Integrated Surveillance Model for the Assessment of Surveillance Architectures and Operations

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ABSTRACT

There exists a requirement to be able to assess the effectiveness of surveillance operations involving multiple assets for the purposes of appraising proposals for capability development through acquisition and upgrades, as well as operationally for evaluating and comparing proposed plans for execution. The provision of a model which supports this assessment capability while accounting for the capabilities of the sensors, the dynamics of the operation, the management of the sensor-derived information in endeavouring to satisfy the surveillance requirement and the various sources of uncertainty is the subject of this report.

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Executive Summary

The work described in this report has been undertaken to fulfil an immediate need for analytical support to decisions on surveillance capability development in Australia. The requirement is for a model which can assess the effectiveness of surveillance operations and architectures involving multiple assets. Such a model would also be useful for evaluating surveillance plans and could conceivably be implemented in some form as a decision support tool for use by surveillance planning staff.

The model represents stochastically the distribution and motion of targets and their detection by sensors on-board platforms with pre-assigned paths as well as more complex surveillance processes designed to satisfy the surveillance information requirement in a given operation. The outputs from the model are measures of how well the information requirement has been met expressed in a probabilistic sense.

Such results will assist decision-makers by providing them with objective quantitative data relating to the effectiveness of a proposed surveillance asset mix, for capability development, or the effectiveness of a surveillance plan proposed for execution. This will contribute to the cost-effective enhancement of surveillance capability development as well as the effective operational employment of current assets.

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Paul studied Mathematics at the University of London, majoring in mathematical physics, before pursuing a PhD on the topic of 'Waves on Flows with Vorticity' which he completed in 1979. For 12 years he was employed at research laboratories in the UK's electricity supply industry, working on fundamental scientific problems associated with the generation and transmission of electrical energy. During this period he became progressively more involved with the modelling and optimisation of large-scale engineering systems for the purposes of investment appraisal, design evaluation, economic operation and risk mitigation. He originally joined DSTO in 1991 to work on the stochastic analysis of telecommunication networks and moved to Surveillance Systems Division in 1998 in order to assume responsibility for the assessment of integrated surveillance operations and architectures.

Contents

1. Introduction

This report describes the development and implementation of a method for scientifically addressing issues of integrated surveillance. The work was undertaken in support of the Integrated Surveillance Assessment (ISA) Task (JNT 99/005) which has focussed on the development of a capability to assess the operational effectiveness of surveillance architectures and operations. To be able to assess surveillance *operations* requires a representation of the physical environment, the targets, platforms and sensors it contains, and their dynamic interaction over time. This need was satisfied by the acquisition of Virtual Prototype's STAGE high fidelity simulation product whose open architecture facilitates the integration of legacy sensor models previously existing within SSD. To be able to assess surveillance *architectures* requires software entities referred to as intelligent agents attached to the physical simulation models which represent the manipulation of sensor-derived information and the responses to that information. This has been provided by Command and Control Division' DICE which has been interfaced with STAGE and to which Agent Oriented Software's JACK intelligent agents can be attached, as well as other agents such as ATTITUDE and Petri Nets.

The approach adopted for the ISA Task has been to provide the necessary models needed to undertake assessments of surveillance operational effectiveness. Such models are very general and are sufficient, but not strictly necessary, for integrated surveillance modelling. The next logical step is to apply these models specifically to issues of integrated surveillance. Up until this point there has been no systematic approach to the question of integrated surveillance. No all-embracing integrated surveillance architectures or integrated surveillance operational concepts have been proposed for which assessments can be routinely undertaken. It is not hard to think of specific integrated surveillance issues which arise in the context of particular scenarios but to catalogue these and provide *ad hoc* solutions on a case by case basis would not be a general solution to the problem. Thinking about integrated surveillance has been constrained by the specifics of scenarios, by the complexities of how to organise multiple surveillance assets, and has been clouded by technological issues. A systematic approach is required which divorces thinking from technological constraints and focuses on the potential of the system and the opportunities for integration. This approach is unconstrained by scenarios by being equally applicable to any scenario, and is able to accommodate the complexities associated with a surveillance system which comprises multiple assets each with unique capabilities and constraints, and which are interacting dynamically over time with each other, the physical environment and the targets of interest.

This report details a method which complements simulation and is specifically targeted at the core issue of integrated surveillance. Simulation aims to emulate reality as closely as possible by representing events which, in the case of surveillance sensors, are generated from probability distributions. The Integrated Surveillance Model, however, works directly with the probability distributions which can be generated from the same sensor models and therefore conforms to the same level of model fidelity as simulation. Working directly with probability distributions for events, rather than with the events themselves, provides a higher degree of computational efficiency because simulation of processes embodying random variables requires the use of the Monte Carlo method to derive statistically significant results. This is not particularly important with the advent of high speed computing unless the simulation is embedded within a combinatorial optimisation which is systematically exploring and identifying opportunities for enhanced resource efficiency and operational effectiveness.

Such is the case for the scientific investigation of integrated surveillance for which a stochastic model is more appropriate than direct simulation. Furthermore, not only does the detection process need to be represented stochastically for each individual sensor but so does the entire *surveillance process*, such as tracking, fusion, classification and identification for the manipulation of information originating from multiple sensors. This does not involve direct implementation of the relevant algorithms but a stochastic representation of the performance of the appropriate sub-system.

We assume for the purposes of this model that integrated surveillance is fundamentally concerned with both *retrospectively* maximising the 'value of information' previously derived from sensors (*correlation*) as well as *prospectively* directing sensors and their platforms to maximise the *expected* information (*coordination*). That is because we believe that from these principles can be derived the greatest payoffs in terms of enhanced effectiveness. Here the word *expected* is used in its strict statistical decision-theoretic sense to mean the average of the utility of information obtained across the range of predicted outcomes. In reality there will be practical constraints arising from the imperfect connectivity of the surveillance architecture, communications bandwidth limitations and processing capabilities. To a degree these technology effects can be emulated but are not explicitly represented because this study is concerned with scientifically developing the fundamental principles of integrated surveillance and determining the global properties of the system which emerge from the application of those principles. Technology issues would need to be considered properly as part of a systems engineering study.

Whereas the generation of information by sensors can easily be represented by the assignment of values to variables representing measurements, the converse which is to determine how sensors and their platforms should respond to sensor-derived information is a highly active area of current research. Examples of real-time environment estimation and sensor control are in robotics and automated air traffic control. In the area of planning and scheduling typical applications are in scheduling transport, manufacturing operations and airline flights. This is partly due to the inherent uncertainty associated with information derived from measurements (i.e. estimation), but more especially because of the uncertainty associated with future measurements (i.e. prediction). Whereas one can assume, for simplicity, perfect sensors and therefore no uncertainty associated with previous knowledge, there is no way of avoiding the fact that nothing at all is known about something which has not yet been measured (or may not yet even be known to exist!).

In summary, in its basic form the model could be used to simply assess effectiveness as an alternative to direct simulation, but more usefully, should be used to derive strategies for the effective employment of surveillance assets by efficiently exploring the parameter space of sensor actions. Surveillance operational concepts can be developed from such strategies.

The principles mentioned above will be referred to as the *Principle of Correlation* and the *Principle of Coordination* respectively. By themselves they do not tell one how to do integrated surveillance, but they provide the opportunity to perform surveillance more effectively and so we refer to their effective application to surveillance as *integrated* surveillance. One of the purposes of the integrated surveillance model is to demonstrate scientifically how their application can lead to more effective surveillance. We shall see that the Principle of Correlation is embodied within the model of the surveillance process whereas the Principle of Coordination is captured by the variables representing the commitment, scheduling and control of the surveillance assets. The representation within the integrated surveillance model of these principles simply *affords* the opportunity of enhancing effectiveness. This opportunity requires a mechanism which enables it to be fully exploited, namely an optimisation framework. This topic will be the subject of future research and will be addressed by extending the present assessment model.

The structure of this paper is as follows. Section 2 discusses the issues surrounding integrated surveillance and argues the case for a systematic scientific approach to the problem and the need for a model to solve it. Section 3 states the requirements for such a model as a prerequisite to mathematically modelling the problem. Section 4 develops the mathematical model in its entirety in the physical domain, which is extended to the conceptual target domain in Section 5. This is necessary in order to maintain the state of surveillance information acquisition with respect to the targets of interest. Section 6 develops the numerical algorithm which provides the solution whereas Section 7 describes its practical implementation. Note that the modelling philosophy adopted in this report is closely related to, and builds upon, other work reported previously (Berry [2], Berry & Hall [3]).

2. The Need to Model Integrated Surveillance

Simple examples should suffice to demonstrate the range of integrated surveillance issues. It should be understood that these issues compound combinatorially over the number of assets, sequences of possible decision points in time, and surveillance information requirements. Consequently, a standardised prescription for how to undertake effective integrated surveillance operations, such as a set of Standard Operating Procedures (SOPs), is not conceivable.

