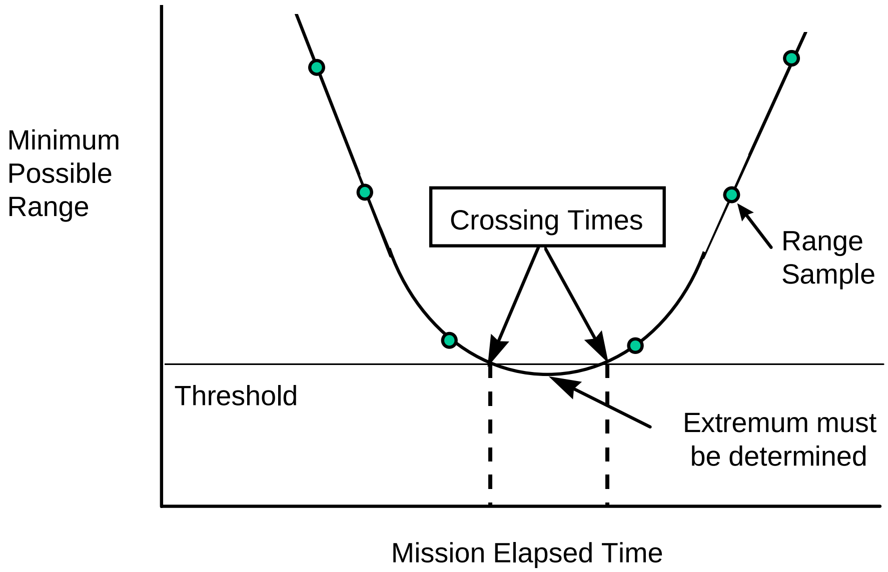

Figure 4. Situation where an extremum is needed for threshold crossing determination

The lowest value of the minimum possible range, the closest possible approach, is then determined within each violation interval. This process may have been accomplished during the threshold crossing detection or may need to be performed separately if a normal sample point was below the threshold value. The solution for the lowest value of the minimum possible range also yields the MET when the closest approach will occur and the corresponding time of launch during the launch window. Since the trajectory of the launch vehicle is referenced to the time of launch, the MET of the closest possible approach is simply the independent variable portion of the extremum solution, see Figure 4. To get the time of launch, in UTC, that corresponds to the closest possible approach, we need to examine the solution of the corresponding minimum possible range, see Figure 5. Since the solution for the minimum possible range involves sampling the position of the candidate object based on the span of the launch window, the independent variable part of the minimum possible range solution corresponds to the time of launch for the closest possible approach.

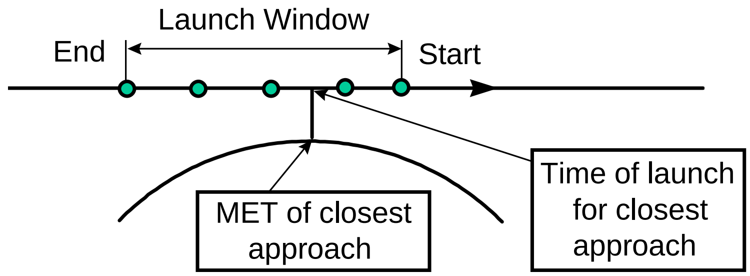

Figure 5. The relationship between the time of launch and MET for the closest possible approach.

COMPUTATION OF LAUNCH WINDOW BLACKOUT PERIODS

The algorithm presented above provides a method for determining which objects in the space catalogue may have close approaches with the launch vehicle. The start and end of the possible conjunction periods are computed in MET. The time of the closest possible approach to each close approach object is computed in MET and is then mapped back into the launch window to give the corresponding time of launch. To complete the picture, the conjunction intervals must now be mapped back to the launch window to yield blackout periods. The blackout periods correspond to launch times that would result in unacceptable conjunctions between the launch vehicle and other orbiting objects.

The relationship between the boundaries of the launch window blackout periods and the conjunction intervals in MET is complicated due to the lack of a one to one relationship between MET and time of launch during a conjunction. A single MET within the conjunction interval corresponds to a range of times within the launch window, see Figure 6. Only the end points of the conjunction interval have a one to one mapping into launch window blackout times. It is tempting to use the mappings of the end points of the conjunction intervals to produce the launch window blackout periods, but this practice can yield blackout periods that are too small. To achieve the correct bounds for the blackout intervals, it is necessary to sample the times computed from the threshold crossing algorithm for the start and end points of the blackout intervals and compute the extrema of their values. It is especially important to sample points close to the boundaries of the conjunction interval due to the shape of the function. Curves representing the mapping of a conjunction interval in MET into launch window blackout start and end times are shown in Figure 7. It is important to note the shapes of the curves shown in Figure 7. The blackout end time increases near the start of the conjunction interval in MET and decreases thereafter. The maximum value of this curve must be determined to yield the proper blackout end time. A similar behavior is seen in the curve for the blackout start time. This behavior is a result of the fact that the slope of the duration of the blackout interval corresponding to a single MET within the conjunction interval with respect to MET is greater than one.