Consider an airborne surveillance platform employed co-operatively with a wide area surveillance radar to find and track all targets of a particular type. To detect all targets in a given area the strategy employed could be to locate all targets using the wide area surveillance radar and cue the airborne surveillance platform to classify them. Each target would be tracked by the wide area surveillance radar until the airborne surveillance radar had successfully classified it. Clearly this would be more efficient than using the airborne surveillance platform alone for wide area search, and efficiency translates to effectiveness through freeing up resources for other uses. However an alternative strategy would be to use the airborne surveillance platform to establish a barrier patrol along the boundary of the region and cue the wide area surveillance radar to track targets of interest as they enter the region. Which of these is best depends on factors such as the size of the region and how close together targets of interest are likely to be.

Consider imperfect sensors co-operatively searching a region for targets. They could deliberately overlap their search areas to improve their combined probability of detection or they could avoid overlap to maximise the total area searched. Which of these strategies is optimal depends upon their individual probabilities of detection and false alarm rates, which depend upon the local environment, and the size of the region to be searched. This is an example of integrated surveillance at its basic level.

Consider the targets of interest being ships emanating from a particular port with an aircraft as the airborne surveillance asset. Whereas more targets could potentially be detected closer to the port, limited fuel forces the search area to be further offshore. Exactly where should the search area be chosen that maximises the targets detected within the fuel constraints taking into account the time taken to reach the search area? This is an example of a resource constraint.

A similar problem is when a particular target is expected to leave the port in a certain time frame and the issue is whether the surveillance asset can get to the vicinity of the port in time to detect and identify it. This is an example of a time constraint.

These are examples of self-contained problems that can often be solved analytically or semi-analytically. Why could one not catalogue all of the situations and decide in each case the optimal response? By doing so one would be able to populate a rule-base that would cover all of the possibilities that could arise in practice. The reason is that the situations which could arise are not identifiably discrete and distinct even though the possibilities for an integrated surveillance system response are. Although there may be a discrete and finite set of surveillance assets, and their availabilities begin and end at discrete instants of time, there is a continuum of possibilities for the target densities and behaviours as well as the spatially and temporally dependent effects of the environment on sensor performances and these interact with the discrete targets and sensors in complex ways which render a rule-based approach as being over simplistic. We conclude that there is such close interdependence between the elements of time, space, surveillance assets (viewed as resources subject to constraints), measurements and consequent surveillance information that a single model is required to encapsulate them all. This model would be used for evaluating the possible options for surveillance operational responses.

3. Integrated Surveillance Model Requirements

At the outset it is clear that any formal representation of integrated surveillance requires the Surveillance Information Requirement (*S.I.R.*) to be captured, which is the ultimate objective of any surveillance operation or mission, and is the ultimate output from a surveillance architecture. It is also clear that the individual sensors and their platforms that comprise the surveillance system and deliver the information needed to satisfy the *S.I.R.* need to be represented in terms of their capabilities and constraints. Consequently the spatial relationships between the sensors and targets, as well as the intervening environment, which impact on their detection capabilities, require representation.

Because targets sought by sensors generally move through space in time, the way in which sensors are employed over time changes, requiring the dimension of time to be explicitly represented. Although these are fairly obvious modelling requirements, precisely how a collection of sensors is employed over time to measure target attributes and how to represent this process in a model is yet to be specified.

This requires representation of what is referred to as the *surveillance process* which provides the essential relationship between, on the one hand, the sensors and the measurements they make subject to their constraints, and on the other hand, the *S.I.R.* which is ultimately sought. To further complicate the issue, a *S.I.R.* could be time-varying and may be required to be satisfied concurrently with other *S.I.R.*s by competing for resources.

The way in which the *S.I.R.* and the supporting surveillance process are specified mathematically is explained in the following section. Needless to say these processes are at the heart of the integrated surveillance issue and account for the fundamental principles referred to previously (needing to both *retrospectively* maximise the value of information previously derived from sensors and *prospectively* direct sensors and their platforms to maximise the *expected* information).

To deal with the first principle from the modelling perspective (as opposed to practical implementation) is straightforward; simply make all sensor-derived information measurements instantly and globally available. In practical terms there will always be technological and operational constraints which limit the extent to which this can be achieved and a judgement will have to be made by the OR (Operations Research) practitioner as to whether this is a reasonable assumption at the level of fidelity which the model strives to attain[1].

Similarly for the second principle, global and centrally-directed decisions instantaneously communicated will achieve the best result overall but may not be achievable nor even desirable from the perspective of robustness. The model of the surveillance process should therefore aim to embody the *structure* of the surveillance architecture in terms of the distributed, sensor-derived information processing, and the sensor tasking and control actions.

With the above discussion in mind we summarise the modelling requirements of an integrated surveillance model as follows:

- The Surveillance Information Requirement (*S.I.R.*)
- The Surveillance Process (incorporating fusion and sensor management)
- 2 or 3 dimensions of space, as required
- Time
- Sensors and their platforms, their capabilities, constraints, locations and kinematics
- Targets, their locations in space, their kinematics and their attributes
- The intervening environment

Constraints on sensors and platforms could take the form of spatial constraints (e.g. cannot fly over a specified area), temporal constraints (e.g. a satellite only has access to a region for a limited portion of its orbit) or resource constraints (e.g. availability of an aircraft dependent on fuel load, maintenance schedule and crew roster). The environment may be viewed as a constraint which limits a sensor's capabilities.

If sensors and platforms are regarded as a set of constrained resources which interact in space and time to deliver a quantifiable product, namely the *S.I.R.*, through the execution of a surveillance process, then we have a classical constrained optimisation problem. The objective is to maximise the quality of surveillance information which conforms to the *S.I.R.* subject to the resource constraints or, alternatively, precisely satisfies the *S.I.R.* with minimal employment of surveillance resources. What makes this problem different from most applications is that we are working with *information*, originating from measurements by sensors and which contributes, in an unspecified way (as yet), to the achievement of an objective, the *S.I.R.* (which has yet to be quantified).

In a simulation we would wish to emulate the measurement process of a sensor by generating a value for the parameter representing a measurement and then determine, by means of an algorithm or rule, an appropriate response to that measured value. For the current model, however, we are interested not in the value arising from a

[1] However it is an attractive assumption due, as will be seen, to its simplicity.

measurement but the fact of its occurrence, its quality and the response to it in a stochastic sense through the execution of the surveillance process.

4. Surveillance Formulated as a Stochastic Model in the Physical Domain

The integrated surveillance assessment problem will initially be formulated in the physical domain as generally as possible without consideration of solution procedures.

4.1 Target representation

A surveillance operation should be assessed within the context of a particular set of target types, their spatial distributions and their motions, and the information which is required of them. This section is concerned with how the target attributes are specified for modelling purposes.

4.1.1 Target density

The targets of interest specified in the *S.I.R.* in the bounded (two or three dimensional) region of interest $\Re$ and during the finite time interval $[0,T]$ are modelled as a density function which, in general, will be a function of space and time, $\rho(\underline{r},t)$. This spatial and temporal dependence will depend upon the initial target distribution (at time $t=0$), the boundary conditions for the target density (on $\partial\Re$, representing targets entering and departing from the region of interest) and the model assumed to represent the target motions (as will be seen, a flow-field $\underline{v}(\underline{r},t)$). This is *a priori* information relating to the nature of the targets of interest as specified in the *S.I.R.* and could be based upon normalcy patterns.

It is important to correctly interpret the meaning of the target density ρ. Koopman [1] describes it as a density expressed in terms of targets per unit area (for maritime search operations). It could however easily be targets per unit volume for the analysis of surveillance for airborne targets. This is a purely deterministic representation although Koopman discusses a Gaussian target motion model that modifies the target density over time if the targets are subject to random perturbations analogous to Brownian motion. An alternative interpretation is (Berry [2]) the probability density function for a single target or (Berry, Pontecorvo & Fogg [6]) the joint probability density function for a number of targets and their locations. This requires that the integral of the target density function interpreted as a probability density function over mutually exclusive and exhaustive states of the system must be unity. This approach effectively deals with targets on an individual basis within a scenario but takes account of all the possible ways in which the target could behave, and the scenario unfold as a consequence as well as stochastic effects.