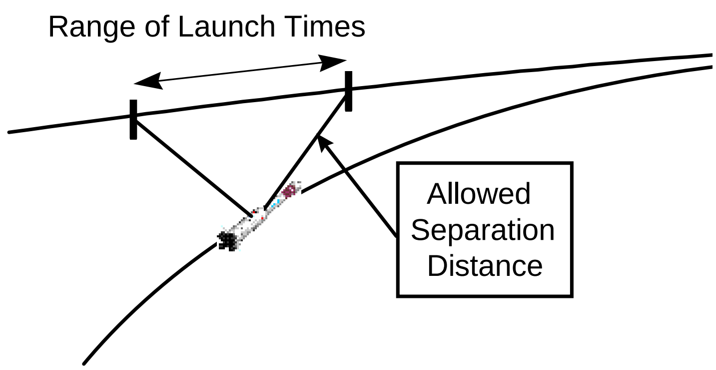

Figure 6. A single MET maps to a range of possible launch times

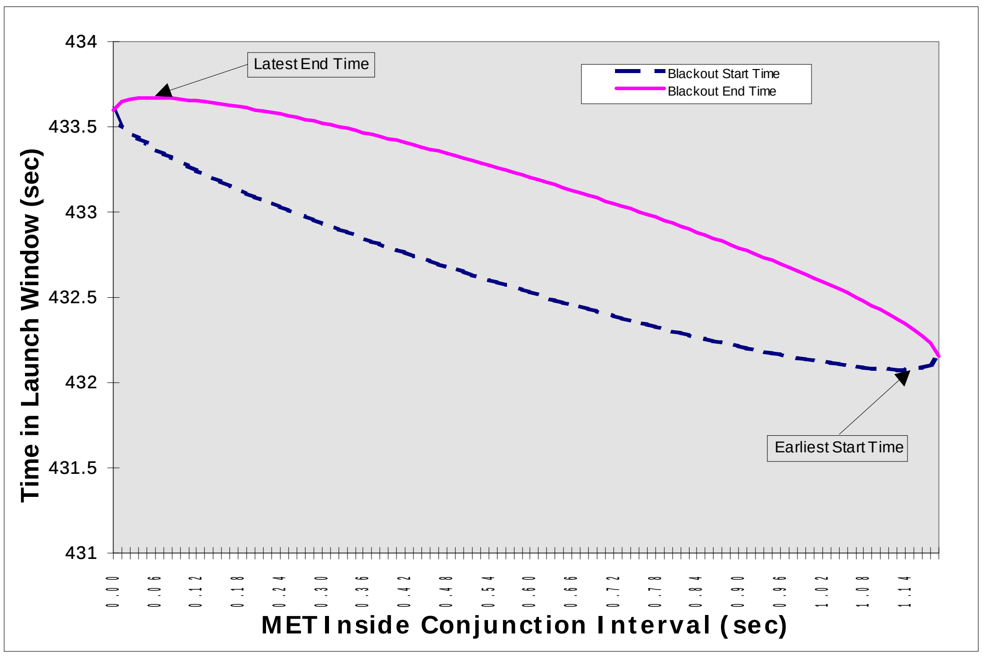

Figure 7. Variation of the mapping of MET to launch window blackout periods

RESULTS

A launch trajectory spanning six hours was generated starting in Florida and inserting a satellite into an orbit with orbital parameters shown in Table 1.

TABLE 1. CLASSICAL ELEMENTS AT ORBIT INSERTION

Epoch	10 Min. after launch
Semi-major Axis Length	7075.698 Km
Eccentricity	0.00199643

Inclination	28.5245 Deg
Argument of Perigee	13.2189 Deg
Earth Fixed Longitude of Ascending Node	335.6467 Deg
True Anomaly	80.1874 Deg