The interpretation in this paper, however, is more general than both of the previous ones in that ρ represents the density of a Poissonian spatial distribution of targets which is appropriate for randomly distributed points in space (Feller [1]). That is, given an infinitesimally small element of area ΔA in two dimensions, or element of volume ΔV in three dimensions, in the vicinity of $\underline{r}$ at time t then the probability of a target existing within that element is $\rho(\underline{r},t)\Delta A$, or $\rho(\underline{r},t)\Delta V$, respectively. This is justified on the grounds that targets may be considered to be completely independent of each other as regards their relative positions in space. A consequence of this is that given an area *A* at time *t* the average number of targets contained within *A* is given by

$$\overline{n}(t) = \int_A \rho(\underline{r},t)dA$$

It is possible, if required, to specify the variation in target numbers in *A* around this mean.

4.1.2 Target detection

As a consequence of this interpretation it is convenient to represent the detection process of a sensor sweeping or scanning the element of area ΔA in time Δt

$$\begin{aligned}&\Pr\{\text{a target exists in } \Delta\text{A} \cap \text{it is detected}\}\\&= \Pr\{\text{target detected} \mid \text{target exists}\}\Pr\{\text{target exists}\}\\&= p_d \rho(\underline{r},t)\Delta A\end{aligned} \qquad \text{....................(1)}$$

where p_d is the probability of detecting a true target at $\underline{r}$ by the sensor at time t. In practice the quantity p_d will be generated by sensor models which will take account of effects such as the range of the target from the sensor, clutter, noise, target cross-section and velocity, power and other environmental effects. The point being that there is no limit to the potential level of fidelity of the detection process nor to any other measuring process whose performance can be expressed in probabilistic terms. As an aside, the point about the joint probability distribution is that it represents the detection of *true* targets (i.e. existing) and not false alarms.

This enables us to construct a target density function for *detected* targets as we shall see. A further generalisation is that it is possible to separate densities for different target types based upon either their detection attributes or target motions.

We shall find that as different sensors scan, dwell and sweep at different rates the probability of detection p_d for a sensor will be specific to a particular time increment Δt which, in general, varies between sensors. In order to provide a consistent basis for representing the detection processes for a suite of sensors with different characteristics we normalise in time by determining for each sensor the *detection rate* λ interpreted as a Poisson process in continuous time. This is appropriate if consecutive detection events for different scans are independent. If $p_d << 1$ (i.e. small) we simply define

$$\lambda = \frac{p_d}{\Delta t}$$

and equation (1) becomes

$$\Pr\{\text{a target exists in } \Delta\text{A} \cap \text{it is detected}\} = \lambda\rho(\underline{r},t)\Delta A \Delta t \text{ ....(2)}$$

However, if p_d is not small then λ is defined as follows. For a Poisson process with detection event occurrence rate λ, the probability of an arrival in time Δt is

$$p_d = \int_0^{\Delta t} \lambda e^{-\lambda t} dt = 1 - e^{-\lambda \Delta t}$$

Hence

$$\lambda = -\frac{\ln(1 - p_d)}{\Delta t}$$

Note that in general λ will be implicitly time dependent because it will be a function of the spatial relationship between sensor and target, as well as the intervening environment, which can be expected to change over time. However over a relatively small increment of time Δt corresponding to a scan it can be assumed to be constant.

As will be seen, the advantage of employing Poisson distributions for spatial target densities, as well as measurement processes in time, is that the resulting expressions are local and not global which simplifies their evaluation. This is a consequence of the so-called 'memoryless' property of events generated by the Poisson process.

4.1.3 Target motion

The next issue with which we need to be concerned is how $\rho(\underline{r},t)$ becomes modified over time as a consequence of target motion. We assume that the targets move according to a prescribed target velocity field $\underline{v}(\underline{r},t)$ which could be derived from normalcy data. Conservation of probability for the existence of targets demands that the density function satisfy

$$\frac{D\rho}{Dt} + \rho \underline{\nabla} \cdot \underline{v} = 0 \text{ ....(3)}$$

for a prescribed target velocity vector field $\underline{v}(\underline{r},t)$ where the differential operator $D/Dt = \partial/\partial t + \underline{v} \cdot \underline{\nabla}$ represents differentiation following the motion as utilised in continuum mechanics (see Appendix A for the derivation). This yields a well-posed problem for the target density function over $\Re$ as the solution of a linear partial differential equation with given initial and boundary conditions.

The formulation of the target density component model as a partial differential equation happens to be a convenient and compact, though abstract, way of specifying the problem in mathematical terms. However it should not imply that an analytical solution is required or even that it is necessary to delve into the theory of partial differential equations! The numerical implementation of this component of the overall model will be discussed in Section 6. If a Gaussian target motion model is to be superimposed upon the deterministic target trajectories then the resulting diffusion term considerably increases the difficulty of their solution.

4.2 Sensor representation

4.2.1 Sensor and platform motion and control

Next we discuss how, within this modelling framework, sensors interact with targets. We suppose that sensors attached to platforms, as opposed to fixed sensors, follow prescribed paths which are referred to as *trajectories*. The motion of targets, on the other hand, is described by the continuous *flow-field* $\underline{v}(\underline{r},t)$. We represent the motion of a surveillance asset s belonging to a suite of assets $\Im$ at time t by a kinematic, in general nonlinear, ordinary differential equation

$$\frac{d\underline{r}_s}{dt} = \underline{g}_s(\underline{r}_s, \underline{u}_s, t) \quad s \in \Im \text{ ....(4)}$$

where $\underline{u}_s(t)$ is the vector of control variables governing its motion over time. Obvious control variables would be, for instance, the speed and direction of the platform and the mode of operation of the sensor. But it could, more generally, include a discrete variable component representing a decision to employ the asset at some future point in time. In this case the decision variable would have two discrete states (on/off) and would be a step function in time.

Systems which have a mix of continuously varying control variables, as in classical control theory, as well as discrete decision variables are known as *hybrid systems*. The control and analysis of hybrid systems is an active area of current research and it is this feature which makes the planning, tasking and control aspects of the integrated surveillance problem so challenging to solve.

4.2.2 Sensor and platform constraints

Constraints on the use of a surveillance asset viewed as a limited resource could be in the form of time-varying spatial constraints:

$$\underline{r}_s(t) \in S(t) \subset \Re \quad \forall t \in [0,T]$$

where *S(t)* is the subset of $\Re$ at time *t* where the platforms are allowed to go; but also as limitations on the available time an asset can be used or distance it can cover before having to return to base, and limitations on the number of flights that can be undertaken in a given time period due, for example, to maintenance requirements and crew rostering requirements. Resource constraints can generally be specified mathematically but the specifics depend very much upon the situation of interest and the characteristics of the surveillance asset (sensor and platform) concerned.

4.3 Sensor and target flow-field interaction over time

For a given target type, the detection rate λ_s for a sensor $s \in \Im$ is a function of time *t*, target location $\underline{r}$ and velocity $\underline{v}$, sensor/platform location $\underline{r}_s$, velocity $\underline{g}_s$ and control variable set $\underline{u}_s$, each of the latter five quantities being themselves functions of *t*. Hence we can express the functional dependence of the detection rate as

$$\lambda_s[\underline{r}_s(t), \underline{g}_s\{\underline{r}_s(t), \underline{u}_s(t), t\}, \underline{u}_s(t), \underline{r}, \underline{v}(\underline{r}, t), t] \equiv \lambda_s(\underline{r}, t)$$

where the implicit time dependence is due to target and sensor motions and controls, and the explicit time dependence is due to varying environmental conditions. Quite often, in simpler analyses, it will be adequate to remove the explicit time dependence and make the spatial dependence purely a function of the relative positions of the target and sensor, hence $\lambda_s\{\underline{r}_s(t) - \underline{r}(t)\}$. It is assumed that suitable models exist for the sensors which compute probability of detection, and hence detection rate, as a function of the dependent variables. These could be either analytical or computational.

Note that this detection rate function applies to a target which is assumed to exist at a location at a given time whereas the stochastic model being developed will model the detection of targets which exist to a prescribed probability within infinitesimally small incremental areas or volumes. The connection is provided by equation (2).

Having now modelled the targets, the sensors and the detection process (which can be generalised to a measurement process as will be seen), we need to combine these into a single representation which estimates the numbers of targets which get detected at a location during an increment of time. Let us therefore propose distinct Poisson density functions for undetected targets ρ_u and for detected targets ρ_d where $\rho = \rho_u + \rho_d$. Then

$$\frac{D\rho_u}{Dt} + \rho_u \underline{\nabla} \cdot \underline{v} = -\lambda \rho_u \qquad (5)$$

$$\frac{D\rho_d}{Dt} + \rho_d \underline{\nabla} \cdot \underline{v} = \lambda \rho_u \qquad (6)$$

which are obtained by considering rates of transition from states of non-detection to states of detection for hypothetical targets within incremental areas or volumes following the target motions as given by $\underline{v}(\underline{r}, t)$. This is proved in the general case in Appendix B. Note that the detection rate is the combined detection rate for all sensors which the incremental area concerned is within range of. Hence

$$\lambda = \sum_{s \in \Im} \lambda_s$$

Sensors out of range simply have $\lambda_s = 0$. In effect, then, we are addressing the issue of the combined effectiveness of the sensor suite for initial detection, irrespective of which sensor does the detection and taking into account overlaps between sensors. By recording the fact that a target has been previously detected by simultaneously reducing ρ_u, we exclude the possibility of recording a second detection of the same target at a later time and thereby accounting for its detection more than once. Note that the combined effectiveness problem serves to illustrate the modelling issues, and that other more sophisticated integrated surveillance issues are capable of being modelled using this formalism as will be seen.