The launch window is defined to be from 9 Feb 1998 00:00:00 UTC to 9 Feb 1998 00:30:00 UTC. A version of the space catalogue containing orbital elements with epochs on 23 Jan 1998 was used as the source of initial conditions for the candidate close approach objects. Of the 8050 sets of initial conditions which were read in from the database, 2471 sets remained after the application of the apogee/perigee filter. Of the remaining satellites, 39 had potential approaches within 5 Km of the launch vehicle depending upon the time of launch. The resulting launch window blackout periods are listed in Table 2. A timeline depiction of the blackout intervals is shown in Figure 8. The 39 close approach objects had a total of 98 close approach solutions in MET. These 98 solutions mapped into 84 distinct intervals in the launch window. The total launch blackout time was 143.75 seconds. This corresponds to an approximately 8% probability of a close approach occurring within the first six hours after launch based on a launch time randomly selected from within the launch window. Launch window blackout periods as small as 0.3 seconds were detected by this new algorithm. Normal close approach analyses were then conducted using selected times throughout the launch window for the start of the trajectory. The results of these test cases agreed with the launch window blackout results in Table 2. The processing time for the new algorithm was about 30 times longer than the processing time for a single run of the traditional method. When the number of runs needed to detect close approaches of 0.3 seconds in duration over the 30 minute launch window is considered, the new algorithm demonstrated an improvement in efficiency of 200x.

TABLE 2. LAUNCH WINDOW BLACKOUT PERIODS IN SECONDS PAST THE START OF THE LAUNCH WINDOW

OBJECT	BLACKOUT START	BLACKOUT END	BLACKOUT DURATION	MIN RANGE LAUNCH TIME	MIN RANGE (KM)
Debris-14555	0.000	1.103	1.10	0.000	2.242
ORBVIEW-02-24883	38.435	41.736	3.30	40.086	0.394
Debris-22399	48.307	48.786	0.48	48.547	4.826
ORBVIEW-02-24883	54.688	57.990	3.30	56.339	0.399
Debris-22578	55.482	56.400	0.92	55.946	3.923
ORBVIEW-02-24883	70.942	74.243	3.30	72.592	0.403
Debris-10240	107.840	109.214	1.37	108.527	1.523
Debris-22399	115.884	116.577	0.69	116.231	4.627
Debris-22578	133.087	133.924	0.84	133.506	4.143
FR1-01814	181.881	183.412	1.53	182.647	1.183
FR1-01814	192.983	194.516	1.53	193.750	1.146
FR1-01814	204.083	205.619	1.54	204.851	1.109

OBJECT	BLACKOUT START	BLACKOUT END	BLACKOUT DURATION	MIN RANGE LAUNCH TIME	MIN RANGE (KM)
Debris-22578	210.700	211.382	0.68	211.065	4.362
FR1-01814	215.183	216.721	1.54	215.952	1.072
Debris-03559	306.315	308.291	1.98	307.303	0.476
Debris-04629	312.861	313.996	1.14	313.429	4.077
Debris-18984	351.095	354.483	3.39	352.786	0.338
Debris-19170	491.672	492.941	1.27	492.307	2.262
CentaurMotor-23590	498.020	498.834	0.81	498.427	4.150
Debris-24517	524.070	525.173	1.10	524.621	3.556
Debris-05432	525.090	526.252	1.16	525.671	3.060
Debris-05280	555.482	557.243	1.76	556.362	2.792
Debris-24239	599.458	600.608	1.15	600.033	3.341
Debris-05028	653.121	654.275	1.15	653.750	2.803
Debris-24517	670.637	671.580	0.94	671.109	3.996
Debris-05432	696.322	697.542	1.22	696.932	2.786
Debris-09819	707.146	709.061	1.91	708.116	2.066
Debris-04726	721.963	723.344	1.38	722.654	1.834
Debris-24239	811.775	813.165	1.39	812.470	2.193
Debris-24517	817.228	817.879	0.65	817.587	4.446
Debris-04726	828.668	829.621	0.95	829.205	3.446
Debris-05432	867.554	868.823	1.27	868.189	2.521
Debris-24517	963.905	964.207	0.30	964.056	4.906
Debris-24239	1024.153	1025.666	1.51	1024.910	1.047
Debris-00211	1061.313	1062.753	1.44	1062.033	1.850
Debris-00211	1062.187	1063.623	1.44	1062.905	1.881
Debris-00211	1063.062	1064.495	1.43	1063.779	1.912
Debris-00211	1063.938	1065.148	1.21	1064.653	1.945
Debris-17120	1065.403	1066.559	1.16	1065.981	3.775
Debris-00211	1093.706	1094.696	0.99	1094.201	3.843
Debris-00211	1094.578	1095.554	0.98	1095.066	3.880
Debris-00211	1095.451	1096.413	0.96	1095.932	3.917
Debris-17120	1101.070	1102.196	1.13	1101.633	3.846
Debris-24234	1110.063	1111.577	1.51	1110.820	1.194
Debris-24234	1133.988	1135.347	1.36	1134.757	0.824
UKX4-07213	1134.221	1136.172	1.95	1135.208	3.048
Debris-17120	1136.736	1137.803	1.07	1137.284	3.918
Debris-24234	1157.916	1159.470	1.55	1158.693	0.453
Debris-17120	1172.402	1173.464	1.06	1172.933	3.989
Debris-11006	1223.782	1225.224	1.44	1224.503	0.815