4.4 The Surveillance Information Requirement

The Surveillance Information Requirement is the information sought about the targets of interest as specified within the model by their type, distribution and motion. The purpose of assessment is to determine how well the requirement is met by the surveillance assets as the requirement will never be perfectly satisfied due to sources of uncertainty and resource constraints. Having selected a stochastic modelling approach to deal with uncertainty, an appropriate Measure of Effectiveness (MoE), *M*, is required in order to quantify the degree to which the *S.I.R.* is satisfied. For a requirement to simply detect targets in a region $\Re$ this measure could, at the simplest level, be the proportion of all targets in $\Re$, or in sub-region A of $\Re$, which have been detected at time *t*.

$$M(t) = \frac{\iint_{\mathrm{A}} \rho_d(x,y,t)\,dxdy}{\iint_{\mathrm{A}} \{\rho_u(x,y,t) + \rho_d(x,y,t)\}\,dxdy}$$

Another simple MoE could be the proportion of all targets crossing an arc *S* which have been detected in a time period $[0,T]$. Hence

$$M = \frac{\int_0^T dt \int_S \rho_d(x,y,t)\underline{v}(x,y)\cdot d\underline{s}}{\int_0^T dt \int_S \{\rho_u(x,y,t) + \rho_d(x,y,t)\}\underline{v}(x,y)\cdot d\underline{s}}$$

For more complex surveillance processes involving, for example, tracking, classification and identification, more general MoEs can be proposed as functions of the target state probability distributions, as will be seen.

Sometimes it may be more meaningful to devise measures that express the operational consequences of receiving surveillance information. For example if the *S.I.R.* is the identification of targets to allow for interception if need be, the measure could be the proportion of all targets successfully identified in time for them to be intercepted. This requires statistical information regarding the times and locations at which the targets are identified as inputs to a function representing the response of an interceptor. If the response of the interceptor is, itself, stochastic, then the measure will be the probability of a target being successfully intercepted.

In general, then, a Measure of Effectiveness will be a function of the joint probability distribution over a state space defined for the problem of interest. As discussed above, the state space is structured so as to record the status of targets in terms of whether and where they exist, what measurements have been made of them and how those measurements have been processed (for example, fusing measurements from different sensors, and tracking). The examples quoted above are probabilistic *expectations* (averages) but as they are derived from probability distributions it would be possible to determine variances.

4.5 The Surveillance Process

The Surveillance Process needs to represent the ways in which information can be processed and combined to satisfy the *S.I.R*. In the combined surveillance effectiveness problem modelled above it was pointed out that the target density could be split into a density for undetected targets and a density for detected targets with a rate of transition between states governed by a Poisson process. In effect then we have a two state, continuous-time Markov chain defined at each point in $\Re$ with a superimposed target drift. This can easily be generalised to multiple state Markov chains to represent more complex surveillance processes other than just search and detection, such as measurement of target attributes, tracking and fusion. It is also possible to represent delays in the transmission and processing of measurements; however, introducing time delay distributions other than negative exponential (i.e. Poisson process), such as a deterministic delay, renders the equations non-local in time and creates data storage requirements.

The spatially continuous target state density functions, defined for the states of interest, record probabilistic information pertinent to a hypothetical target occupying an incremental area or volume, such as

- whether it has previously been detected by a particular sensor,
- whether it has previously been detected by any sensor,
- whether it is currently being tracked,
- whether the tracks from different sensors have been successfully fused,
- whether a previously detected target has been classified, and
- whether it has been identified.

Such a function could conceivably be defined to record the error associated with a measurement, such as location estimate, and its degradation over time as a target moves in an unpredictable fashion. This could be implemented as spatially continuous functions of Gaussian means and variances evolving over time in accordance with a suitable target motion model. From such functions can be extracted statistics relating to the effectiveness of the surveillance process, such as the average time a target was tracked for, as a measure of tracking performance, and the distribution of time until a target is successfully identified. However, as will be seen, time statistics for particular targets or sets of targets will require the results to be interpreted from the perspective of the targets rather than in physical space.

In general, the target densities for the occupation of the N discrete states of the surveillance process evolve over time according to

$$\frac{D\underline{\rho}}{Dt} + \underline{\rho}(\underline{\nabla} \cdot \underline{v}) = \underline{\underline{A}}(\underline{\lambda})\underline{\rho} \quad \text{.....................(7)}$$

where $\underline{\underline{A}}$ is an $N \times N$ matrix whose elements are a function of $\underline{\lambda}(\underline{r},t)$, the given vector of discrete state transition rates, each of which will in general be a complicated function of target and sensor attributes which are, in turn, functions of $\underline{r}$ and *t* (see Appendix B for the derivation). Note that the summation of the state densities at a point must equal the actual target density, irrespective of state i.e. $\sum_i \rho_i = \rho$.

As a simple example of a surveillance process consider the situation in which it is desired to assess the performance of a system in being able to track a field of targets. Suppose that the tracker concerned is a 3-in-5 PDA tracker which means that a track is initiated if the SNR exceeds a track initiation threshold with probability p_i, and an initiated track is subsequently maintained if the the SNR exceeds a lower track maintenance threshold with probability p_m at least twice in the following four dwells or scans and, in general, at least three times in any five dwells. It is possible to model this tracker in its entirety using a Markov Chain but it requires a large number of states. If the only issue of interest is whether such a tracker succeeds in establishing and maintaining a track at least once then the matter is considerably simplified as only two states are required: a state of never having previously been tracked and a state of having established a track at least once. The probability of transition between the states is simply

$$P_T = p_i \{ {}^4C_2 p_m^2 (1-p_m)^2 + {}^4C_3 p_m^3 (1-p_m) + {}^4C_4 p_m^4 \} .$$

4.6 Discussion of the physical domain model formulation

We have derived the system of equations governing the time evolution of the target density, the dynamical equations for the sensors, and the equations governing the interactions between the sensors and targets as well as demonstrating how this can be generalised to more complex surveillance processes. We have seen how to specify the *S.I.R.* and quantify it probabilistically. Having derived all necessary equations for the time evolution of the surveillance system, the principle underlying their simultaneous solution, for the purpose of estimating its effectiveness in terms of how well the *S.I.R.* is satisfied, is for the equations to be integrated over time, assuming all control functions have been given at the outset. That is, the plan for the deployment of the sensors has been prescribed. In principle this is possible for a well-posed problem and techniques for solving linear partial differential equations are well established.

The equations have been developed and specified in the physical domain as generally as possible so as to exploit physical insight and facilitate any further extension of the model specification which may be required. The solution $\underline{\rho}(\underline{r},t)$ gives the spatial densities for the targets in their various states as a function of time. It enables conclusions to be drawn regarding the effectiveness of sensor coverage in given areas over given time periods. For instance, since $\underline{\rho}(\underline{r},t)$ is defined over physical space, once computed it would be easy to deduce the proportion of all targets in a given area at an instant of time which have been previously been detected or are being tracked, or the proportion of all targets crossing a given boundary during a time period which remain undetected. It does not, however, give any indication of the history of a specific target, or collection of targets, such as when a particular target was first detected and for how long it was tracked. Hence this is a *physical-domain-oriented* perspective of surveillance system effectiveness. When we speak of the history of a target, what is meant is that of a *hypothetical* target which occupies a volume of space with a prescribed probability. What is needed to achieve this *target-domain-oriented perspective* which enables surveillance effectiveness to be summarised in terms of how successfully and timely information is collected on specific targets, is to transform in some sense to a target space. This idea will be developed further below.

When attempting to solve this system of equations in its present form for practical problems, various technical difficulties can be encountered. These are listed as follows:

1. When target paths converge on a point or targets cease moving, their density can become unbounded.
2. Where target paths cross there is no longer a one-to-one correspondence between physical space and target space, which means that a density defined at a point in space does not uniquely refer to a target moving in a single direction.
3. Solving simultaneous sets of partial differential equations requires specialised techniques. If simpler solution techniques can be found then they should be considered in the first instance.
4. The equations require explicit computation of the target density in physical space whereas this quantity is not of primary interest. If it can be eliminated, or at least not require updating at each time step of the dynamic interaction between targets and sensors, then computation is simplified.
5. The solution $\underline{\rho}(\underline{r},t)$ is not target oriented as discussed above. In other words the evolution equations are an Eulerian formulation whereas a Lagrangian formulation would sometimes be more convenient. What is required is a transformation to a frame of reference in which quantities are intimately associated with the hypothetical targets as they move through the physical domain and interact with the sensors. Suppose that such a transformation exists from $\underline{r} \in \Re$ to $\underline{q} \in \Omega$ then $\underline{\rho}(\underline{q},t)$ tracks the target densities associated with the surveillance process states following the target motion. This transformation will be seen to resolve points 1, 2 and 5, and to contribute to the resolution of point 3.

These difficulties are addressed in the next section.

5. Extension of the Model to the Target Domain

This section constructs a transformation from physical space to a more convenient frame of reference referred to as the target space. In view of $\sum_i \rho_i = \rho$ we can define $f_i(\underline{r},t), \quad i = 1,\ldots,N$ with $0 \le f_i \le 1$ by $\rho_i = f_i \rho$. Then $\underline{\rho} = \underline{f}\rho$ and $\sum_i f_i = 1$. Note that if ρ vanishes anywhere or at any time then $\underline{f}$ is undefined. This possibility will be excluded from consideration for the present as it can be demonstrated that if $\rho(\underline{r},0) \ne 0$ for $\underline{r} \in \Re$ and $\rho(\underline{r},t) \ne 0$ for $\underline{r} \in \partial\Re$ then $\rho(\underline{r},t) \ne 0$ for $\underline{r} \in \Re$.

Substituting for $\underline{\rho}$ in equation (7) yields

$$\rho \frac{D\underline{f}}{Dt} + \underline{f}\{\frac{D\rho}{Dt} + \rho(\underline{\nabla} \cdot \underline{v})\} = \rho \underline{\underline{A}}(\underline{\lambda})\underline{f}$$

which, on using equation (3), simplifies to

$$\frac{D\underline{f}}{Dt} = \underline{\underline{A}}(\underline{\lambda})\underline{f} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

Hence we have decoupled the target density and target state representations and as a consequence no longer need to explicitly compute target state densities. However, as will be seen, the overall target density is still required for the transformation to target space. Equation (8) is, in effect, the Chapman-Kolmogorov equation for the state of an element of area containing a target and following the target flow field, considered as a continuous-time Markov chain. This deals with point 4 in Section 4.6.

The next step is transform equation (8) to a target-oriented domain. The following derivation is for a time-steady target velocity vector field $\underline{v}(\underline{r})$ (i.e. time-independent) in two dimensions.

Now, consider any target moving along a trajectory in accordance with the prescribed velocity vector field $\underline{v}(\underline{r})$, then its position $\underline{r}$ as a function of *t* satisfies

$$\frac{d\underline{r}}{dt} = \underline{v}(\underline{r})$$

Suppose $\underline{v} = (u,v)$ then in time increment dt along the target trajectory

$$dt = \frac{dx}{u} = \frac{dy}{v}$$

Define a function ψ of $\underline{r}$ which is constant along all such trajectories by

$$d\psi = \rho v dx - \rho u dy = 0$$

which is a total differential. If $\psi_x = \rho v$ and $\psi_y = -\rho u$ then for *any* element of arc $d\underline{r} = (dx,dy)$

$$d\psi = \underline{\nabla}\psi \cdot d\underline{r} = \rho(-v,u) \cdot d\underline{r},$$

which can be integrated to give

$$\psi(\underline{r}) = \int_C \underline{\nabla}\psi \cdot d\underline{r} \;\forall\; \underline{r} \in \Re = \int_C \rho(v,-u) \cdot d\underline{r}$$

where the integral is along some curve C which begins at some arbitrary reference point and terminates at $\underline{r}$.

Since $(v,-u).d\underline{r} = (v,-u)\cdot(dx,dy) = (u,v)\cdot(-dy,dx) = \underline{v}\cdot d\underline{s}$, where $d\underline{s}$ is normal to the curve *C*, the function $\psi(\underline{r})$ can be written

$$\psi(\underline{r}) = \int_C \rho\underline{v}\cdot d\underline{s},$$

which is easily seen to be the target flux between the trajectory which passes through the origin and that which passes through $\underline{r}$. Also, there can be no target flux *through* a trajectory, hence ψ is constant along a trajectory and its value can be used to uniquely characterise it. We shall use ψ as one of the variables in the transformed target space. Next we define the second variable.

Let ξ be the time at which a conceptual target *would* have entered $\Re$, if it is subsequently found to be at location $\underline{r}$ at time *t*. Then, given a trajectory ψ and start time ξ, it is possible, in principle, to determine the location $\underline{r}$ of a target at any subsequent time $t > \xi$ from the target velocity vector field $\underline{v}(\underline{r})$. Define $\underline{q} = (\xi,\psi)$, then we have a time-dependent coordinate transformation $\underline{r} = \hat{T}(\underline{q};t)$. If $\hat{T}$ is one-to-one then an inverse transformation $\hat{T}^{-1}$ exists such that $\underline{q} = \hat{T}^{-1}(\underline{r};t)$. However, in view of point 2 in Section 4.6 this cannot generally be guaranteed. Consequently, we shall almost always transform from $\underline{q} \in \Omega$ to $\underline{r} \in \Re$ so as to avoid this problem.

Ideally we should represent $\underline{v}$ in target space because in physical space it can be multi-valued. It needs then to be specified as a function of $\underline{q}$ and *t*, i.e. $\underline{v} = \underline{v}(t-\xi,\psi)$ for a steady velocity vector field. Then

$$\underline{r}(\xi,\psi,t) = \underline{r}(\xi,\psi,\xi) + \int_{\xi}^{t} \underline{v}(t'-\xi,\psi)dt'.$$

Recognising that $\underline{r}$ has the functional dependence $\underline{r} = \underline{r}(t-\xi,\psi)$ and changing the integration parameter we obtain

$$\underline{r}(t-\xi,\psi) = \underline{r}(0,\psi) + \int_{0}^{t-\xi} \underline{v}(t',\psi)dt'.$$

A convenient feature of this transformation is that targets do not cross the contours of constant ξ and ψ in $\Re$. These lines form a rectangular grid in Ω such that an element of transformed area $\Delta\xi\Delta\psi$ contains, on average, the same number of targets irrespective of its location in the grid. Hence for an area $A \subset \Re$

$$\iint_A \rho(x,y,t)dxdy = \iint_{\hat{T}^{-1}(A)} d\xi d\psi$$

which simplifies the computation of Measures of Effectiveness.

This can be proven as follows. A standard result for coordinate transformations states

$$\iint_{\hat{T}^{-1}(A)} d\xi d\psi = \iint_A \frac{\partial(\xi,\psi)}{\partial(x,y)} dxdy$$

where $\frac{\partial(\xi,\psi)}{\partial(x,y)} = \xi_x\psi_y - \xi_y\psi_x$ is the Jacobian of the transformation for fixed *t*. From above

$$\psi_x = \rho v \text{ and } \psi_y = -\rho u$$

We need to deduce ξ_x and ξ_y. Since

$$\underline{r}(\psi,\xi,t) = \underline{r}(\psi,\xi+\Delta\xi,t) + \underline{v}(\psi,t-\xi)\Delta\xi$$

we have

$$x_\xi = -u \text{ and } y_\xi = -v .$$

Using

$$\begin{pmatrix} \psi_x & \psi_y \\ \xi_x & \xi_y \end{pmatrix} \begin{pmatrix} x_\psi & y_\psi \\ x_\xi & y_\xi \end{pmatrix} = I$$

we can obtain

$$\xi_x = -\rho u \text{ and } \xi_y = \frac{\rho u^2 - 1}{v}$$

and hence

$$\frac{\partial(\xi,\psi)}{\partial(x,y)} = \xi_x\psi_y - \xi_y\psi_x = \rho$$

as required.

Finally, we transform equation (8) to the target domain. Equation (8) can be written in the form

$$\frac{\partial \underline{f}}{\partial t} + u\frac{\partial \underline{f}}{\partial x} + v\frac{\partial \underline{f}}{\partial y} = \underline{\underline{A}}(\underline{\lambda})\underline{f}$$

for $\underline{f}(x,y,t)$ in the physical domain.

Now $\frac{\partial}{\partial x} = \xi_x\frac{\partial}{\partial \xi} + \psi_x\frac{\partial}{\partial \psi}$, $\frac{\partial}{\partial y} = \xi_y\frac{\partial}{\partial \xi} + \psi_y\frac{\partial}{\partial \psi}$ and $\frac{\partial}{\partial t} = \frac{\partial}{\partial t} + \xi_t\frac{\partial}{\partial \xi} + \psi_t\frac{\partial}{\partial \psi}$.