OBJECT	BLACKOUT START	BLACKOUT END	BLACKOUT DURATION	MIN RANGE LAUNCH TIME	MIN RANGE (KM)
Debris-22357	1227.995	1229.667	1.67	1228.831	1.038
UKX4-07213	1240.283	1242.107	1.82	1241.227	3.269
Debris-22357	1303.437	1305.121	1.68	1304.279	0.792
2nd Stage-22232	1309.541	1310.562	1.02	1310.051	3.501
UKX4-07213	1346.383	1348.174	1.79	1347.279	3.494
Debris-24605	1352.428	1353.954	1.53	1353.191	2.285
Debris-04983	1357.984	1361.370	3.39	1359.677	0.600
HELIOS-01A-23605	1363.817	1365.481	1.66	1364.649	0.760
Debris-22357	1378.861	1380.554	1.69	1379.708	0.549
Debris-04983	1378.977	1382.378	3.40	1380.677	0.369
HELIOS-01A-23605	1382.681	1384.345	1.66	1383.513	0.761
Debris-04983	1399.972	1403.382	3.41	1401.677	0.138
HELIOS-01A-23605	1401.546	1403.209	1.66	1402.377	0.762
Debris-24605	1415.034	1416.510	1.48	1415.772	2.554
Debris-04983	1420.971	1424.381	3.41	1422.676	0.092
2nd Stage-22232	1426.315	1427.190	0.88	1426.921	2.653
2nd Stage-10954	1427.373	1428.372	1.00	1427.935	3.494
UKX4-07213	1452.524	1454.201	1.68	1453.363	3.721
Debris-22357	1454.269	1455.967	1.70	1455.118	0.308
Debris-24605	1477.644	1479.062	1.42	1478.353	2.824
Debris-05028	1479.112	1480.316	1.20	1479.714	3.089
Debris-24081	1522.730	1522.840	0.11	1522.785	4.986
Debris-06216	1525.065	1528.297	3.23	1526.680	0.066
Debris-14817	1538.925	1540.318	1.39	1539.622	1.995
2nd Stage-22232	1543.118	1544.453	1.33	1543.785	1.786
Debris-24081	1562.230	1562.772	0.54	1562.501	4.696
Debris-14817	1575.433	1576.824	1.39	1576.128	2.003
Debris-24977	1586.852	1588.427	1.58	1587.640	3.639
Debris-06085	1597.939	1599.323	1.38	1598.631	3.737
Debris-14817	1611.940	1613.331	1.39	1612.636	2.012
LANDSAT04-13367	1617.410	1619.819	2.41	1618.613	0.745
LANDSAT04-13367	1626.755	1628.954	2.20	1627.855	0.869
Debris-20851	1630.380	1633.130	2.75	1631.755	0.953
Debris-24373	1634.496	1635.306	0.81	1634.901	4.281
Debris-05520	1634.580	1636.066	1.49	1635.323	2.642
LANDSAT04-13367	1635.767	1637.968	2.20	1636.868	0.854
LANDSAT04-13367	1644.780	1646.983	2.20	1645.881	0.839
Debris-05520	1647.653	1649.128	1.47	1648.390	2.693

OBJECT	BLACKOUT START	BLACKOUT END	BLACKOUT DURATION	MIN RANGE LAUNCH TIME	MIN RANGE (KM)
Debris-14817	1648.448	1649.838	1.39	1649.143	2.021
Debris-05520	1660.724	1662.050	1.33	1661.456	2.744
Debris-24373	1667.955	1668.920	0.97	1668.464	3.801
Debris-05520	1673.793	1675.245	1.45	1674.519	2.795
Debris-20851	1682.379	1685.104	2.73	1683.741	1.173
Debris-24373	1701.438	1702.610	1.17	1702.024	3.321
Debris-06085	1725.562	1727.363	1.80	1726.484	2.319
Debris-20851	1734.384	1737.052	2.67	1735.732	1.393
Debris-06216	1760.059	1762.921	2.86	1761.553	1.952
Debris-17124	1760.519	1762.990	2.47	1762.049	1.758