Using the above expressions for ξ_x, ξ_y, ψ_x and ψ_y as well as $\xi_t = 1$ and $\psi_t = 0$ gives

$$\frac{\partial \underline{f}}{\partial t} = \underline{\underline{A}}(\underline{\lambda})\underline{f}$$

where the partial differentiation w.r.t. *t* is for fixed ξ and ψ.

This is a mixed Eulerian-Lagrangian formulation of the problem in which $\underline{f}(\xi,\psi,t)$ is integrated forward in time in the target domain while $\underline{\lambda}(\underline{r},t)$ is obtained by transforming to the physical domain to compute the surveillance process state transitions.

As will be seen there is a further generalisation in which different states can be associated with different target velocity vector fields, and hence require different transformations to physical space.

6. Integrated Surveillance Model Algorithmic Development

Having developed the theoretical foundations for the technique in the previous sections, this section is concerned with the development of an algorithm for its practical implementation by computer. For ease of explanation this is broken down into a series of steps.

Step 1: Define the physical domain, the target flow field and the conceptual target domain. The latter, without loss of generality, may be assumed to be rectangular.

Step 2: Derive the transformation between the physical domain and the conceptual target domain for each instant of time *t*. which depends upon not just the target trajectories but also their probabilistically defined densities.

Step 3: Construct a rectangular grid in the conceptual target domain, within each cell of which the probability of a target existing is conserved over time.

Step 4: Integrate forward in time from $t = 0$ in increments of Δt, and for each cell in the conceptual target space, update the localised surveillance process state space probabilities over the time increment. This involves mapping across to the physical domain, updating the deterministic sensor/platform locations, computing the revised spatial relationships between the conceptual target and all sensors, and deducing from this and the prevailing environmental conditions the appropriate probability of detection which drives the stochastic representation of the surveillance process. The detection probabilities are derived from computational models specific to the sensor in question.

Step 5: From the surveillance process state space probabilities expressed as functions of space and time, compute the appropriate Measures of Effectiveness as functions of these probability distributions.

Each of these steps will be expressed in terms of algorithms which can be implemented computationally. This requires the continuous functions of time and space to be discretised.

6.1 Discretising the conceptual target domain

We refer to the portion of the boundary of the region $\Re$ of interest from which the targets emanate as the 'start line'. All conceptual targets are uniquely referred to by the time ξ at which they cross this line and the trajectory ψ they move along. Without loss of generality we assume the origin $\underline{r} = \underline{0}$ to be at one end of this line and the trajectory passing through the origin to be labelled $\psi = 0$. All targets which cross the start line at time $t = 0$ are labelled $\xi = 0$, of course, since ξ refers to the time at which they cross the start line. Time t is discretised into increments of length Δt and we shall refer to the discretised instants of time $t_n = n\Delta t$ where $n = 0,1,\ldots,N$. At time $t = 0$ the targets in $\Re$ which crossed the start line first did so at time $-L\Delta t$. This naturally discretises the ξ variable also into values $\xi_l = l\Delta t$ where $l = -L,\ldots,N$.

The variable ψ represents the target flux between a target trajectory labelled ψ and that for which $\psi = 0$ which passes through the origin in the physical domain. Recall that along any curve *C* passing through the origin

$$\psi(\underline{r}) = \int_C \rho \underline{v} \cdot d\underline{s}$$

If *C* is the start line then ψ can be discretised into $\psi_m = m\Delta\psi$ along it, where

$$\Delta\psi = \rho_m \underline{v}_m \cdot \Delta\underline{s}_m$$

which is made constant (ie independent of *m*) through judicious choice of $\Delta\underline{s}_m$. We assume that $m = 0,\ldots,M$ where *M* is chosen to ensure that $\Re$ is adequately 'covered' by target trajectories.

Each target is now uniquely characterised by its value of (ξ_l, ψ_m) or, equivalently, by the integer pair (l,m). At each time t_n for $n = 0,\ldots,N$ there are corresponding points in the physical domain which we need to compute. These are denoted $\underline{r}_{l,m}^n$.

6.2 Transforming from the conceptual target domain to the physical domain

Note that the point in the physical domain corresponding to $\underline{q}_{l,m} = (\xi_l, \psi_m)$ at time t_n is the same as that corresponding to $\underline{q}_{l-1,m} = (\xi_{l-1}, \psi_m)$ at time t_{n-1}, i.e. $\underline{r}_{l,m}^n = \underline{r}_{l-1,m}^{n-1}$. This is a consequence of the fact that the target flow-field is steady in time, that is the trajectories have a fixed pattern. Applying this recursively we have $\underline{r}^{n}_{l,m} = \underline{r}^{0}_{l-n,m}$ and hence we need only compute the transformed points at time $t = 0$.

Using the previous notation for the transformation from the target to physical domain, we have

$$\underline{r}^{n}_{l,m} = \underline{r}^{0}_{l-n,m} = \hat{T}(\underline{q}_{l,m}; n\Delta t).$$

Note that although $n = 0,\ldots,N$, $l = -L,\ldots,N$ and $m = 0,\ldots,M$, the transformation is defined only for $-L \le l - n \le 0$ for given n. This excludes from $\Re$ targets which have not yet entered or have previously departed. Since the surveillance operation is assumed to be entirely contained within $\Re$ what happens to the targets before they enter or after they depart is of no consequence.

We now determine how to compute the points $\underline{r}^{0}_{l,m}$ for $-L \le l \le 0$ and $0 \le m \le M$ from which $\underline{r}^{n}_{l,m}$ can be computed for any *n*. Firstly, $\underline{r}^{0}_{0,0} = \underline{0}$ as previously defined. The points along the start line, $\underline{r}^{0}_{0,m}$ for $m = 1,\ldots,M$, are chosen so as to satisfy $\Delta\psi = \rho_m \underline{v}_m \cdot \Delta\underline{s}_m$ for given $\Delta\psi$ with $\Delta\underline{s}_m = \underline{r}^{0}_{0,m+1} - \underline{r}^{0}_{0,m}$. Next, the points in the interior are defined by

$$\underline{r}^{0}_{l-1,m} = \underline{r}^{0}_{l,m} + \underline{v}(\underline{r}^{0}_{l,m})\Delta t \ \text{ for } m = 0,\ldots,M$$

and can be computed recursively for $l = 0,-1,\ldots,-L+1$.

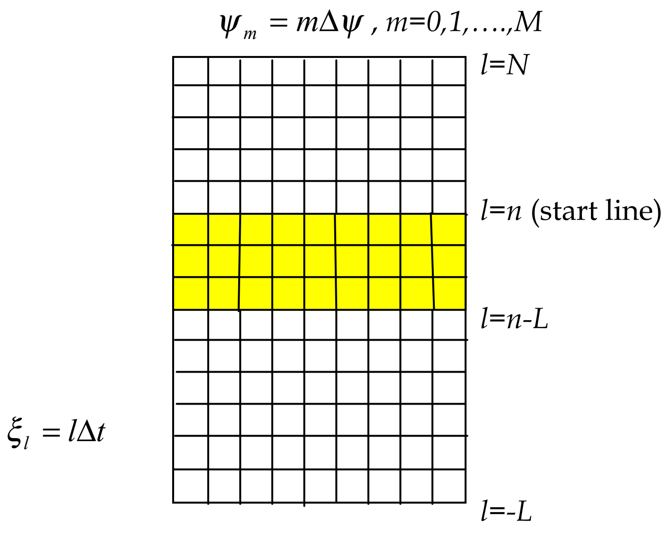


Figure 1: Position of sliding window over conceptual target space at time $t_n = n\Delta t$

Figure 1 shows the conceptual target space as a grid with each grid point corresponding to a target which is uniquely labelled by *l* and *m*. At any instant of time the region containing the sensors can be regarded as a sliding window (shaded yellow) which slides over the target space as time progresses.

6.3 Integrating over time

For each time step $t_n = n\Delta t$ it is now possible to transform from the conceptual target domain to the physical domain, within which the probabilities of detection for hypothetical targets by all sensors can be computed. The vector of these detection, and more generally surveillance process state transition, probabilities $\underline{p}^n_{l,m} = \underline{\lambda}(\underline{r}^n_{l,m}, t_n)\Delta t$ enables the surveillance process state occupation probability vector $\underline{f}(\xi_l, \psi_m, t_n) = \underline{f}^n_{l,m}$ to be updated thus

$$\underline{f}^{n+1}_{l,m} = \underline{f}^n_{l,m} + \Delta t \underline{\underline{A}}(\underline{p}^n_{l,m})\underline{f}^n_{l,m}.$$

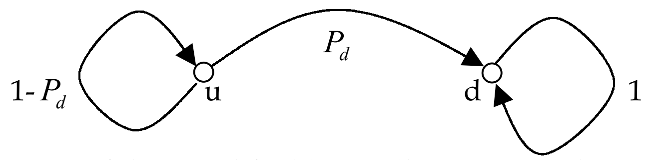
In order to illustrate some of the practical issues arising from the implementation, we consider the example of combined sensor effectiveness for detection with multiple sensors. Essentially, we are only interested in whether a target has been detected or not at time $t_n = n\Delta t$ and hence define

$$\underline{f}(\xi, \psi, t_n) = \begin{pmatrix} f_u \\ f_d \end{pmatrix}^n$$

where f_u is the probability that the target (ξ, ψ) has not yet been detected, f_d is the probability it has been detected, and $f_u + f_d = 1$. Then the probability of the target being detected by *at least one sensor* in time interval Δt between t_n and t_{n+1} is

$$P_d = 1 - \prod_{s \in \Im}(1 - p_s).$$

This is the simple 'OR' model of sensor fusion. In order to determine how detections affect the probabilities of being in states of having been detected or undetected we construct a two state Markov chain and indicate the transition probabilities



From this specification of the simplified 'surveillance process' we can write

$$f_u^{n+1} = (1 - P_d) f_u^n$$

and

$$f_d^{n+1} = f_d^n + P_d f_u^n.$$

Account can be taken of different scan rates and dwell times by adjusting the transition probabilities appropriately.

It is important to note at this point that the preceding development assumes that time has been discretised using a single time step length for the target motion and sensor detection events. If the timescale of the target motion is commensurate with the scan and dwell rate then the time step chosen should correspond with the individual scans and dwells. However there will be situations when the target speed may be so slow that many sensor scans, sweeps or dwells could occur during the time step representing the movement of a target. Furthermore, different sensors will scan and dwell at different rates. In this case it will be necessary to subdivide the time step for the target motion, Δt_{target}, into smaller time increment, Δt_s, for each sensor $s \in \Im$. For these shorter timescales on which sensing is done the target is assumed to be stationary. The sensing is modelled asynchronously for each sensor and information fused from multiple sensors at any convenient point in time or after the time integration altogether.

There are two limiting cases which require special consideration. The first is when the scan rate is so fast that it would be computationally infeasible to represent the individual scans between target motion time steps Δt_{target}. In this case it is appropriate to regard the sensing process as time-continuous with detection rate $\lambda(t)$ and numerically integrate. The second is when the range of a sensor is sufficiently precisely defined with respect to the length and time scales of interest that to all practical purposes a target is detected with certainty if it is within range of the sensor and is undetected if it is beyond range. In this case it is necessary to explicitly represent the transition from an undetected state to a detected state for a given target by a given sensor within the Markov chain, based upon whether the target is, or is not, within range of the sensor during a time increment Δt_{target}.

7. Integrated Surveillance Model Implementation Architecture

A scenario may be defined stochastically in terms of target distributions and behaviours. All platform motions and sensor taskings are predetermined in accordance with a plan designed for the scenario under consideration. The Integrated Surveillance Model (ISM) comprises a module written in MATLAB which is linked to AGI's Satellite Toolkit (STK). Initially, MATLAB is used to compute the target to physical domain transformation and define the points in physical space representing the locations of targets at each time step. STK is then used to determine when and for how long sensors have access to those target locations. This data is returned to MATLAB which is used to compute probabilities of detections of targets by sensors and update the probabilities of the discrete states comprising the surveillance process. Each sensor model computes a probability of detection as a function of target attributes, environmental factors and spatial relationship, normalised for the time increment Δt_{target} which may differ from the scan or dwell time for the sensor concerned, as previously discussed.

The steps in the procedure executed by the ISM are as follows:

1. The target flow-field and target density distribution is used to define a time increment and a corresponding spatial grid of hypothetical target locations such that at each time step targets translate from one grid point to the next in the MATLAB ISM module
2. The predetermined motions of the sensor platforms are specified as a series of waypoints and speeds between waypoints in the MATLAB ISM module. The sensor's maximum ranges are also entered.
3. The spatial grid and sensor motions and ranges are communicated to Satellite Toolkit which executes the scenario and determines the accesses of the sensors to the grid-points. These are the start and end times at which each sensor has line of sight to each grid point for its maximum range. Note that STK does not time step through the scenario.
4. This information is returned to the MATLAB ISM module which then determines, for each time step, at which grid point a hypothetical target would be located.
5. At each time step and for each hypothetical target, the probability of detection is computed for each sensor it is within range of, as determined by STK, using appropriate probabilistic sensor performance models within MATLAB. Some of these models are written in other languages such as C or FORTRAN but can be made accessible to MATLAB.
6. At each time step and for each hypothetical target the state space probabilities of the Markov chain describing the surveillance process are updated based upon the detection probabilities which govern the state transitions.
7. After integrating forward in time to the end of the scenario the overall Measures of Effectiveness (MoEs) are computed as functions of the probabilities of the target states and as functions of space and time.

STK was used as the engine for computing the sensor to target accesses as it happens to be convenient to use for this purpose, is accessible from MATLAB and offers the benefits of easily being able to incorporate satellites if required. It may be determined that for greater flexibility this should be undertaken within MATLAB but this would require further code development.

The implementation of the Integrated Surveillance Model has two key stages:

1. Sensor-projected Capability Definition: definition of the surveillance scenario/operation and generate statistics on the capability that is projected onto the area of interest.
2. Target-field Analysis: analysis of the interaction between the effective sensor coverage and the target-field.

In the first stage, the surveillance operation is defined in STK. Custom-written applications or Commercial Off-The-Shelf (COTS) products like MATLAB can be used to connect to the STK Application Programming Interface (API) to build up the scenario with information such as simulation details, coverage definitions and surveillance assets (sensors, movement profiles, etc).

STK provides two main sets of outputs: a Coverage Access report and Asset Ephemeris data files. Each coverage definition contains a set of grid points for which STK can generate an access report. The access report contains information on the periods when the grid point is within any sensor's coverage envelope. STK generates an ephemeris data file for every asset in the scenario. The information about locations of assets in time is used to compute actual probabilities of detection based on the range values.

The second stage is the analysis of the coverage of the target-field. Here, MATLAB is used to provide an interactive environment for users to graphically define a target field to analyse. The access reports and the asset ephemeris data are then retrieved into MATLAB and a data structure containing the intermediate sensor-projected capabilities is generated. This data structure forms the basis of the target field analysis. Combining this with sensor-target-environment information, the probability of detection is computed for each sensor scan of during the access periods computed by STK. The overall process is shown schematically in Figure 2.