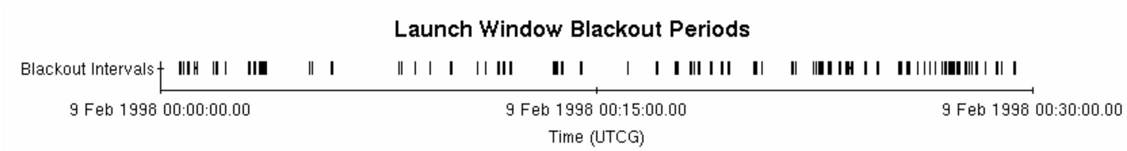

Figure 8. Launch window blackout periods

It should be noted that there are sequential entries in Table 2 with overlapping or nearly overlapping blackout periods resulting from the same candidate object. This type of pattern is the result of the existence of candidate objects whose period and nodal precession rates interact with the period and nodal precession rate of the primary to produce a repeating pattern of close approaches. The most obvious case of this behavior occurs for the candidate object Debris-00211. The movement of the launch time for the closest possible approach indicates that the trajectories, while well synchronized, are not in perfect resonance. The ground tracks of the newly launched satellite and Debris-00211 are shown in Figure 9. The initial four possible close approaches, which map to nearly the same times in the launch window, are seen to occur on the first four revolutions of the new satellite.

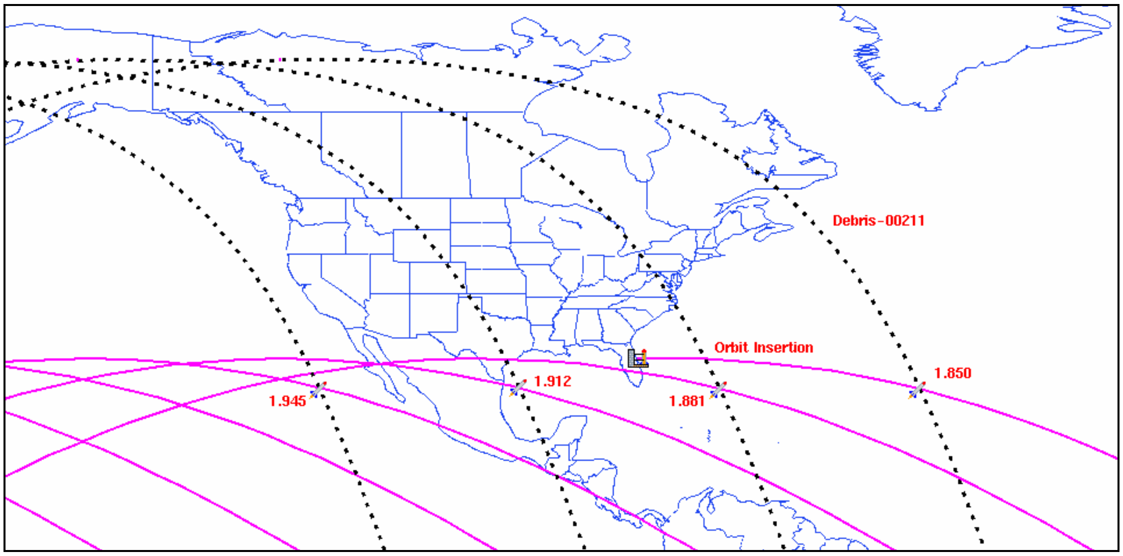

Figure 9. Locations of close approaches for Debris-00211 designated by the closest possible approach distance.

CONCLUSIONS

A new algorithm for predicting close approaches to vehicles with fixed ECEF trajectories in the presence of a launch time uncertainty has been developed. This algorithm has been shown to require less computation time and to be more rigorous than the simple technique of creating many sample trajectories throughout the launch window and performing individual close approach analyses. The computations required by the algorithm are simple but the implementation of the algorithm requires two generic numerical methods. The first method must be able to find the minimum of a function and the second must be able to detect threshold crossings. It has also been shown that careful attention must be paid to the relationship between MET and mappings into the launch window during periods of conjunction.

RECOMMENDATIONS FOR FUTURE WORK

The minimum miss distance is the measure of risk used in the development of this algorithm, however, other measures of risk such as an ellipsoidal boundary or probability of collision could be implemented in the same manner. The restriction on the trajectory of the launch vehicle being fixed in the ECEF frame could be extended to trajectories fixed in the Earth-Centered Inertial (ECI) frame with a starting point at the orbital insertion point for satellites being launched into a specific orbital plane. A post filter could also be applied to concatenate overlapping blackout periods due to the same candidate satellite if the individual close approach information is not of interest.

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