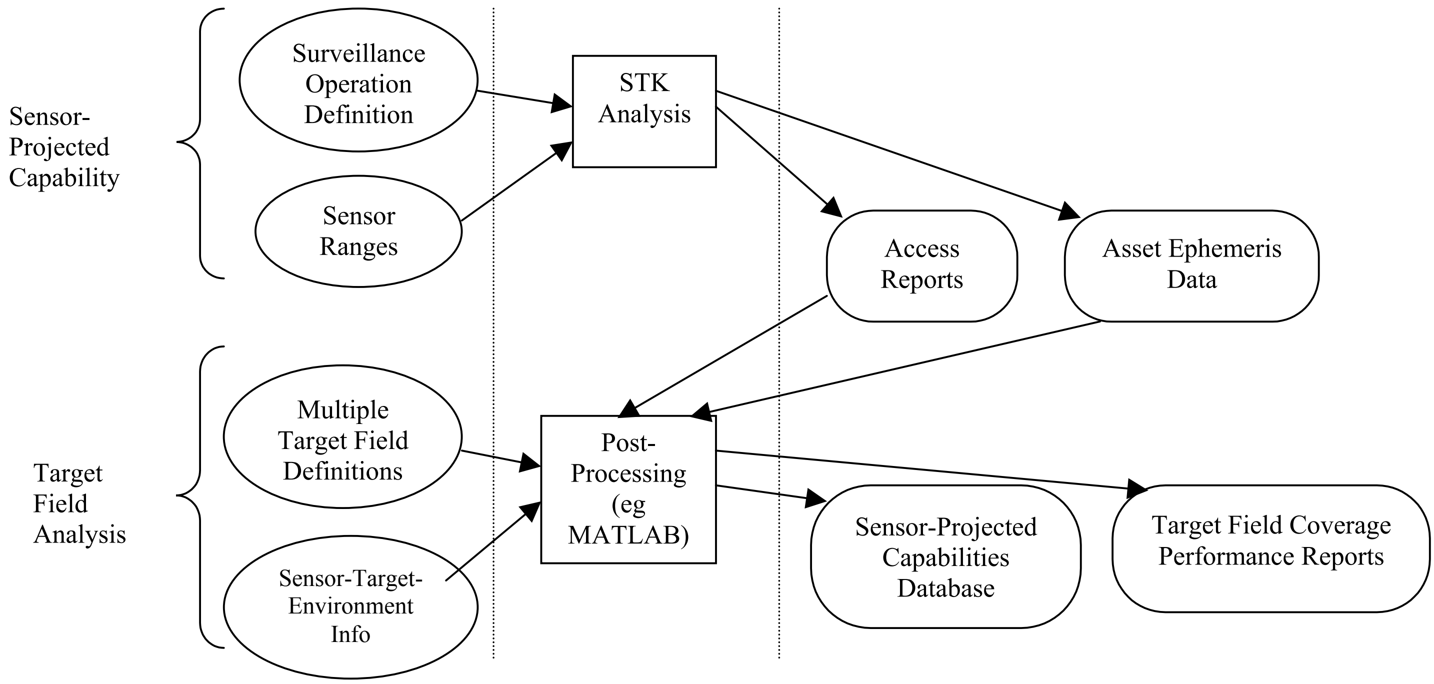


Figure 2: Integrated Surveillance Model architecture

The Integrated Surveillance Model has been used as part of the Surveillance Capability Assessment Study for the JNT 99/005 Task sponsor, DGISREW, with the purpose of gauging the potential contributions of proposed sensor systems to the effectiveness of the overall surveillance system (Berry, Fok & Hall [4]; Hall & Berry [5]).

8. Conclusions

In this paper a model has been specified and practically implemented which assesses the effectiveness of a surveillance operation involving

- multiple sensors and platforms taking into account patterns of target motion,
- non-deterministic spatial target distributions,
- non-deterministic sensor measurement capabilities, and
- the process leading from the sensor measurements to the satisfaction of the surveillance information requirement.

This enables surveillance integration issues to be addressed through quantitative analysis.

The Integrated Surveillance model can, in principle, be embedded within an optimisation framework which will enable the question of how to enhance effectiveness through the exploitation of integration subject to constraints to be investigated systematically. This is a subject for future research.

9. References

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Appendix A: Derivation of the target density evolution equations

Two derivations are given: the first is semi-intuitive while the second employs probabilistic reasoning.

A.1. 1st derivation

It is well known that discrete events in time which are generated by a Poisson process with negative exponential inter-arrival distributions can be thought of in terms of event occurrence rates in continuous time. Consider then an arbitrary closed volume in space V with surface ∂V through which targets pass according to the velocity vector field $\underline{v}(\underline{r},t)$. Then the average number of targets in *V* at time *t* is given by

$$\overline{n}_V(t) = \iiint_V \rho(\underline{r},t)dV$$

After an infinitesimal increment of time Δt the new average is given by

$$\overline{n}_V(t+\Delta t) = \overline{n}_V(t) - \Delta t \iint_{\partial V} \rho(\underline{r},t)\underline{v}(\underline{r},t)\cdot d\underline{S}$$

taking into account the rate of flow of targets out of *V*. Note that this holds true only for the assumption of targets spatially distributed according to a Poisson distribution and therefore arriving at the fixed boundary ∂V of *V* according to a Poisson arrival process with velocity, and hence arrival rate, determined by $\underline{v}$. It also holds true, incidentally, for a locally uniform, and hence deterministic, spatial target distribution treated as a continuous media.

Rearranging and taking the limit $\Delta t \to 0$, we obtain

$$\frac{d\overline{n}_V}{dt} = \frac{d}{dt}\iiint_V \rho(\underline{r},t)dV = -\iint_{\partial V} \rho(\underline{r},t)\underline{v}(\underline{r},t)\cdot d\underline{S}$$

which, taking the time derivative through the integral over the fixed volume and applying the Divergence Theorem (otherwise known as Gauss' theorem), may be written as

$$\iiint_V \left\{\frac{\partial \rho}{\partial t} + \underline{\nabla}\cdot(\rho\underline{v})\right\}dV = 0$$

for all arbitrary volumes of space *V*. Hence

$$\frac{\partial \rho}{\partial t} + \underline{\nabla}\cdot(\rho\underline{v}) = 0$$

everywhere. Expanding the del operation gives

$$\frac{\partial \rho}{\partial t} + (\underline{v}\cdot\underline{\nabla})\rho + \rho\underline{\nabla}\cdot\underline{v} = 0$$

which is equation (3).

A.2. 2nd derivation

Consider an arbitrary closed volume in space ΔV with surface $\partial\Delta V$ through which targets pass according to the velocity field $\underline{v}(\underline{r},t)$. If ΔV is infinitesimally small then the probability of it containing a target at time *t* is given by

$$P(t) = \Pr\{\exists \text{ target in } \Delta V \text{ at } t\} = \iiint_{\Delta V} \rho(\underline{r},t)dV$$

In an infinitesimal interval of time Δt the probability that a target leaves ΔV is

$$p^-(t) = \Pr\{\text{target leaves } \Delta \text{V}\} = \Delta t \iint_{\partial\Delta \text{V}^+} \rho(\underline{r},t)\underline{v}(\underline{r},t)\cdot d\underline{S}$$

where $\partial\Delta V^+$ is that part of $\partial\Delta V$ on which $\underline{v}\cdot d\underline{S} > 0$. Similarly

$$p^+(t) = \Pr\{\text{target enters } \Delta \text{V}\} = -\Delta t \iint_{\partial\Delta \text{V}^-} \rho(\underline{r},t)\underline{v}(\underline{r},t)\cdot d\underline{S}$$

where $\underline{v}\cdot d\underline{S} \le 0$ on $\partial\Delta V^-$. Hence $(\partial\Delta V^-)\cup(\partial\Delta V^+) = \partial\Delta V$.

At the end of an interval of time Δt we can say that the probability of a target existing in ΔV is related to the possibility of a target not existing in ΔV at the beginning of the interval and a target arriving, and the possibility of a target already existing and not departing:

$$P(t+\Delta t) = (1-P(t))p^+ + P(t)(1-\frac{p^-}{P(t)})$$

using Bayes' rule, or

$$P(t+\Delta t) = P(t) + p^+ - p^- - P(t)p^+$$

which, upon substitution, gives

$$\iiint_V \rho(\underline{r},t+\Delta t)dV = \iiint_V \rho(\underline{r},t)dV - \Delta t\iint_{\partial V} \rho(\underline{r},t)\underline{v}(\underline{r},t)\cdot d\underline{S} - \Delta t \iint_{\partial\Delta V-} \rho\underline{v}\cdot d\underline{S}\iiint_{\Delta V}\rho dV$$

Dividing by Δt, taking the limit of $\Delta t \to 0$ and applying Gauss' theorem as before we obtain

$$\iiint_{\Delta V}\left\{\frac{\partial\rho}{\partial t} + \underline{\nabla}\cdot(\rho\underline{v})\right\}dV = O(\Delta V^2)$$

which, on applying the Mean Value theorem and taking the limit of $\|\Delta V\| \to 0$, yields, as before,

$$\frac{\partial\rho}{\partial t} + \underline{\nabla}\cdot(\rho\underline{v}) = 0$$

Appendix B: Derivation of the target state density evolution equations

Define the probability of there existing a target in state *i* within an incremental volume ΔV about the location $\underline{r}$ at time *t* as

$$\rho_i(\underline{r},t)\Delta V$$

Let $\underline{\rho}(\underline{r},t)$ be the vector of densities for the N states comprising the state space of the surveillance process:

$$\underline{\rho} = \begin{bmatrix} \rho_1 \\ \vdots \\ \rho_N \end{bmatrix}$$

Following derivation 1 of Appendix A we can add a term representing the change in states of targets contained in a volume V during a time step Δt and take the limit $\Delta t \to 0$ to obtain

$$\frac{d}{dt}\iiint_V \underline{\rho}(\underline{r},t)dV = -\iint_{\partial V} \underline{\rho}(\underline{r},t)\underline{v}(\underline{r},t)\cdot d\underline{S} + \iiint_V \underline{\underline{A}}(\underline{\lambda}(\underline{r},t))\underline{\rho}(\underline{r},t)dV$$

and proceed as before to obtain

$$\frac{D\underline{\rho}}{Dt} + \underline{\rho}(\underline{\nabla}\cdot\underline{v}) = \underline{\underline{A}}(\underline{\lambda})\underline{\rho}$$

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Paul E. Berry

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19. ABSTRACT

There exists a requirement to be able to assess the effectiveness of surveillance operations involving multiple assets for the purposes of appraising proposals for capability development through acquisition and upgrades, as well as operationally for evaluating and comparing proposed plans for execution. The provision of a model which supports this assessment capability while accounting for the capabilities of the sensors, the dynamics of the operation, the management of the sensor-derived information in endeavouring to satisfy the surveillance requirement and the various sources of uncertainty is the subject of this report.

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